Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.8-P-x-c-x-^m-a+b-x^2-^p

Nasser M. Abbasi

May 13, 2020 Compiled on May 13, 2020 at 10:41am

Contents

1 Introduction **9**

1.1 Listing of CAS systems tested . . . 9
1.2 Results . . . 10
1.3 Performance . . . 13
1.4 list of integrals that has no closed form antiderivative . . . 14
1.5 list of integrals solved by CAS but has no known antiderivative . . . 14
1.6 list of integrals solved by CAS but failed verification . . . 14
1.7 Timing . . . 15
1.8 Verification . . . 15
1.9 Important notes about some of the results . . . 15
1.10 Design of the test system . . . 17

2 detailed summary tables of results **19**

2.1 List of integrals sorted by grade for each CAS . . . 19
2.2 Detailed conclusion table per each integral for all CAS systems . . . 21
2.3 Detailed conclusion table specific for Rubi results . . . 57

3 Listing of integrals **63**

3.1 $\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$ . . . 63
3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$ . . . 68
3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$ . . . 73

3.4 $\int (A+Bx)\sqrt{a+bx^2}\,dx$ 77
3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$ 81
3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$ 86
3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx$ 91
3.8 $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$ 96
3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$ 101
3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$ 106
3.11 $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$ 110
3.12 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx$ 114
3.13 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$ 119
3.14 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$ 124
3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$ 129
3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$ 134
3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$ 139
3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$ 144
3.19 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx$ 148
3.20 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx$ 154
3.21 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx$ 160
3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$ 166
3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx$ 170
3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx$ 174
3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$ 178
3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$ 182
3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$ 186
3.28 $\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx$ 190
3.29 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$ 195
3.30 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$ 199
3.31 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$ 203

3.32 $\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx$ 207
3.33 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx$ 210
3.34 $\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx$ 215
3.35 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx$ 220
3.36 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$ 225
3.37 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$ 229
3.38 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$ 233
3.39 $\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx$ 237
3.40 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx$ 241
3.41 $\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx$ 246
3.42 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx$ 252
3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx$ 258
3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx$ 261
3.45 $\int \frac{3+x^2}{-3+x^2}\,dx$ 264
3.46 $\int \frac{-1+x^2}{1+x^2}\,dx$ 267
3.47 $\int \frac{x^7\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$ 270
3.48 $\int \frac{x^6\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$ 275
3.49 $\int \frac{x^5\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$ 280
3.50 $\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$ 284
3.51 $\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$ 288
3.52 $\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$ 293
3.53 $\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$ 298
3.54 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}}\,dx$ 303

3.55 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$ 308

3.56 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx$ 313

3.57 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx$ 319

3.58 $\int \frac{A(cx)^m}{a+bx^2}\,dx$ 325

3.59 $\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx$ 328

3.60 $\int \frac{(cx)^m\left(A+Cx^2\right)}{a+bx^2}\,dx$ 332

3.61 $\int \frac{(cx)^m\left(A+Bx+Cx^2\right)}{a+bx^2}\,dx$ 336

3.62 $\int x^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 340

3.63 $\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 343

3.64 $\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 346

3.65 $\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 349

3.66 $\int \frac{\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$ 352

3.67 $\int \frac{\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx$ 355

3.68 $\int \frac{\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)}{x^3}\,dx$ 358

3.69 $\int \frac{\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)}{x^4}\,dx$ 361

3.70 $\int x^3\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 364

3.71 $\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 367

3.72 $\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 370

3.73 $\int \left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 374

3.74 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$ 378

3.75 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx$ 382

3.76 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^3}\,dx$ 386

3.77 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^4}\,dx$ 389

3.78 $\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 392

3.79 $\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 396

3.80 $\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 400

3.81 $\int \left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 404

3.82 $\int \frac{\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$ 408

3.83 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$ 412
3.84 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$ 416
3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$ 419
3.86 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$ 422
3.87 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$ 426
3.88 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$ 430
3.89 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$ 434
3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$ 438
3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$ 442
3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$ 446
3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx$ 450
3.94 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$ 455
3.95 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$ 460
3.96 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$ 465
3.97 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$ 470
3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx$ 474
3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2}\,dx$ 478
3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx$ 483
3.101 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^2}\,dx$ 488
3.102 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$ 493
3.103 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$ 498
3.104 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$ 503
3.105 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$ 507

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$ 511
3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx$ 515
3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx$ 520
3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx$ 526
3.110 $\int \frac{-x+4x^3}{(5+x^2)^2}\,dx$ 532
3.111 $\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx$ 536
3.112 $\int \frac{-x^2+2x^4}{1+2x^2}\,dx$ 539
3.113 $\int \frac{x^3+x^4}{1+x^2}\,dx$ 543
3.114 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$ 547
3.115 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$ 551
3.116 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$ 555
3.117 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx$ 559
3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx$ 563
3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$ 567
3.120 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx$ 571
3.121 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx$ 575
3.122 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$ 579
3.123 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx$ 583
3.124 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$ 587
3.125 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$ 593
3.126 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$ 598
3.127 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx$ 603
3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2}\,dx$ 607
3.129 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx$ 611

3.130 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$ . . . 615

3.131 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$ . . . 620

3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$ . . . 625

3.133 $\int \frac{x^8\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$ . . . 630

3.134 $\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$ . . . 636

3.135 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$ . . . 642

3.136 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$ . . . 648

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3}\,dx$ . . . 654

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$ . . . 659

3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx$ . . . 664

3.140 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$ . . . 669

3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$ . . . 674

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$ . . . 679

3.143 $\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$ . . . 684

3.144 $\int \frac{x^3\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$ . . . 688

3.145 $\int \frac{x\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$ . . . 692

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$ . . . 696

3.147 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$ . . . 700

3.148 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$ . . . 705

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$ . . . 710

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$ . . . 716

3.151 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$ . . . 722

3.152 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$ . . . 728

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$ 733
3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$ 738
3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$ 743
3.156 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$ 748
3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$ 753
3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$ 758
3.159 $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 763
3.160 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 770
3.161 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 777
3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 784
3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx$ 790
3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{9/2}}\,dx$ 794
3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx$ 799
3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx$ 804
3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$ 809
3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$ 815
3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$ 821
3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$ 825
3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$ 829
3.172 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$ 833
3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$ 840
3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$ 845

4 Listing of Grading functions **851**

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [174]. This is test number [24].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (174)	% 0. (0)
Mathematica	% 100. (174)	% 0. (0)
Maple	% 97.7 (170)	% 2.3 (4)
Maxima	% 24.71 (43)	% 75.29 (131)
Fricas	% 69.54 (121)	% 30.46 (53)
Sympy	% 87.36 (152)	% 12.64 (22)
Giac	% 97.7 (170)	% 2.3 (4)

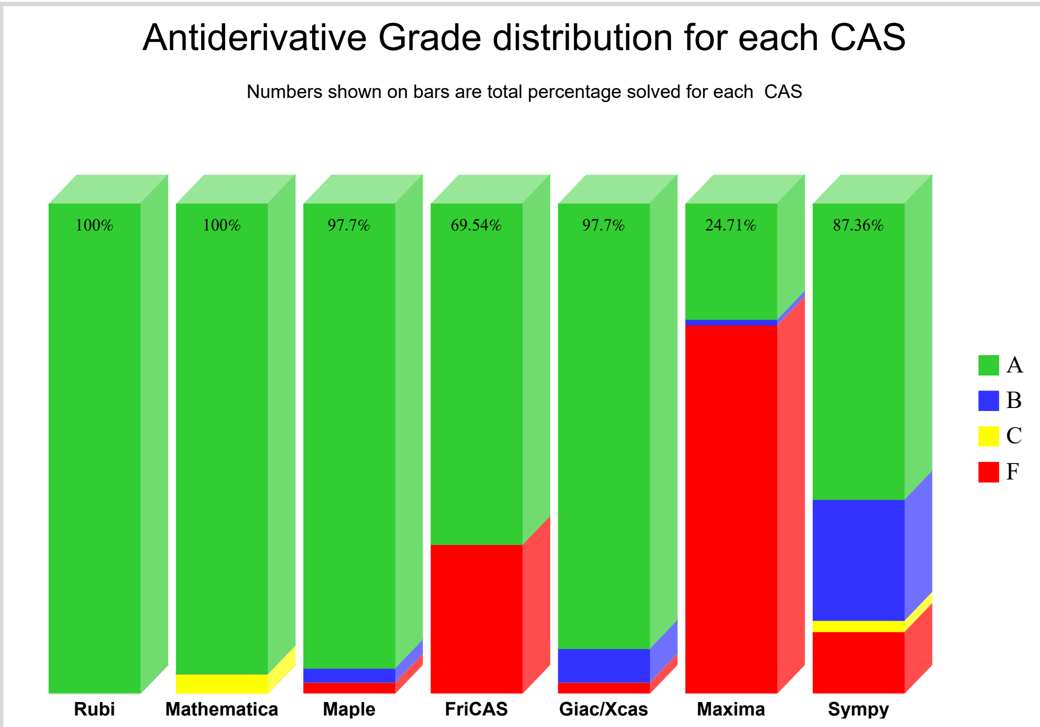
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

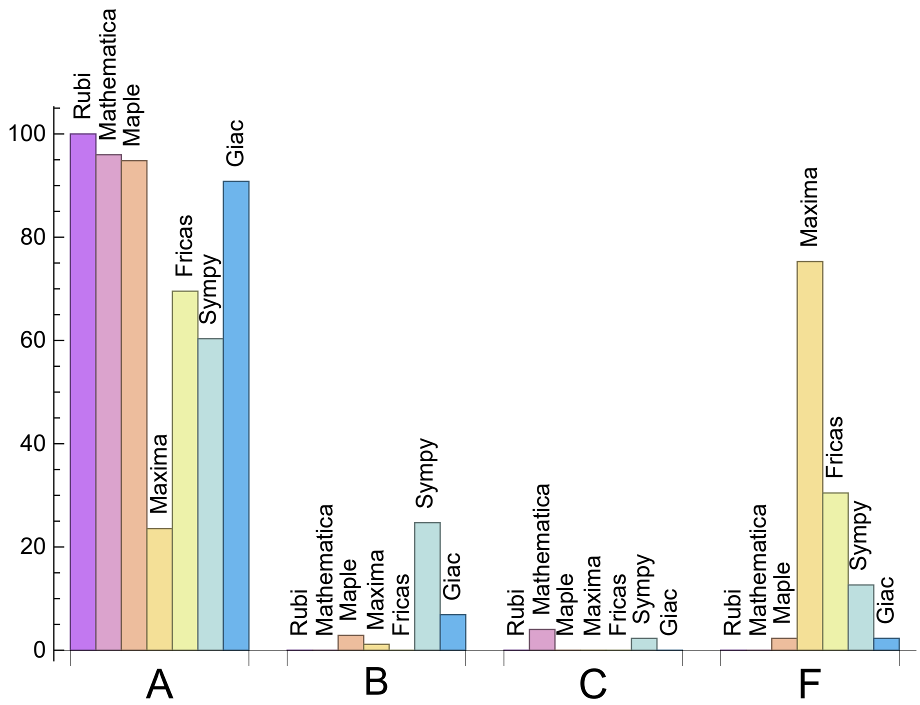
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	95.98	0.	4.02	0.
Maple	94.83	2.87	0.	2.3
Maxima	23.56	1.15	0.	75.29
Fricas	69.54	0.	0.	30.46
Sympy	60.34	24.71	2.3	12.64
Giac	90.8	6.9	0.	2.3

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	129.82	1.	121.	1.
Mathematica	0.12	113.94	0.91	105.5	0.9
Maple	0.01	154.84	1.13	133.5	1.1
Maxima	1.12	131.16	1.45	130.	1.34
Fricas	1.56	606.12	4.84	500.	4.58
Sympy	15.91	319.35	2.73	214.	1.79
Giac	1.21	205.72	1.49	169.5	1.34

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

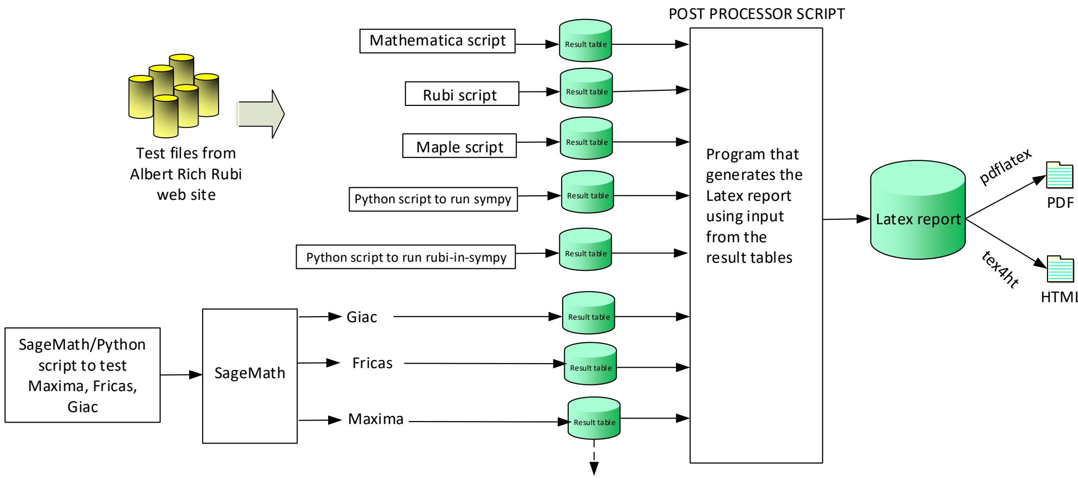
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { }

C grade: { 13, 14, 20, 21, 148, 149, 150 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174 }

B grade: { 48, 161, 162, 172, 173 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.4 Maxima

A grade: { 32, 37, 38, 39, 43, 44, 45, 46, 50, 51, 52, 53, 54, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 113

B grade: { 49, 163 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 40, 41, 42, 47, 48, 55, 56, 57, 58, 59, 60, 61, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 70, 71, 72, 73, 78, 79, 80, 81, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 169, 170, 171 }

B grade: { }

C grade: { }

F grade: { 58, 59, 60, 61, 66, 67, 68, 69, 74, 75, 76, 77, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 35, 36, 38, 43, 44, 45, 46, 51, 53, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 147, 148, 153, 154, 155, 156, 169, 170, 171 }

B grade: { 25, 33, 34, 37, 39, 40, 41, 42, 52, 54, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 118, 119, 121, 122, 131, 149, 150, 151, 152, 157, 158 }

C grade: { 58, 59, 60, 61 }

F grade: { 47, 48, 49, 50, 55, 56, 57, 141, 142, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174 }

B grade: { 7, 14, 28, 35, 45, 150, 156, 157, 158, 166, 167, 168 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	107	115	0	501	192	126
normalized size	1	1.	0.84	0.91	0.	3.94	1.51	0.99
time (sec)	N/A	0.082	0.187	0.008	0.	1.578	6.529	1.19

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	94	0	435	165	109
normalized size	1	1.	0.89	0.9	0.	4.18	1.59	1.05
time (sec)	N/A	0.049	0.174	0.006	0.	1.555	4.431	1.182

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	75	0	381	124	92
normalized size	1	1.	1.08	0.94	0.	4.76	1.55	1.15
time (sec)	N/A	0.026	0.141	0.005	0.	1.586	4.38	1.171

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	53	0	316	70	74
normalized size	1	1.	1.	0.79	0.	4.72	1.04	1.1
time (sec)	N/A	0.019	0.05	0.004	0.	1.546	2.726	1.179

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	100	78	0	859	107	105
normalized size	1	1.	1.27	0.99	0.	10.87	1.35	1.33
time (sec)	N/A	0.061	0.208	0.004	0.	1.661	5.044	1.19

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	99	97	0	821	124	138
normalized size	1	1.	1.32	1.29	0.	10.95	1.65	1.84
time (sec)	N/A	0.059	0.159	0.006	0.	1.621	3.552	1.194

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	108	121	0	934	107	220
normalized size	1	1.	1.35	1.51	0.	11.68	1.34	2.75
time (sec)	N/A	0.06	0.091	0.008	0.	1.62	3.755	1.224

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	126	134	0	630	318	155
normalized size	1	1.	0.84	0.89	0.	4.2	2.12	1.03
time (sec)	N/A	0.095	0.228	0.008	0.	1.603	15.786	1.221

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	113	113	0	559	287	139
normalized size	1	1.	0.89	0.89	0.	4.4	2.26	1.09
time (sec)	N/A	0.062	0.215	0.007	0.	1.648	10.718	1.255

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	107	94	0	500	223	120
normalized size	1	1.	1.04	0.91	0.	4.85	2.17	1.17
time (sec)	N/A	0.033	0.184	0.005	0.	1.613	10.032	1.171

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	69	0	425	219	103
normalized size	1	1.	1.01	0.79	0.	4.89	2.52	1.18
time (sec)	N/A	0.026	0.07	0.004	0.	1.63	6.309	1.181

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	118	107	0	1094	218	135
normalized size	1	1.	1.11	1.01	0.	10.32	2.06	1.27
time (sec)	N/A	0.094	0.247	0.006	0.	1.743	14.508	1.19

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	105	126	0	1033	184	167
normalized size	1	1.	0.97	1.17	0.	9.56	1.7	1.55
time (sec)	N/A	0.089	0.167	0.007	0.	1.672	5.773	1.217

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	90	150	0	1048	182	258
normalized size	1	1.	0.81	1.35	0.	9.44	1.64	2.32
time (sec)	N/A	0.085	0.036	0.007	0.	1.715	6.862	1.241

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	145	153	0	775	469	189
normalized size	1	1.	0.84	0.88	0.	4.48	2.71	1.09
time (sec)	N/A	0.105	0.276	0.008	0.	1.7	30.433	1.237

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	131	132	0	687	442	173
normalized size	1	1.	0.87	0.88	0.	4.58	2.95	1.15
time (sec)	N/A	0.069	0.259	0.006	0.	1.62	21.051	1.23

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	112	113	0	635	354	154
normalized size	1	1.	0.89	0.9	0.	5.04	2.81	1.22
time (sec)	N/A	0.044	0.516	0.006	0.	1.66	19.883	1.228

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	108	85	0	549	348	136
normalized size	1	1.	1.01	0.79	0.	5.13	3.25	1.27
time (sec)	N/A	0.037	0.081	0.005	0.	1.59	12.57	1.214

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	139	138	0	1353	323	169
normalized size	1	1.	1.05	1.05	0.	10.25	2.45	1.28
time (sec)	N/A	0.159	0.329	0.005	0.	1.745	24.432	1.186

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	117	158	0	1319	318	203
normalized size	1	1.	0.86	1.16	0.	9.7	2.34	1.49
time (sec)	N/A	0.132	0.222	0.009	0.	1.704	10.135	1.215

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	92	181	0	1316	279	296
normalized size	1	1.	0.65	1.28	0.	9.33	1.98	2.1
time (sec)	N/A	0.117	0.026	0.007	0.	1.702	11.318	1.186

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	76	96	0	385	150	100
normalized size	1	1.	0.73	0.92	0.	3.7	1.44	0.96
time (sec)	N/A	0.077	0.044	0.007	0.	1.588	5.284	1.231

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	75	0	320	94	82
normalized size	1	1.	0.79	0.93	0.	3.95	1.16	1.01
time (sec)	N/A	0.042	0.036	0.005	0.	1.576	3.528	1.212

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	55	0	275	70	68
normalized size	1	1.	1.02	0.98	0.	4.91	1.25	1.21
time (sec)	N/A	0.023	0.032	0.004	0.	1.793	3.293	1.183

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	0	223	102	53
normalized size	1	1.	1.07	0.86	0.	5.19	2.37	1.23
time (sec)	N/A	0.015	0.029	0.004	0.	1.776	1.016	1.181

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	0	683	99	78
normalized size	1	1.	1.	0.98	0.	12.89	1.87	1.47
time (sec)	N/A	0.04	0.014	0.006	0.	1.585	2.407	1.217

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	49	0	240	41	88
normalized size	1	1.	1.	1.04	0.	5.11	0.87	1.87
time (sec)	N/A	0.033	0.015	0.007	0.	1.556	2.149	1.186

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	68	0	306	66	197
normalized size	1	1.	0.88	0.94	0.	4.25	0.92	2.74
time (sec)	N/A	0.053	0.102	0.008	0.	1.564	3.156	1.263

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	72	93	0	448	117	95
normalized size	1	1.	0.89	1.15	0.	5.53	1.44	1.17
time (sec)	N/A	0.043	0.052	0.008	0.	1.6	7.672	1.217

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	72	0	369	83	78
normalized size	1	1.	1.02	1.09	0.	5.59	1.26	1.18
time (sec)	N/A	0.036	0.034	0.006	0.	1.595	5.857	1.229

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	64	54	0	336	66	65
normalized size	1	1.	1.33	1.12	0.	7.	1.38	1.35
time (sec)	N/A	0.02	0.054	0.005	0.	1.581	4.694	1.196

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	26	42	69	46	31
normalized size	1	1.	0.96	0.93	1.5	2.46	1.64	1.11
time (sec)	N/A	0.007	0.014	0.001	0.992	1.605	3.635	1.204

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	60	0	338	206	80
normalized size	1	1.	1.	1.28	0.	7.19	4.38	1.7
time (sec)	N/A	0.038	0.027	0.007	0.	1.85	5.909	1.189

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	72	80	0	381	235	130
normalized size	1	1.	1.03	1.14	0.	5.44	3.36	1.86
time (sec)	N/A	0.057	0.038	0.008	0.	1.885	8.088	1.223

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	101	0	474	124	231
normalized size	1	1.	0.79	1.06	0.	4.99	1.31	2.43
time (sec)	N/A	0.078	0.16	0.008	0.	1.878	8.607	1.255

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	69	91	0	536	400	95
normalized size	1	1.	0.87	1.15	0.	6.78	5.06	1.2
time (sec)	N/A	0.042	0.076	0.008	0.	1.74	12.579	1.416

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	41	95	128	141	49
normalized size	1	1.	0.83	0.77	1.79	2.42	2.66	0.92
time (sec)	N/A	0.022	0.017	0.004	1.013	1.549	10.608	1.183

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	32	29	69	99	95	35
normalized size	1	1.	0.68	0.62	1.47	2.11	2.02	0.74
time (sec)	N/A	0.014	0.017	0.004	0.992	1.548	9.037	1.168

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	65	126	146	50
normalized size	1	1.	0.84	0.78	1.27	2.47	2.86	0.98
time (sec)	N/A	0.01	0.02	0.003	1.009	1.589	8.949	1.223

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	92	0	537	840	111
normalized size	1	1.	0.91	1.21	0.	7.07	11.05	1.46
time (sec)	N/A	0.063	0.055	0.007	0.	1.692	22.992	1.196

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	95	112	0	589	910	161
normalized size	1	1.	0.91	1.08	0.	5.66	8.75	1.55
time (sec)	N/A	0.088	0.055	0.008	0.	1.688	20.084	1.203

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	106	134	0	683	1034	266
normalized size	1	1.	0.82	1.04	0.	5.29	8.02	2.06
time (sec)	N/A	0.119	0.149	0.008	0.	1.69	35.277	1.245

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	29	38	82	24	26
normalized size	1	1.	0.89	1.07	1.41	3.04	0.89	0.96
time (sec)	N/A	0.008	0.015	0.007	1.851	1.448	0.206	1.181

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	29	38	82	24	26
normalized size	1	1.	0.89	1.07	1.41	3.04	0.89	0.96
time (sec)	N/A	0.017	0.007	0.003	1.529	1.525	0.214	1.234

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	15	30	70	27	41
normalized size	1	1.	1.94	0.88	1.76	4.12	1.59	2.41
time (sec)	N/A	0.006	0.007	0.002	1.45	1.418	0.082	1.13

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	23	5	8
normalized size	1	1.	1.	1.17	1.33	3.83	0.83	1.33
time (sec)	N/A	0.003	0.004	0.001	1.603	1.447	0.08	1.153

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	165	265	0	1160	0	275
normalized size	1	1.	0.77	1.24	0.	5.45	0.	1.29
time (sec)	N/A	0.324	0.433	0.043	0.	1.813	0.	1.217

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	147	277	0	1044	0	186
normalized size	1	1.	0.98	1.85	0.	6.96	0.	1.24
time (sec)	N/A	0.165	0.268	0.012	0.	1.745	0.	1.225

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	89	95	324	290	0	151
normalized size	1	1.	0.67	0.72	2.45	2.2	0.	1.14
time (sec)	N/A	0.166	0.079	0.005	1.081	1.604	0.	1.2

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	78	76	342	254	0	109
normalized size	1	1.	0.52	0.51	2.3	1.7	0.	0.73
time (sec)	N/A	0.181	0.082	0.006	1.022	1.632	0.	1.19

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	84	85	242	271	660	128
normalized size	1	1.	0.6	0.61	1.74	1.95	4.75	0.92
time (sec)	N/A	0.151	0.09	0.005	0.988	1.66	142.489	1.218

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	87	88	266	277	904	127
normalized size	1	1.	0.63	0.63	1.91	1.99	6.5	0.91
time (sec)	N/A	0.131	0.095	0.006	1.034	1.672	110.567	1.185

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	75	73	166	250	796	111
normalized size	1	1.	0.63	0.61	1.39	2.1	6.69	0.93
time (sec)	N/A	0.088	0.058	0.005	1.174	1.699	71.287	1.228

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	92	96	207	289	1880	151
normalized size	1	1.	0.72	0.76	1.63	2.28	14.8	1.19
time (sec)	N/A	0.069	0.063	0.003	1.025	1.627	111.682	1.196

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	120	169	0	1040	0	205
normalized size	1	1.	0.87	1.22	0.	7.54	0.	1.49
time (sec)	N/A	0.162	0.156	0.009	0.	1.856	0.	1.221

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	158	240	0	1172	0	323
normalized size	1	1.	0.84	1.28	0.	6.23	0.	1.72
time (sec)	N/A	0.381	0.156	0.009	0.	1.883	0.	1.174

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	178	288	0	1527	0	439
normalized size	1	1.	0.81	1.32	0.	6.97	0.	2.
time (sec)	N/A	0.48	0.377	0.011	0.	2.123	0.	1.209

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	0	97	0
normalized size	1	1.	0.96	0.	0.	0.	2.16	0.
time (sec)	N/A	0.017	0.009	0.03	0.	0.	1.515	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	192	0
normalized size	1	1.	0.9	0.	0.	0.	2.11	0.
time (sec)	N/A	0.043	0.029	0.031	0.	0.	4.736	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	56	0	0	0	204	0
normalized size	1	1.	0.74	0.	0.	0.	2.68	0.
time (sec)	N/A	0.039	0.054	0.036	0.	0.	5.379	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	99	0	0	0	298	0
normalized size	1	1.	0.82	0.	0.	0.	2.46	0.
time (sec)	N/A	0.122	0.075	0.032	0.	0.	6.414	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	150	60	77
normalized size	1	1.	1.	0.83	1.11	2.31	0.92	1.18
time (sec)	N/A	0.074	0.015	0.002	1.063	1.235	0.065	1.561

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	150	60	77
normalized size	1	1.	1.	0.83	1.11	2.31	0.92	1.18
time (sec)	N/A	0.076	0.016	0.002	1.053	1.23	0.065	1.53

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	150	60	77
normalized size	1	1.	1.	0.83	1.11	2.31	0.92	1.18
time (sec)	N/A	0.061	0.009	0.	1.224	1.222	0.064	1.533

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	142	56	73
normalized size	1	1.	1.	0.85	1.13	2.37	0.93	1.22
time (sec)	N/A	0.041	0.009	0.001	1.039	1.289	0.063	1.152

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	65	0	54	72
normalized size	1	1.	1.	0.95	1.16	0.	0.96	1.29
time (sec)	N/A	0.04	0.015	0.004	1.03	0.	0.285	1.165

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	50	65	0	49	68
normalized size	1	1.	1.	0.93	1.2	0.	0.91	1.26
time (sec)	N/A	0.048	0.033	0.007	1.218	0.	0.302	1.18

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	48	65	0	49	65
normalized size	1	1.	0.94	0.89	1.2	0.	0.91	1.2
time (sec)	N/A	0.048	0.026	0.005	1.062	0.	0.416	1.152

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	55	51	66	0	53	68
normalized size	1	1.	1.02	0.94	1.22	0.	0.98	1.26
time (sec)	N/A	0.049	0.017	0.005	1.007	0.	0.805	1.194

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	98	102	136	263	110	142
normalized size	1	1.	0.9	0.94	1.25	2.41	1.01	1.3
time (sec)	N/A	0.124	0.037	0.001	1.129	1.227	0.077	1.125

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	92	102	136	261	110	142
normalized size	1	1.	0.84	0.94	1.25	2.39	1.01	1.3
time (sec)	N/A	0.112	0.048	0.001	0.971	1.253	0.078	1.131

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	102	136	258	110	142
normalized size	1	1.	0.88	0.98	1.31	2.48	1.06	1.37
time (sec)	N/A	0.074	0.038	0.001	0.99	1.29	0.079	1.165

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	132	250	107	138
normalized size	1	1.	0.89	1.	1.33	2.53	1.08	1.39
time (sec)	N/A	0.072	0.036	0.002	1.058	1.267	0.075	1.157

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	88	100	130	0	104	135
normalized size	1	1.	0.96	1.09	1.41	0.	1.13	1.47
time (sec)	N/A	0.069	0.043	0.003	0.971	0.	0.366	1.163

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	98	130	0	99	132
normalized size	1	1.	0.98	1.09	1.44	0.	1.1	1.47
time (sec)	N/A	0.08	0.05	0.006	0.994	0.	0.379	1.224

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	97	130	0	99	131
normalized size	1	1.	0.89	0.99	1.33	0.	1.01	1.34
time (sec)	N/A	0.086	0.038	0.006	0.988	0.	0.511	1.161

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	83	97	131	0	99	131
normalized size	1	1.	0.85	0.99	1.34	0.	1.01	1.34
time (sec)	N/A	0.086	0.047	0.006	0.984	0.	0.975	1.165

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	150	196	382	163	207
normalized size	1	1.	1.	1.01	1.32	2.56	1.09	1.39
time (sec)	N/A	0.186	0.027	0.003	1.011	1.371	0.091	1.155

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	150	196	377	165	207
normalized size	1	1.	0.84	1.01	1.32	2.53	1.11	1.39
time (sec)	N/A	0.141	0.06	0.002	1.477	1.316	0.086	1.143

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	124	150	196	371	163	207
normalized size	1	1.	0.9	1.09	1.42	2.69	1.18	1.5
time (sec)	N/A	0.095	0.05	0.001	1.001	1.28	0.085	1.229

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	121	147	192	355	158	201
normalized size	1	1.	0.91	1.11	1.44	2.67	1.19	1.51
time (sec)	N/A	0.093	0.047	0.003	1.014	1.357	0.085	1.125

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	121	148	189	0	158	200
normalized size	1	1.	0.94	1.15	1.47	0.	1.22	1.55
time (sec)	N/A	0.09	0.061	0.003	0.977	0.	0.444	1.171

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	123	145	188	0	150	196
normalized size	1	1.	0.99	1.17	1.52	0.	1.21	1.58
time (sec)	N/A	0.109	0.08	0.006	1.007	0.	0.456	1.154

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	144	188	0	150	194
normalized size	1	1.	0.92	1.07	1.39	0.	1.11	1.44
time (sec)	N/A	0.112	0.061	0.007	1.029	0.	0.593	1.149

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	124	146	192	0	153	197
normalized size	1	1.	0.89	1.05	1.38	0.	1.1	1.42
time (sec)	N/A	0.114	0.048	0.008	1.001	0.	1.061	1.168

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	130	176	0	0	308	217
normalized size	1	1.	0.86	1.17	0.	0.	2.04	1.44
time (sec)	N/A	0.145	0.073	0.005	0.	0.	1.115	1.199

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	152	0	0	269	185
normalized size	1	1.	0.88	1.17	0.	0.	2.07	1.42
time (sec)	N/A	0.124	0.09	0.006	0.	0.	1.077	1.187

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	128	0	0	243	151
normalized size	1	1.	0.86	1.15	0.	0.	2.19	1.36
time (sec)	N/A	0.113	0.053	0.004	0.	0.	1.033	1.183

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	81	106	0	0	211	119
normalized size	1	1.	0.88	1.15	0.	0.	2.29	1.29
time (sec)	N/A	0.085	0.064	0.004	0.	0.	0.968	1.188

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	83	0	0	219	89
normalized size	1	1.	0.93	1.14	0.	0.	3.	1.22
time (sec)	N/A	0.065	0.04	0.004	0.	0.	0.96	1.196

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	80	0	0	1268	89
normalized size	1	1.	1.01	1.11	0.	0.	17.61	1.24
time (sec)	N/A	0.098	0.051	0.006	0.	0.	25.449	1.123

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	83	0	0	1258	92
normalized size	1	1.	0.99	1.09	0.	0.	16.55	1.21
time (sec)	N/A	0.098	0.046	0.007	0.	0.	23.343	1.158

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	102	0	0	1686	108
normalized size	1	1.	0.91	1.11	0.	0.	18.33	1.17
time (sec)	N/A	0.109	0.08	0.009	0.	0.	24.608	1.164

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	139	201	0	0	333	215
normalized size	1	1.	0.79	1.14	0.	0.	1.89	1.22
time (sec)	N/A	0.268	0.126	0.01	0.	0.	3.681	1.18

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	128	177	0	0	287	177
normalized size	1	1.	0.83	1.15	0.	0.	1.86	1.15
time (sec)	N/A	0.241	0.077	0.008	0.	0.	3.464	1.204

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	100	154	0	0	284	150
normalized size	1	1.	0.75	1.15	0.	0.	2.12	1.12
time (sec)	N/A	0.226	0.075	0.008	0.	0.	3.332	1.194

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	92	127	0	0	212	109
normalized size	1	1.	0.91	1.26	0.	0.	2.1	1.08
time (sec)	N/A	0.118	0.049	0.008	0.	0.	2.656	1.146

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	83	97	0	0	233	119
normalized size	1	1.	0.89	1.04	0.	0.	2.51	1.28
time (sec)	N/A	0.065	0.081	0.006	0.	0.	2.154	1.174

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	125	0	0	797	126
normalized size	1	1.	0.89	1.32	0.	0.	8.39	1.33
time (sec)	N/A	0.122	0.071	0.01	0.	0.	8.108	1.205

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	136	0	0	782	139
normalized size	1	1.	1.	1.24	0.	0.	7.11	1.26
time (sec)	N/A	0.142	0.07	0.012	0.	0.	8.452	1.186

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	112	169	0	0	1807	170
normalized size	1	1.	0.83	1.25	0.	0.	13.39	1.26
time (sec)	N/A	0.204	0.102	0.015	0.	0.	39.576	1.168

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	139	235	0	0	357	212
normalized size	1	1.	0.75	1.27	0.	0.	1.93	1.15
time (sec)	N/A	0.338	0.116	0.01	0.	0.	23.307	1.197

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	126	206	0	0	282	165
normalized size	1	1.	0.81	1.33	0.	0.	1.82	1.06
time (sec)	N/A	0.232	0.076	0.01	0.	0.	21.829	1.139

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	122	133	0	0	303	173
normalized size	1	1.	0.9	0.98	0.	0.	2.23	1.27
time (sec)	N/A	0.158	0.093	0.009	0.	0.	18.111	1.191

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	110	0	0	177	131
normalized size	1	1.	0.83	0.92	0.	0.	1.49	1.1
time (sec)	N/A	0.114	0.112	0.008	0.	0.	13.417	1.196

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	104	111	0	0	184	143
normalized size	1	1.	0.9	0.96	0.	0.	1.59	1.23
time (sec)	N/A	0.068	0.099	0.007	0.	0.	7.954	1.218

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	117	184	0	0	872	173
normalized size	1	1.	0.9	1.42	0.	0.	6.71	1.33
time (sec)	N/A	0.135	0.103	0.013	0.	0.	16.359	1.671

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	141	195	0	0	860	190
normalized size	1	1.	0.98	1.35	0.	0.	5.97	1.32
time (sec)	N/A	0.228	0.096	0.013	0.	0.	20.804	1.703

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	147	250	0	0	1904	219
normalized size	1	1.	0.84	1.44	0.	0.	10.94	1.26
time (sec)	N/A	0.311	0.15	0.017	0.	0.	60.189	1.199

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	63	15	34
normalized size	1	1.	1.	0.95	1.2	3.15	0.75	1.7
time (sec)	N/A	0.023	0.007	0.007	0.987	1.395	0.093	1.152

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	15	30	39	22	23
normalized size	1	1.	0.78	0.65	1.3	1.7	0.96	1.
time (sec)	N/A	0.021	0.008	0.005	0.97	1.428	0.343	1.144

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	27	61	20	27
normalized size	1	1.	1.	0.84	1.08	2.44	0.8	1.08
time (sec)	N/A	0.024	0.01	0.003	1.529	1.41	0.088	1.18

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	73	22	32
normalized size	1	1.	1.	0.83	1.07	2.43	0.73	1.07
time (sec)	N/A	0.025	0.005	0.002	1.522	1.452	0.09	1.136

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	210	278	0	953	366	338
normalized size	1	1.	1.	1.32	0.	4.54	1.74	1.61
time (sec)	N/A	0.161	0.146	0.006	0.	1.519	0.9	1.2

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	162	230	0	770	325	270
normalized size	1	1.	0.94	1.34	0.	4.48	1.89	1.57
time (sec)	N/A	0.123	0.114	0.004	0.	1.507	0.867	1.175

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	182	0	606	180	205
normalized size	1	1.	0.94	1.34	0.	4.46	1.32	1.51
time (sec)	N/A	0.105	0.089	0.004	0.	1.562	0.813	1.169

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	135	0	505	158	143
normalized size	1	1.	0.98	1.35	0.	5.05	1.58	1.43
time (sec)	N/A	0.062	0.08	0.002	0.	1.504	0.927	1.145

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	114	0	444	150	116
normalized size	1	1.	0.99	1.36	0.	5.29	1.79	1.38
time (sec)	N/A	0.094	0.063	0.005	0.	1.507	1.304	1.19

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	115	0	455	151	109
normalized size	1	1.	1.01	1.4	0.	5.55	1.84	1.33
time (sec)	N/A	0.087	0.08	0.007	0.	1.565	2.428	1.167

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	103	142	0	522	167	142
normalized size	1	1.	0.99	1.37	0.	5.02	1.61	1.37
time (sec)	N/A	0.102	0.083	0.007	0.	1.501	5.304	1.151

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	139	190	0	624	301	204
normalized size	1	1.	1.01	1.39	0.	4.55	2.2	1.49
time (sec)	N/A	0.13	0.115	0.009	0.	1.51	13.263	1.182

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	174	238	0	788	354	271
normalized size	1	1.	0.99	1.36	0.	4.5	2.02	1.55
time (sec)	N/A	0.146	0.133	0.008	0.	1.504	27.46	1.184

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	211	286	0	976	398	336
normalized size	1	1.	1.	1.36	0.	4.63	1.89	1.59
time (sec)	N/A	0.175	0.161	0.011	0.	1.54	51.001	1.175

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	227	309	0	1260	430	340
normalized size	1	1.	0.95	1.29	0.	5.25	1.79	1.42
time (sec)	N/A	0.294	0.124	0.011	0.	1.587	2.449	1.155

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	187	258	0	1041	250	271
normalized size	1	1.	0.93	1.28	0.	5.15	1.24	1.34
time (sec)	N/A	0.235	0.101	0.01	0.	1.492	2.522	1.153

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	148	212	0	902	216	205
normalized size	1	1.	0.91	1.3	0.	5.53	1.33	1.26
time (sec)	N/A	0.229	0.082	0.009	0.	1.496	2.431	1.164

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	122	177	0	761	199	170
normalized size	1	1.	1.03	1.5	0.	6.45	1.69	1.44
time (sec)	N/A	0.12	0.088	0.009	0.	1.701	1.943	1.164

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	115	165	0	714	197	165
normalized size	1	1.	1.03	1.47	0.	6.38	1.76	1.47
time (sec)	N/A	0.132	0.062	0.013	0.	1.835	4.611	1.166

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	125	182	0	786	212	166
normalized size	1	1.	1.03	1.5	0.	6.5	1.75	1.37
time (sec)	N/A	0.158	0.072	0.011	0.	1.465	12.21	1.254

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	151	219	0	926	226	204
normalized size	1	1.	0.99	1.44	0.	6.09	1.49	1.34
time (sec)	N/A	0.213	0.082	0.014	0.	1.394	30.535	1.173

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	190	268	0	1057	394	271
normalized size	1	1.	1.01	1.42	0.	5.59	2.08	1.43
time (sec)	N/A	0.293	0.102	0.016	0.	1.38	76.736	1.197

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	230	318	0	1284	449	340
normalized size	1	1.	1.	1.38	0.	5.58	1.95	1.48
time (sec)	N/A	0.376	0.114	0.016	0.	1.303	144.033	1.243

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	272	394	0	1748	491	406
normalized size	1	1.	0.95	1.37	0.	6.09	1.71	1.41
time (sec)	N/A	0.492	0.155	0.014	0.	1.253	16.19	1.18

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	232	343	0	1517	311	338
normalized size	1	1.	0.94	1.39	0.	6.14	1.26	1.37
time (sec)	N/A	0.411	0.137	0.013	0.	1.306	16.214	1.214

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	176	294	0	1362	279	270
normalized size	1	1.	0.85	1.42	0.	6.58	1.35	1.3
time (sec)	N/A	0.334	0.159	0.013	0.	1.315	15.871	1.205

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	156	259	0	1195	258	234
normalized size	1	1.	0.93	1.55	0.	7.16	1.54	1.4
time (sec)	N/A	0.262	0.121	0.011	0.	1.5	12.917	1.228

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	141	234	0	1062	243	201
normalized size	1	1.	0.96	1.59	0.	7.22	1.65	1.37
time (sec)	N/A	0.15	0.114	0.01	0.	1.508	8.284	1.167

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	155	237	0	1076	250	207
normalized size	1	1.	1.01	1.55	0.	7.03	1.63	1.35
time (sec)	N/A	0.176	0.124	0.013	0.	1.527	22.808	1.171

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	169	264	0	1219	270	230
normalized size	1	1.	1.01	1.57	0.	7.26	1.61	1.37
time (sec)	N/A	0.242	0.138	0.016	0.	1.514	58.453	1.17

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	196	300	0	1386	284	267
normalized size	1	1.	1.	1.53	0.	7.07	1.45	1.36
time (sec)	N/A	0.35	0.111	0.016	0.	1.546	135.007	1.235

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	234	351	0	1539	0	338
normalized size	1	1.	1.	1.5	0.	6.58	0.	1.44
time (sec)	N/A	0.486	0.136	0.016	0.	1.468	0.	1.213

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	276	401	0	1778	0	406
normalized size	1	1.	1.	1.45	0.	6.42	0.	1.47
time (sec)	N/A	0.603	0.148	0.02	0.	1.317	0.	1.216

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	158	193	0	433	442	387
normalized size	1	1.	0.74	0.9	0.	2.02	2.07	1.81
time (sec)	N/A	0.252	0.166	0.008	0.	1.313	4.459	1.215

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	122	145	0	316	340	296
normalized size	1	1.	0.73	0.87	0.	1.89	2.04	1.77
time (sec)	N/A	0.194	0.121	0.007	0.	1.358	2.752	1.244

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	89	99	0	221	238	207
normalized size	1	1.	0.74	0.82	0.	1.83	1.97	1.71
time (sec)	N/A	0.133	0.082	0.005	0.	1.372	1.63	1.171

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	86	134	0	483	102	171
normalized size	1	1.	0.83	1.3	0.	4.69	0.99	1.66
time (sec)	N/A	0.14	0.124	0.008	0.	1.394	24.939	1.229

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	131	127	0	482	138	154
normalized size	1	1.	1.31	1.27	0.	4.82	1.38	1.54
time (sec)	N/A	0.203	0.382	0.009	0.	1.46	40.125	1.23

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	141	162	0	501	194	190
normalized size	1	1.	1.24	1.42	0.	4.39	1.7	1.67
time (sec)	N/A	0.233	0.345	0.01	0.	1.478	78.677	1.17

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	162	238	0	605	303	313
normalized size	1	1.	1.11	1.63	0.	4.14	2.08	2.14
time (sec)	N/A	0.276	0.962	0.01	0.	1.628	110.921	1.213

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	140	320	0	803	444	487
normalized size	1	1.	0.72	1.64	0.	4.12	2.28	2.5
time (sec)	N/A	0.35	0.328	0.014	0.	2.054	176.308	1.224

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	184	368	0	983	586	302
normalized size	1	1.	0.75	1.5	0.	4.01	2.39	1.23
time (sec)	N/A	0.258	0.224	0.022	0.	2.07	33.212	1.187

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	149	284	0	779	444	236
normalized size	1	1.	0.77	1.46	0.	4.02	2.29	1.22
time (sec)	N/A	0.208	0.158	0.008	0.	1.716	21.593	1.224

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	118	203	0	581	362	174
normalized size	1	1.	0.81	1.4	0.	4.01	2.5	1.2
time (sec)	N/A	0.119	0.109	0.006	0.	1.523	12.11	1.191

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	103	140	0	489	250	163
normalized size	1	1.	0.88	1.2	0.	4.18	2.14	1.39
time (sec)	N/A	0.136	0.11	0.007	0.	1.411	6.778	1.21

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	93	117	0	479	197	238
normalized size	1	1.	0.85	1.06	0.	4.35	1.79	2.16
time (sec)	N/A	0.127	0.105	0.009	0.	1.47	3.979	1.204

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	95	136	0	509	456	437
normalized size	1	1.	0.81	1.15	0.	4.31	3.86	3.7
time (sec)	N/A	0.133	0.107	0.007	0.	1.405	3.2	1.295

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	103	111	0	232	891	748
normalized size	1	1.	0.74	0.79	0.	1.66	6.36	5.34
time (sec)	N/A	0.184	0.078	0.005	0.	1.664	4.744	1.23

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	134	157	0	327	1642	900
normalized size	1	1.	0.71	0.83	0.	1.73	8.69	4.76
time (sec)	N/A	0.254	0.086	0.006	0.	2.269	6.782	1.243

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	273	517	0	0	0	462
normalized size	1	1.	0.72	1.36	0.	0.	0.	1.21
time (sec)	N/A	0.662	0.534	0.301	0.	0.	0.	1.248

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	229	460	0	0	0	358
normalized size	1	1.	0.82	1.65	0.	0.	0.	1.28
time (sec)	N/A	0.455	0.431	0.008	0.	0.	0.	1.204

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	194	405	0	0	0	274
normalized size	1	1.	0.92	1.93	0.	0.	0.	1.3
time (sec)	N/A	0.387	0.441	0.012	0.	0.	0.	1.203

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	192	168	363	0	0	0	216
normalized size	1	1.07	0.94	2.03	0.	0.	0.	1.21
time (sec)	N/A	0.314	0.41	0.008	0.	0.	0.	1.343

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	98	109	452	0	0	177
normalized size	1	1.	0.73	0.81	3.37	0.	0.	1.32
time (sec)	N/A	0.21	0.101	0.005	1.058	0.	0.	1.202

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	179	133	157	0	0	0	285
normalized size	1	0.97	0.72	0.85	0.	0.	0.	1.54
time (sec)	N/A	0.246	0.152	0.006	0.	0.	0.	1.216

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	165	205	0	0	0	471
normalized size	1	1.	0.68	0.85	0.	0.	0.	1.95
time (sec)	N/A	0.322	0.129	0.007	0.	0.	0.	1.216

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	275	202	253	0	0	0	799
normalized size	1	0.98	0.72	0.9	0.	0.	0.	2.84
time (sec)	N/A	0.429	0.146	0.007	0.	0.	0.	1.259

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	234	301	0	0	0	1266
normalized size	1	1.	0.7	0.9	0.	0.	0.	3.79
time (sec)	N/A	0.48	0.146	0.008	0.	0.	0.	1.337

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	380	270	349	0	0	0	1569
normalized size	1	0.97	0.69	0.89	0.	0.	0.	4.
time (sec)	N/A	0.546	0.163	0.008	0.	0.	0.	1.323

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	158	193	0	433	442	387
normalized size	1	1.	0.74	0.9	0.	2.02	2.07	1.81
time (sec)	N/A	0.222	0.179	0.007	0.	1.423	4.555	1.228

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	122	145	0	316	340	296
normalized size	1	1.	0.73	0.87	0.	1.89	2.04	1.77
time (sec)	N/A	0.175	0.126	0.006	0.	1.291	2.785	1.18

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	89	99	0	221	238	207
normalized size	1	1.	0.74	0.82	0.	1.83	1.97	1.71
time (sec)	N/A	0.15	0.088	0.004	0.	1.366	1.611	1.162

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	257	221	478	0	0	0	302
normalized size	1	0.98	0.85	1.83	0.	0.	0.	1.16
time (sec)	N/A	0.716	0.505	0.01	0.	0.	0.	1.427

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	250	197	427	0	0	0	275
normalized size	1	1.17	0.92	2.	0.	0.	0.	1.29
time (sec)	N/A	0.41	0.446	0.008	0.	0.	0.	1.243

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	138	166	0	0	0	297
normalized size	1	1.	0.72	0.86	0.	0.	0.	1.54
time (sec)	N/A	0.341	0.165	0.006	0.	0.	0.	1.24

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [13] had the largest ratio of [0.4]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.	20	0.25
2	A	5	5	1.	20	0.25
3	A	4	4	1.	18	0.222
4	A	4	4	1.	17	0.235
5	A	7	7	1.	20	0.35
6	A	7	7	1.	20	0.35
7	A	7	7	1.	20	0.35
8	A	7	5	1.	20	0.25
9	A	6	5	1.	20	0.25
10	A	5	4	1.	18	0.222
11	A	5	4	1.	17	0.235
12	A	8	7	1.	20	0.35
13	A	8	8	1.	20	0.4
14	A	8	7	1.	20	0.35
15	A	8	5	1.	20	0.25
16	A	7	5	1.	20	0.25
17	A	6	4	1.	18	0.222
18	A	6	4	1.	17	0.235
19	A	9	7	1.	20	0.35
20	A	9	8	1.	20	0.4
21	A	9	8	1.	20	0.4
22	A	5	4	1.	20	0.2
23	A	4	4	1.	20	0.2

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	3	3	1.	18	0.167
25	A	3	3	1.	17	0.176
26	A	6	6	1.	20	0.3
27	A	4	4	1.	20	0.2
28	A	5	5	1.	20	0.25
29	A	4	4	1.	20	0.2
30	A	4	4	1.	20	0.2
31	A	3	3	1.	18	0.167
32	A	1	1	1.	17	0.059
33	A	5	5	1.	20	0.25
34	A	5	5	1.	20	0.25
35	A	6	6	1.	20	0.3
36	A	4	4	1.	20	0.2
37	A	2	2	1.	20	0.1
38	A	2	2	1.	18	0.111
39	A	2	2	1.	17	0.118
40	A	6	5	1.	20	0.25
41	A	6	5	1.	20	0.25
42	A	7	6	1.	20	0.3
43	A	2	2	1.	18	0.111
44	A	3	3	1.	19	0.158
45	A	2	2	1.	13	0.154
46	A	2	2	1.	13	0.154
47	A	7	5	1.	25	0.2
48	A	6	5	1.	25	0.2
49	A	5	4	1.	25	0.16
50	A	4	4	1.	25	0.16
51	A	4	4	1.	25	0.16
52	A	4	4	1.	25	0.16
53	A	4	4	1.	23	0.174
54	A	5	4	1.	22	0.182
55	A	8	6	1.	25	0.24

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	8	5	1.	25	0.2
57	A	9	6	1.	25	0.24
58	A	2	2	1.	16	0.125
59	A	3	2	1.	20	0.1
60	A	2	2	1.	22	0.091
61	A	5	3	1.	25	0.12
62	A	2	1	1.	26	0.038
63	A	2	1	1.	26	0.038
64	A	2	1	1.	24	0.042
65	A	2	1	1.	23	0.043
66	A	2	1	1.	26	0.038
67	A	2	1	1.	26	0.038
68	A	2	1	1.	26	0.038
69	A	2	1	1.	26	0.038
70	A	2	1	1.	28	0.036
71	A	2	1	1.	28	0.036
72	A	3	2	1.	26	0.077
73	A	3	2	1.	25	0.08
74	A	3	2	1.	28	0.071
75	A	3	2	1.	28	0.071
76	A	2	1	1.	28	0.036
77	A	2	1	1.	28	0.036
78	A	2	1	1.	28	0.036
79	A	2	1	1.	28	0.036
80	A	3	2	1.	26	0.077
81	A	3	2	1.	25	0.08
82	A	3	2	1.	28	0.071
83	A	3	2	1.	28	0.071
84	A	2	1	1.	28	0.036
85	A	2	1	1.	28	0.036
86	A	5	4	1.	28	0.143
87	A	5	4	1.	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	4	1.	28	0.143
89	A	5	4	1.	26	0.154
90	A	5	4	1.	25	0.16
91	A	5	4	1.	28	0.143
92	A	5	4	1.	28	0.143
93	A	5	4	1.	28	0.143
94	A	6	5	1.	28	0.179
95	A	6	5	1.	28	0.179
96	A	6	5	1.	28	0.179
97	A	6	5	1.	26	0.192
98	A	4	4	1.	25	0.16
99	A	6	5	1.	28	0.179
100	A	6	5	1.	28	0.179
101	A	6	5	1.	28	0.179
102	A	7	5	1.	28	0.179
103	A	6	5	1.	28	0.179
104	A	5	4	1.	28	0.143
105	A	4	4	1.	26	0.154
106	A	3	3	1.	25	0.12
107	A	7	6	1.	28	0.214
108	A	7	5	1.	28	0.179
109	A	7	5	1.	28	0.179
110	A	4	3	1.	17	0.176
111	A	4	3	1.	17	0.176
112	A	4	4	1.	21	0.19
113	A	6	5	1.	15	0.333
114	A	3	2	1.	30	0.067
115	A	3	2	1.	30	0.067
116	A	3	2	1.	30	0.067
117	A	3	2	1.	27	0.074
118	A	3	2	1.	30	0.067
119	A	3	2	1.	30	0.067

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	2	1.	30	0.067
121	A	3	2	1.	30	0.067
122	A	3	2	1.	30	0.067
123	A	3	2	1.	30	0.067
124	A	5	4	1.	30	0.133
125	A	5	4	1.	30	0.133
126	A	5	4	1.	30	0.133
127	A	4	3	1.	27	0.111
128	A	4	3	1.	30	0.1
129	A	4	3	1.	30	0.1
130	A	4	3	1.	30	0.1
131	A	4	3	1.	30	0.1
132	A	4	3	1.	30	0.1
133	A	6	5	1.	30	0.167
134	A	6	5	1.	30	0.167
135	A	6	5	1.	30	0.167
136	A	6	5	1.	30	0.167
137	A	4	4	1.	27	0.148
138	A	4	4	1.	30	0.133
139	A	5	4	1.	30	0.133
140	A	5	3	1.	30	0.1
141	A	5	3	1.	30	0.1
142	A	5	3	1.	30	0.1
143	A	3	2	1.	32	0.062
144	A	3	2	1.	32	0.062
145	A	3	2	1.	30	0.067
146	A	5	4	1.	32	0.125
147	A	6	5	1.	32	0.156
148	A	6	6	1.	32	0.188
149	A	6	6	1.	32	0.188
150	A	7	7	1.	32	0.219
151	A	7	6	1.	32	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	6	6	1.	32	0.188
153	A	5	5	1.	29	0.172
154	A	6	6	1.	32	0.188
155	A	6	6	1.	32	0.188
156	A	6	6	1.	32	0.188
157	A	5	3	1.	32	0.094
158	A	6	4	1.	32	0.125
159	A	11	9	1.	32	0.281
160	A	10	9	1.	32	0.281
161	A	9	9	1.	32	0.281
162	A	8	8	1.07	32	0.25
163	A	5	4	1.	29	0.138
164	A	6	5	0.97	32	0.156
165	A	7	5	1.	32	0.156
166	A	8	4	0.98	32	0.125
167	A	9	5	1.	32	0.156
168	A	10	5	0.97	32	0.156
169	A	4	3	1.	33	0.091
170	A	4	3	1.	33	0.091
171	A	4	3	1.	31	0.097
172	A	10	9	0.98	37	0.243
173	A	6	5	1.17	34	0.147
174	A	6	4	1.	37	0.108

Chapter 3

Listing of integrals

3.1 $\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=127

$$\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5*b) + (B*x
^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b*x^2)^(3/2))/(120*b^
2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] time = 0.0819759, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {833, 780, 195, 217, 206}

$$\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5*b) + (B*x
^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b*x^2)^(3/2))/(120*b^
2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx = \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} + \frac{\int x^2(-3aB+6Abx)\sqrt{a+bx^2}\,dx}{6b}$$
$$= \frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} + \frac{\int x(-12aAb-15abBx)\sqrt{a+bx^2}\,dx}{30b^2}$$
$$= \frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{\left(a^2B\right)\int\sqrt{a+bx^2}\,dx}{8b^2}$$
$$= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2} + \frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{(a^3}{}$$
$$= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2} + \frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{(a^3}{}$$
$$= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2} + \frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{a^3}{}$$

Mathematica [A] time = 0.186739, size = 107, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(-a^2(32A+15Bx)+2abx^2(8A+5Bx)+8b^2x^4(6A+5Bx)\right)+\frac{15a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{240b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(8*A + 5*B*x
) - a^2*(32*A + 15*B*x)) + (15*a^(5/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[1 + (b*x^2)/a]))/(240*b^(5/2))
```

Maple [A] time = 0.008, size = 115, normalized size = 0.9

$$\frac{Bx^3}{6\,b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{Bax}{8\,b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{a^2Bx}{16\,b^2}\sqrt{bx^2+a} + \frac{Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}} + \frac{Ax^2}{5\,b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{2\,Aa}{15\,b^2}\left(bx^2+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/8*B/b^2*a*x*(b*x^2+a)^(3/2)+1/16*B/b^2*a^2*x*
(b*x^2+a)^(1/2)+1/16*B/b^(5/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/5*A*x^2*
(b*x^2+a)^(3/2)/b-2/15*A*a/b^2*(b*x^2+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57808, size = 501, normalized size = 3.94

$$\left[\frac{15\,Ba^3\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\left(40\,Bb^3x^5 + 48\,Ab^3x^4 + 10\,Bab^2x^3 + 16\,Aab^2x^2 - 15\,Ba^2bx - 32\,Aa^2b\right)}{480\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3 + 16*A*a*b^2*x^2 - 15*B*a^2
*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3, -1/240*(15*B*a^3*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3
+ 16*A*a*b^2*x^2 - 15*B*a^2*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 6.52921, size = 192, normalized size = 1.51

$$A\left(\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases}\right) - \frac{Ba^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}} + \frac{5B\sqrt{a}x^5}{24\sqrt{1+\frac{bx^2}{a}}} + \frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(
15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) - B*a**
(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(3/2)*x**3/(48*b*sqrt(1 + b*x**
2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + B*a**3*asinh(sqrt(b)*x/s
qrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.19043, size = 126, normalized size = 0.99

$$-\frac{Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}+\frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Bx+6\,A)x+\frac{5\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)x-\frac{15\,Ba^2}{b^2}\right)x-\frac{32\,Aa^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/240*sqrt(b*x
^2 + a)*((2*((4*(5*B*x + 6*A)*x + 5*B*a/b)*x + 8*A*a/b)*x - 15*B*a^2/b^2)*x
 - 32*A*a^2/b^2)
```

3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2 A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

```
[Out] -(a*A*x*Sqrt[a + b*x^2])/(8*b) + (B*x^2*(a + b*x^2)^(3/2))/(5*b) - ((8*a*B
- 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(3/2))
```

Rubi [A] time = 0.0490937, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {833, 780, 195, 217, 206}

$$-\frac{a^2 A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*A*x*Sqrt[a + b*x^2])/(8*b) + (B*x^2*(a + b*x^2)^(3/2))/(5*b) - ((8*a*B
- 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(3/2))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2(A+Bx)\sqrt{a+bx^2}\,dx &= \frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{\int x(-2aB+5Abx)\sqrt{a+bx^2}\,dx}{5b}\\
&= \frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{(aA)\int\sqrt{a+bx^2}\,dx}{4b}\\
&= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\right.}{8b}\\
&= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.174386, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(-\frac{15a^{3/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-16a^2B+abx(15A+8Bx)+6b^2x^3(5A+4Bx)\right)}{120b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^2*B + 6*b^2*x^3*(5*A + 4*B*x) + a*b*x*(15*A + 8*B*x
) - (15*a^(3/2)*A*Sqrt[b]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]
))/(120*b^2)
```

Maple [A] time = 0.006, size = 94, normalized size = 0.9

$$\frac{Bx^2}{5\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{2\,Ba}{15\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Ax}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{aAx}{8\,b}\sqrt{bx^2+a}-\frac{Aa^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*B*x^2*(b*x^2+a)^(3/2)/b-2/15*B*a/b^2*(b*x^2+a)^(3/2)+1/4*A*x*(b*x^2+a)^
(3/2)/b-1/8*A/b*a*x*(b*x^2+a)^(1/2)-1/8*A/b^(3/2)*a^2*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55456, size = 435, normalized size = 4.18

$$\left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+a}}{240\,b^2}, \frac{15\,A\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*A*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqrt(
b*x^2 + a))/b^2, 1/120*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqr
t(b*x^2 + a))/b^2]
```

Sympy [A] time = 4.43068, size = 165, normalized size = 1.59

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(1 + b*x**2
/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A*b*x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a)) + B*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x*
*2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x
**4/4, True))
```

Giac [A] time = 1.18154, size = 109, normalized size = 1.05

$$\frac{Aa^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Bx+5\,A)x+\frac{4\,Ba}{b}\right)x+\frac{15\,Aa}{b}\right)x-\frac{16\,Ba^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/120*sqrt(b*x^2
 + a)*((2*(3*(4*B*x + 5*A)*x + 4*B*a/b)*x + 15*A*a/b)*x - 16*B*a^2/b^2)
```

3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

```
[Out] -(a*B*x*Sqrt[a + b*x^2])/(8*b) + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/(12*b) -
 (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] time = 0.0256785, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*B*x*Sqrt[a + b*x^2])/(8*b) + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/(12*b) -
 (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x(A+Bx)\sqrt{a+bx^2}\,dx &= \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{(aB)\int\sqrt{a+bx^2}\,dx}{4b}\\
&= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{\left(a^2B\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{\left(a^2B\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\
&= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.140984, size = 86, normalized size = 1.08

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(8aA+3aBx+8Abx^2+6bBx^3\right)-\frac{3a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) - (3*a^
(3/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(24*b^(3/2))
```

Maple [A] time = 0.005, size = 75, normalized size = 0.9

$$\frac{Bx}{4b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Bax}{8b}\sqrt{bx^2+a}-\frac{Ba^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{A}{3b}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x*(b*x^2+a)^(3/2)/b-1/8*B/b*a*x*(b*x^2+a)^(1/2)-1/8*B/b^(3/2)*a^2*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))+1/3*A*(b*x^2+a)^(3/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58638, size = 381, normalized size = 4.76

$$\left[\frac{3\,Ba^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6\,Bb^2x^3+8\,Ab^2x^2+3\,Babx+8\,Aab\right)\sqrt{bx^2+a}}{48\,b^2},\frac{3\,Ba^2\sqrt{-b}\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*B*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2, 1/2
4*(3*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*B*b^2*x^3 + 8*A
*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 4.37961, size = 124, normalized size = 1.55

$$A\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
+ B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x**3/(8*sqrt(1 + b*x*
*2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + B*b*x**5/(4*sqrt(a)
*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.17109, size = 92, normalized size = 1.15

$$\frac{Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\,(3\,Bx+4\,A)x+\frac{3\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/24*sqrt(b*x^2
+ a)*((2*(3*B*x + 4*A)*x + 3*B*a/b)*x + 8*A*a/b)
```

3.4 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi [A] time = 0.0193309, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*Sqrt[a + b*x^2],x]
```


```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```


Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (A+Bx)\sqrt{a+bx^2}\,dx &= \frac{B\left(a+bx^2\right)^{3/2}}{3b}+A\int \sqrt{a+bx^2}\,dx\\ &= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0496676, size = 67, normalized size = 1.

$$\frac{\sqrt{a+bx^2}(2aB+bx(3A+2Bx))+3aA\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(2*a*B + b*x*(3*A + 2*B*x)) + 3*a*A*Sqrt[b]*Log[b*x + Sqrt
[b]*Sqrt[a + b*x^2]])/(6*b)
```

Maple [A] time = 0.004, size = 53, normalized size = 0.8

$$\frac{B}{3\,b}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Ax}{2}\sqrt{bx^2+a}+\frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b+1/2*A*x*(b*x^2+a)^(1/2)+1/2*A*a/b^(1/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54647, size = 316, normalized size = 4.72

$$\left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\left(2\,Bbx^2 + 3\,Abx + 2\,Ba\right)\sqrt{bx^2+a}}{12\,b}, -\frac{3\,Aa\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - \left(2\,Bb\right.}{6\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a
))/b]
```

Sympy [A] time = 2.72642, size = 70, normalized size = 1.04

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}} + B\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b = 0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
 + B*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True
))
```

Giac [A] time = 1.17924, size = 74, normalized size = 1.1

$$-\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}+\frac{1}{6}\sqrt{bx^2+a}\left((2Bx+3A)x+\frac{2Ba}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0605239, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{\int \frac{2aAb+abBx}{x\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+(aA)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+\frac{1}{2}(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{1}{2}(aA)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)+\frac{1}{2}(aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{(aA)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.208145, size = 100, normalized size = 1.27

$$
\frac{1}{2}\left(\frac{a^{3/2}B\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx^2}}+\sqrt{a+bx^2}(2A+Bx)-2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2] + (a^(3/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt
[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[a + b*x^2]) - 2*Sqrt[a]*A*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/2
```

Maple [A] time = 0.004, size = 78, normalized size = 1.

$$
\frac{Bx}{2}\sqrt{bx^2+a}+\frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-A\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+A\sqrt{bx^2+a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*x*B*(b*x^2+a)^(1/2)+1/2*B*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-A*a^(
1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+A*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66101, size = 859, normalized size = 10.87

$$\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b},-\frac{Ba\sqrt{-b}\operatorname{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*A*sqr
t(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*b*x + 2*A
*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (B*b
*x + 2*A*b)*sqrt(b*x^2 + a))/b, 1/4*(4*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - 2*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x
+ 2*A*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 5.04436, size = 107, normalized size = 1.35

$$-A\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)
) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + B
*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A] time = 1.19014, size = 105, normalized size = 1.33

$$\frac{2\,Aa\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{2}\,\sqrt{bx^2+a}(Bx+2\,A)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*B*a*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/2*sqrt(b*x^2 + a)*(B*x + 2
*A)
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{a+bx^2}(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0586444, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^2}(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = -\frac{(A-Bx)\sqrt{a+bx^2}}{x} - \frac{1}{2}\int \frac{-2aB-2Abx}{x\sqrt{a+bx^2}}\,dx$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + (Ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx + (aB)\int \frac{1}{x\sqrt{a+bx^2}}\,dx$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + (Ab)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}(aB)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\right.$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{(aB)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.158958, size = 99, normalized size = 1.32

$$\frac{\sqrt{a+bx^2}(Bx-A)}{x} + \frac{\sqrt{a}A\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a+bx^2}} - \sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] ((-A + B*x)*Sqrt[a + b*x^2])/x + (Sqrt[a]*A*Sqrt[b]*Sqrt[1 + (b*x^2)/a]*Arc
Sinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a + b*x^2] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]]
```

Maple [A] time = 0.006, size = 97, normalized size = 1.3

$$-B\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + B\sqrt{bx^2+a} - \frac{A}{ax}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{Abx}{a}\sqrt{bx^2+a} + A\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -B*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*x^2+a)^(1/2)-A*(b*x^2
+a)^(3/2)/a/x+A*b/a*x*(b*x^2+a)^(1/2)+A*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62062, size = 821, normalized size = 10.95

$$\left[\frac{A\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+B\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}(Bx-A)}{2\,x},-\frac{2\,A\sqrt{-b}x\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + B*sqrt(
a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 + a
)*(B*x - A))/x, -1/2*(2*A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B
*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(b*x
^2 + a)*(B*x - A))/x, 1/2*(2*B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a))
+ A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*x^
2 + a)*(B*x - A))/x, -(A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B*
sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(B*x - A))/x]
```

Sympy [A] time = 3.55214, size = 124, normalized size = 1.65

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B
*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A] time = 1.19394, size = 138, normalized size = 1.84

$$\frac{2\,Ba\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - A\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \sqrt{bx^2+a}B + \frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - A*sqrt(b)*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*B + 2*A*a*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3} \, dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -((A + 2*B*x)*Sqrt[a + b*x^2])/(2*x^2) + Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rubi [A] time = 0.0604033, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {811, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```


```
[Out] -((A + 2*B*x)*Sqrt[a + b*x^2])/(2*x^2) + Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```


Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} - \frac{\int \frac{-2aAb-4abBx}{x\sqrt{a+bx^2}}\,dx}{4a}$$
$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \frac{1}{2}(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + (bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \frac{1}{4}(Ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + (bB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.$$
$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)$$
$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.0911153, size = 108, normalized size = 1.35

$$-\frac{\sqrt{a+bx^2}\left(a\sqrt{\frac{bx^2}{a}+1}(A+2Bx) + Abx^2\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right) - 2\sqrt{a}\sqrt{b}Bx^2\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{2ax^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^2]*(a*(A + 2*B*x)*Sqrt[1 + (b*x^2)/a] - 2*Sqrt[a]*Sqrt[b]*B*
x^2*ArcSinh[(Sqrt[b]*x)/Sqrt[a]] + A*b*x^2*ArcTanh[Sqrt[1 + (b*x^2)/a]]))/(
2*a*x^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.008, size = 121, normalized size = 1.5

$$-\frac{A}{2\,ax^2}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{Ab}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}} + \frac{Ab}{2\,a}\sqrt{bx^2+a} - \frac{B}{ax}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{bBx}{a}\sqrt{bx^2+a} + B\sqrt{b}\ln\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(3/2)-1/2*A*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1
/2))/x)+1/2*A*b/a*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(3/2)+B*b/a*x*(b*x^2+a)^(
1/2)+B*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62037, size = 934, normalized size = 11.68

$$\left[\frac{2\,Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,ax^2},-\frac{4\,Ba\sqrt{-bx}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A
*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*B
*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), -1/4*(4*B*a*sqrt(-b)*x^2*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*(A*sqrt
(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*x^2*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^
2), -1/2*(2*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(-a
)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))
/(a*x^2)]
```

Sympy [A] time = 3.75492, size = 107, normalized size = 1.34

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt
(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [B] time = 1.22417, size = 220, normalized size = 2.75

$$\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-B\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{}\right.\cdots}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*sqrt(b)*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b
 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2
+ a))*A*a*b - 2*B*a^2*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

3.8 $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}-\frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(\right.}{}$$

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2))/(64*b^2
) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a*
(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2) + (3*a^4*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi [A] time = 0.0950358, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {833, 780, 195, 217, 206}

$$\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}-\frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2))/(64*b^2
) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a*
(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2) + (3*a^4*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}dx = \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b} + \frac{\int x^2(-3aB+8Abx)\left(a+bx^2\right)^{3/2}dx}{8b}$$
$$= \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b} + \frac{\int x(-16aAb-21abBx)\left(a+bx^2\right)^{3/2}dx}{56b^2}$$
$$= \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b} - \frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2} + \frac{\left(a^2B\right)\int\left(a+bx^2\right)^{3}}{16b^2}$$
$$= \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2} + \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b} - \frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2}$$
$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2} + \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b} - \frac{a(32A+3}{}$$
$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2} + \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b} - \frac{a(32A+3}{}$$
$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2} + \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b} - \frac{a(32A+3}{}$$

Mathematica [A] time = 0.228028, size = 126, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(2a^2bx^2(64A+35Bx)-a^3(256A+105Bx)+8ab^2x^4(128A+105Bx)+80b^3x^6(8A+7Bx)\right)+\frac{105a^{7/2}B\sinh^{-1}}{\sqrt{\frac{bx^2}{a}+1}}\right.}{4480b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(80*b^3*x^6*(8*A + 7*B*x) + 2*a^2*b*x^2*(64*A + 3
5*B*x) + 8*a*b^2*x^4*(128*A + 105*B*x) - a^3*(256*A + 105*B*x)) + (105*a^(7
/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(4480*b^(5/2))
```

Maple [A] time = 0.008, size = 134, normalized size = 0.9

$$\frac{Bx^3}{8\,b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{Bax}{16\,b^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{a^2Bx}{64\,b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,a^3Bx}{128\,b^2}\sqrt{bx^2+a} + \frac{3\,Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}} + \frac{Ax^2}{7\,b}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/16*B/b^2*a*x*(b*x^2+a)^(5/2)+1/64*B/b^2*a^2*x
*(b*x^2+a)^(3/2)+3/128*B/b^2*a^3*x*(b*x^2+a)^(1/2)+3/128*B/b^(5/2)*a^4*ln(x
*b^(1/2)+(b*x^2+a)^(1/2))+1/7*A*x^2*(b*x^2+a)^(5/2)/b-2/35*A*a/b^2*(b*x^2+a
)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60339, size = 630, normalized size = 4.2

$$\left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+70\,Ba^2b^2x^3\right.}{8960\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8960*(105*B*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3, -1/4480*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3]
```

Sympy [A] time = 15.7861, size = 318, normalized size = 2.12

$$Aa\left(\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases}\right) + Ab\left(\begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3} - \frac{4a^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{ax^4\sqrt{a+bx^2}}{35b} + \frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^6}{6} & \text{otherwis} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + A*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True)) - 3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**
2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64
*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*B*a**
4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + B*b**2*x**9/(8*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

Giac [A] time = 1.22078, size = 155, normalized size = 1.03

$$-\frac{3\,Ba^4\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}} - \frac{1}{4480}\,\sqrt{bx^2+a}\left(\frac{256\,Aa^3}{b^2} + \left(\frac{105\,Ba^3}{b^2} - 2\left(\frac{64\,Aa^2}{b} + \left(\frac{35\,Ba^2}{b} + 4\,(128\,Aa + 5\,(21\,Ba -\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/4480*sqrt(b
*x^2 + a)*(256*A*a^3/b^2 + (105*B*a^3/b^2 - 2*(64*A*a^2/b + (35*B*a^2/b + 4
*(128*A*a + 5*(21*B*a + 2*(7*B*b*x + 8*A*b)*x)*x)*x)*x)*x)*x)
```

3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

```
[Out] -(a^2*A*x*Sqrt[a + b*x^2])/(16*b) - (a*A*x*(a + b*x^2)^(3/2))/(24*b) + (B*x
^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a + b*x^2)^(5/2))/(210*
b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.0620431, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {833, 780, 195, 217, 206}

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2*A*x*Sqrt[a + b*x^2])/(16*b) - (a*A*x*(a + b*x^2)^(3/2))/(24*b) + (B*x
^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a + b*x^2)^(5/2))/(210*
b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx &= \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{\int x(-2aB+7Abx)\left(a+bx^2\right)^{3/2}\,dx}{7b}\\
&= \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\frac{(aA)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}\\
&= -\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\frac{\left(a^2A\right)\int\sqrt{a+b}}{8b}\\
&= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}\\
&= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}\\
&= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}
\end{aligned}
$$

Mathematica [A] time = 0.214517, size = 113, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(3a^2bx(35A+16Bx)-\frac{105a^{5/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-96a^3B+2ab^2x^3(245A+192Bx)+40b^3x^5(7A+6Bx)\right)}{1680b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-96*a^3*B + 40*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(35*A +
16*B*x) + 2*a*b^2*x^3*(245*A + 192*B*x) - (105*a^(5/2)*A*Sqrt[b]*ArcSinh[(S
qrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(1680*b^2)
```

Maple [A] time = 0.007, size = 113, normalized size = 0.9

$$\frac{Bx^2}{7\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{2\,Ba}{35\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ax}{6\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{aAx}{24\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{a^2Ax}{16\,b}\sqrt{bx^2+a}-\frac{Aa^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*B*x^2*(b*x^2+a)^(5/2)/b-2/35*B*a/b^2*(b*x^2+a)^(5/2)+1/6*A*x*(b*x^2+a)^
(5/2)/b-1/24*A/b*a*x*(b*x^2+a)^(3/2)-1/16*A/b*a^2*x*(b*x^2+a)^(1/2)-1/16*A/
b^(3/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.648, size = 559, normalized size = 4.4

$$\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+490\,Aab^2x^3+48\,Ba^2bx^2+1\right.}{3360\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*A*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48
*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2, 1/1680*(105*
A*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*B*b^3*x^6 + 280*A*
b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a^2*b*
x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 10.7177, size = 287, normalized size = 2.26

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Aa^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11A\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx}}{5} \\ \frac{\sqrt{a}x^4}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*sqrt(1 + b*
x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) +
B*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [A] time = 1.25538, size = 139, normalized size = 1.09

$$\frac{Aa^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}-\frac{1}{1680}\sqrt{bx^2+a}\left(\frac{96\,Ba^3}{b^2}-\left(\frac{105\,Aa^2}{b}+2\left(\frac{24\,Ba^2}{b}+(245\,Aa+4\,(48\,Ba+5\,(6\,Bbx+7\,Ab\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/1680*sqrt(b*x
^2 + a)*(96*B*a^3/b^2 - (105*A*a^2/b + 2*(24*B*a^2/b + (245*A*a + 4*(48*B*a
 + 5*(6*B*b*x + 7*A*b)*x)*x)*x)*x)*x)
```

3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

```
[Out] -(a^2*B*x*Sqrt[a + b*x^2])/(16*b) - (a*B*x*(a + b*x^2)^(3/2))/(24*b) + ((6*
A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.0332214, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2*B*x*Sqrt[a + b*x^2])/(16*b) - (a*B*x*(a + b*x^2)^(3/2))/(24*b) + ((6*
A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(16*b^(3/2))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx &= \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{(aB)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}\\
&=-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^2B\right)\int\sqrt{a+bx^2}\,dx}{8b}\\
&=-\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^3B\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{16b}\\
&=-\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^3B\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\right.}{16b}\\
&=-\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}
\end{aligned}
$$

Mathematica [A]   time = 0.183505, size = 107, normalized size = 1.04

$$
\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(3a^2(16A+5Bx)+2abx^2(48A+35Bx)+8b^2x^4(6A+5Bx)\right)-\frac{15a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{240b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*b^2*x^4*(6*A + 5*B*x) + 3*a^2*(16*A + 5*B*x) +
 2*a*b*x^2*(48*A + 35*B*x)) - (15*a^(5/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/S
```

```
qrt[1 + (b*x^2)/a]))/(240*b^(3/2))
```

Maple [A] time = 0.005, size = 94, normalized size = 0.9

$$\frac{Bx}{6\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bax}{24\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{a^2Bx}{16\,b}\sqrt{bx^2+a}-\frac{Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{A}{5\,b}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)/b-1/24*a*B*x*(b*x^2+a)^(3/2)/b-1/16*a^2*B*x*(b*x^2+
a)^(1/2)/b-1/16*B/b^(3/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/5*A/b*(b*x^2+
a)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61261, size = 500, normalized size = 4.85

$$\left[\frac{15\,Ba^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,Aab^2x^2+15\,Ba^2bx+48\,Aa^2b\right)}{480\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 + 96*A*a*b^2*x^2 + 15*B*a^2
```

```
*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2, 1/240*(15*B*a^3*sqrt(-b)*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) + (40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 +
 96*A*a*b^2*x^2 + 15*B*a^2*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 10.0323, size = 223, normalized size = 2.17

$$Aa\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)
) + A*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x
**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
 B*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b
*x**2/a)) + 11*B*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt
(b)*x/sqrt(a))/(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.17139, size = 120, normalized size = 1.17

$$\frac{Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}+\frac{1}{240}\sqrt{bx^2+a}\left(\frac{48\,Aa^2}{b}+\left(\frac{15\,Ba^2}{b}+2\,(48\,Aa+(35\,Ba+4\,(5\,Bbx+6\,Ab)x)x)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/240*sqrt(b*x^
2 + a)*(48*A*a^2/b + (15*B*a^2/b + 2*(48*A*a + (35*B*a + 4*(5*B*b*x + 6*A*b
)*x)*x)*x)*x)
```

3.11 $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=87

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a + b*x^2)^(5
/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] time = 0.0261586, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a + b*x^2)^(5
/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (A+Bx)\left(a+bx^2\right)^{3/2} dx &= \frac{B\left(a+bx^2\right)^{5/2}}{5b}+A\int \left(a+bx^2\right)^{3/2} dx \\
&= \frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{4}(3aA)\int \sqrt{a+bx^2}\,dx \\
&= \frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{8}\left(3a^2A\right)\int \frac{1}{\sqrt{a+bx^2}}dx \\
&= \frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{8}\left(3a^2A\right)\text{Subst}\left(\int \frac{1}{1-bx^2}dx,x,
\end{aligned}
$$

$$
= \frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}
$$

Mathematica [A] time = 0.0696007, size = 88, normalized size = 1.01

$$
\frac{\sqrt{a+bx^2}\left(8a^2B+abx(25A+16Bx)+2b^2x^3(5A+4Bx)\right)+15a^2A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{40b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x)
) + 15*a^2*A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(40*b)
```

Maple [A] time = 0.004, size = 69, normalized size = 0.8

$$\frac{B}{5b}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ax}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,aAx}{8}\sqrt{bx^2+a}+\frac{3\,Aa^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)/b+1/4*A*x*(b*x^2+a)^(3/2)+3/8*a*A*x*(b*x^2+a)^(1/2)+3
/8*A*a^2/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63008, size = 425, normalized size = 4.89

$$\left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2\right)\sqrt{bx^2+a}}{80\,b},-\frac{15\,Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*
x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*x
^2 + a))/b]
```

Sympy [A] time = 6.30866, size = 219, normalized size = 2.52

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Ab^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3
*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a)
)/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a*Piecewise(
(sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*b*Piecew
ise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) +
x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

Giac [A] time = 1.18071, size = 103, normalized size = 1.18

$$-\frac{3\,Aa^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{40}\,\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+(25\,Aa+2\,(8\,Ba+(4\,Bbx+5\,Ab)x)x)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^2
 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

3.12 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=106

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/12
+ (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]) - a^(3/2)*A*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0940031, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/12
+ (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]) - a^(3/2)*A*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{(4aAb+3abBx)\sqrt{a+bx^2}}{x}\,dx}{4b}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x\sqrt{a+bx^2}}\,dx}{8b^2}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \left(a^2A\right)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + \frac{1}{8}\left(3a^2B\right.$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{2}\left(a^2A\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x\right.$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} + \frac{\left(a^2A\right)\text{Su}}{}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} - a^{3/2}A\tan$$

Mathematica [A] time = 0.247058, size = 118, normalized size = 1.11

$$\frac{1}{24}\left(-24a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{9a^{5/2}B\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx^2}} + \sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) + (9*a^(5/2)*B
*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[a + b*x^2]
) - 24*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/24
```

Maple [A] time = 0.006, size = 107, normalized size = 1.

$$\frac{Bx}{4}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,Bax}{8}\sqrt{bx^2+a} + \frac{3\,Ba^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} + \frac{A}{3}\left(bx^2+a\right)^{\frac{3}{2}} - Aa^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/4*B*x*(b*x^2+a)^(3/2)+3/8*B*a*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))+1/3*A*(b*x^2+a)^(3/2)-A*a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)+A*(b*x^2+a)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7431, size = 1094, normalized size = 10.32

$$\left[\frac{9\,Ba^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+24\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\right.}{48\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 24
*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*B*b
^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, -1/24*(9*
B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*A*a^(3/2)*b*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*B*b^2*x^3 + 8*A*b^2*x^2 +
15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, 1/48*(48*A*sqrt(-a)*a*b*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + 9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(6*B*b^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqr
t(b*x^2 + a))/b, -1/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - 24*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8*A*
b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 14.5079, size = 218, normalized size = 2.06

$$-Aa^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\ldots}{8\sqrt{1+\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
```

```
[Out] -A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*a**(3/2)*x*sqrt(1 + b*x*
*2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*b*x**3/(8*sqrt(
1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + B*b**2*x**
5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.19011, size = 135, normalized size = 1.27

$$\frac{2\,Aa^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\,Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\sqrt{b}}+\frac{1}{24}\sqrt{bx^2+a}(32\,Aa+(15\,Ba+2\,(3\,Bbx+4\,Ab)x)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/8*B*a^
2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/24*sqrt(b*x^2 + a)*(32
*A*a + (15*B*a + 2*(3*B*b*x + 4*A*b)*x)*x)
```

3.13 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=108

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^(3/2))/(3*
x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - a^(3/2)*B*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.0890723, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^(3/2))/(3*
x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - a^(3/2)*B*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = -\frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{(-2aB-6Abx)\sqrt{a+bx^2}}{x}\,dx$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{\int \frac{-4a^2bB-6aAb^2x}{x\sqrt{a+bx^2}}\,dx}{4b}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\int \frac{1}{\sqrt{a+bx^2}}\,dx + \left(a^2B\right)\int$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, -\right.$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{\left(a^2B\right.}{}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - a^{3/2}B$$

Mathematica [C] time = 0.166813, size = 105, normalized size = 0.97

$$-\frac{a^2A\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{a+bx^2}} - a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{1}{3}B\sqrt{a+bx^2}\left(4a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] (B*Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]] - (a^2*A*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*
x^2)/a)])/(x*Sqrt[a + b*x^2])
```

Maple [A] time = 0.007, size = 126, normalized size = 1.2

$$\frac{B}{3}\left(bx^2+a\right)^{\frac{3}{2}} - Ba^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right) + B\sqrt{bx^2+a}a - \frac{A}{ax}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{Abx}{a}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,Abx}{2}\sqrt{bx^2+a} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)-B*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*
x^2+a)^(1/2)*a-A/a/x*(b*x^2+a)^(5/2)+A*b/a*x*(b*x^2+a)^(3/2)+3/2*A*b*x*(b*x
^2+a)^(1/2)+3/2*A*b^(1/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67224, size = 1033, normalized size = 9.56

$$\left[\frac{9\,Aa\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+6\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa\right)\sqrt{b}}{12\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 6*
B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*b*
x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt(-b)
*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*
sqrt(b*x^2 + a))/x, 1/12*(12*B*sqrt(-a)*a*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)
) + 9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*
B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt
(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*B*sqrt(-a)*a*x*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 +
```

```
a))/x]
```

Sympy [A] time = 5.77296, size = 184, normalized size = 1.7

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{A\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}-Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - A
*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
- B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt(a)*x**2/2
, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

Giac [A] time = 1.21717, size = 167, normalized size = 1.55

$$\frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Aa\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{6}\,\sqrt{bx^2+a}(8\,Ba+(2\,Bb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*A*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^2*sqrt(b)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 + a)*(8*B*a + (2*B*b*x + 3*A*b
)*x)
```

3.14 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)^(3/2))/(2
*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - (3*Sqrt[a]
*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.0845428, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {813, 844, 217, 206, 266, 63, 208}

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)^(3/2))/(2
*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - (3*Sqrt[a]
*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = -\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{(-4aB-4Abx)\sqrt{a+bx^2}}{x^2}\,dx$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{8aAb+8abBx}{x\sqrt{a+bx^2}}\,dx$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{2}(3aAb)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + \frac{1}{2}(3abB)\int$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{4}(3aAb)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right.$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}(3aA)\,\mathrm{S}$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{3}{2}\sqrt{a}Ab\,\mathrm{t}$$

Mathematica [C] time = 0.0356247, size = 90, normalized size = 0.81

$$\frac{Ab\left(a+bx^2\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^2} - \frac{aB\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] -((a*B*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a])) + (A*b*(a + b*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7
/2, 1 + (b*x^2)/a])/(5*a^2)
```

Maple [A] time = 0.007, size = 150, normalized size = 1.4

$$-\frac{A}{2\,ax^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{Ab}{2\,a}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{3\,Ab}{2}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right) + \frac{3\,Ab}{2}\sqrt{bx^2+a} - \frac{B}{ax}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{bBx}{a}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(5/2)+1/2*A*b/a*(b*x^2+a)^(3/2)-3/2*A*b*a^(1/2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/2*A*b*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(
5/2)+B*b/a*x*(b*x^2+a)^(3/2)+3/2*B*b*x*(b*x^2+a)^(1/2)+3/2*B*b^(1/2)*a*ln(x
*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71483, size = 1048, normalized size = 9.44

$$\left[\frac{3\,Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+3\,A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(Bbx^3+2\,Abx^2-2\,Bax-Aa\right)\cdots}{4\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3
*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B
*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/4*(6*B*a*sqrt(
-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(a)*b*x^2*log(-(b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x
- A*a)*sqrt(b*x^2 + a))/x^2, 1/4*(6*A*sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b
*x^2 + a)) + 3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/2*(3
*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(-a)*b*x^2*a
rctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqr
t(b*x^2 + a))/x^2]
```

Sympy [A] time = 6.86172, size = 182, normalized size = 1.64

$$-\frac{3A\sqrt{a}b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{B\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x*
*2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/
sqrt(a))/2
```

Giac [B] time = 1.24109, size = 258, normalized size = 2.32

$$\frac{3\,Aab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Ba\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{1}{2}\,(Bbx+2\,Ab)\sqrt{bx^2+a}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aab+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 3*A*a*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*B*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(B*b*x + 2*A*b)*sqrt(b
*x^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b + 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b - 2*B*a^3
*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=173

$$\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx)}{5040b^2}+$$

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2))/(128*b^
2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a + b*x^2)^(7/2))/(9*b
) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(160*A + 189*B*x)*(a + b*x^2)^(7/
2))/(5040*b^2) + (3*a^5*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2
))
```

Rubi [A] time = 0.104559, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {833, 780, 195, 217, 206}

$$\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx)}{5040b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2))/(128*b^
2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a + b*x^2)^(7/2))/(9*b
) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(160*A + 189*B*x)*(a + b*x^2)^(7/
2))/(5040*b^2) + (3*a^5*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2
))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}dx = \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} + \frac{\int x^2(-3aB+10Abx)\left(a+bx^2\right)^{5/2}dx}{10b}$$
$$= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} + \frac{\int x(-20aAb-27abBx)\left(a+bx^2\right)^{5/2}dx}{90b^2}$$
$$= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{\left(3a^2B\right)\int\left(a+\right.}{80b^2}$$
$$= \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)}{5040b^2}$$
$$= \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} - \frac{a(160A}{}$$
$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a\right.}{1}$$
$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a\right.}{1}$$
$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a\right.}{1}$$

Mathematica [A] time = 0.276433, size = 145, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(24a^2b^2x^4(800A+651Bx)+10a^3bx^2(128A+63Bx)-5a^4(512A+189Bx)+16ab^3x^6(1520A+1323Bx)\right.\right.}{80640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(896*b^4*x^8*(10*A + 9*B*x) + 10*a^3*b*x^2*(128*A
 + 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*b^2*x^4*(800*A + 651*B*x) + 1
6*a*b^3*x^6*(1520*A + 1323*B*x)) + (945*a^(9/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[1 + (b*x^2)/a]))/(80640*b^(5/2))
```

Maple [A] time = 0.008, size = 153, normalized size = 0.9

$$\frac{Bx^3}{10\,b}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{3\,Bax}{80\,b^2}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{a^2Bx}{160\,b^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{a^3Bx}{128\,b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,Ba^4x}{256\,b^2}\sqrt{bx^2+a} + \frac{3\,Ba^5}{256}\ln\left(x\sqrt{b}+\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/10*B*x^3*(b*x^2+a)^(7/2)/b-3/80*B/b^2*a*x*(b*x^2+a)^(7/2)+1/160*B/b^2*a^2
*x*(b*x^2+a)^(5/2)+1/128*B/b^2*a^3*x*(b*x^2+a)^(3/2)+3/256*B/b^2*a^4*x*(b*x
^2+a)^(1/2)+3/256*B/b^(5/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/9*A*x^2*(b*
x^2+a)^(7/2)/b-2/63*A*a/b^2*(b*x^2+a)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7001, size = 775, normalized size = 4.48

$$\left[\frac{945\,Ba^5\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6+15624\,B\right.}{161280\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/161280*(945*B*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(8064*B*b^5*x^9 + 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*
x^6 + 15624*B*a^2*b^3*x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*
A*a^3*b^2*x^2 - 945*B*a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3, -1/8064
0*(945*B*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*B*b^5*x^9
+ 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*x^6 + 15624*B*a^2*b^3*
x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*A*a^3*b^2*x^2 - 945*B*
a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 30.4327, size = 469, normalized size = 2.71

$$Aa^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for } \\ \frac{\sqrt{a}x^6}{6} & \text{oth}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x*
*2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
2*A*a*b*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a
+ b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**
2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + A*b**2*Piecewise((-16*a**4*sqrt(
a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x
**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqr
t(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)) - 3*B*a**(9/2)*x/(256*b
**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a)) + 129*
B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*sqrt(1
 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*a**5*a
sinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqrt(1 +
b*x**2/a))
```

Giac [A] time = 1.23683, size = 189, normalized size = 1.09

$$-\frac{3\,Ba^5\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}-\frac{1}{80640}\left(\frac{2560\,Aa^4}{b^2}+\left(\frac{945\,Ba^4}{b^2}-2\left(\frac{640\,Aa^3}{b}+\left(\frac{315\,Ba^3}{b}+4\left(2400\,Aa^2+\left(1953\,Ba^2\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*B*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/80640*(2560
*A*a^4/b^2 + (945*B*a^4/b^2 - 2*(640*A*a^3/b + (315*B*a^3/b + 4*(2400*A*a^2
 + (1953*B*a^2 + 2*(1520*A*a*b + 7*(189*B*a*b + 8*(9*B*b^2*x + 10*A*b^2)*x)
*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{B}{}$$

```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/2))/(192*b
) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^2)^(7/2))/(9*b) - ((
16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^2) - (5*a^4*A*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.0694944, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {833, 780, 195, 217, 206}

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{B}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```


```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/2))/(192*b
) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^2)^(7/2))/(9*b) - ((
16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^2) - (5*a^4*A*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```


Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{\int x(-2aB+9Abx)\left(a+bx^2\right)^{5/2}\,dx}{9b}$$
$$= \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b} - \frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2} - \frac{(aA)\int\left(a+bx^2\right)^{5/2}\,dx}{8b}$$
$$= -\frac{aAx\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b} - \frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2} - \frac{\left(5a^2A\right)\int\left(a+b\right.}{48b}$$
$$= -\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b} - \frac{aAx\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b} - \frac{(16aB-63Abx)\left(a+bx^2\right)}{504b^2}$$
$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b} - \frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b} - \frac{aAx\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b} - \frac{(16aB}{}$$
$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b} - \frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b} - \frac{aAx\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b} - \frac{(16aB}{}$$
$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b} - \frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b} - \frac{aAx\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b} - \frac{(16aB}{}$$

Mathematica [A] time = 0.259423, size = 131, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(6a^2b^2x^3(413A+320Bx)+a^3bx(315A+128Bx)-\frac{315a^{7/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-256a^4B+8ab^3x^5(357A+304Bx)+\right.}{8064b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-256*a^4*B + 112*b^4*x^7*(9*A + 8*B*x) + a^3*b*x*(315*A +
 128*B*x) + 8*a*b^3*x^5*(357*A + 304*B*x) + 6*a^2*b^2*x^3*(413*A + 320*B*x)
 - (315*a^(7/2)*A*Sqrt[b]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]
))/(8064*b^2)
```

Maple [A] time = 0.006, size = 132, normalized size = 0.9

$$\frac{Bx^2}{9\,b}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{2\,Ba}{63\,b^2}\left(bx^2+a\right)^{\frac{7}{2}} + \frac{Ax}{8\,b}\left(bx^2+a\right)^{\frac{7}{2}} - \frac{aAx}{48\,b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{5\,a^2Ax}{192\,b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{5\,a^3Ax}{128\,b}\sqrt{bx^2+a} - \frac{5\,Aa^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/9*B*x^2*(b*x^2+a)^(7/2)/b-2/63*B*a/b^2*(b*x^2+a)^(7/2)+1/8*A*x*(b*x^2+a)^
(7/2)/b-1/48*A/b*a*x*(b*x^2+a)^(5/2)-5/192*A/b*a^2*x*(b*x^2+a)^(3/2)-5/128*
A/b*a^3*x*(b*x^2+a)^(1/2)-5/128*A/b^(3/2)*a^4*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62046, size = 687, normalized size = 4.58

$$\left[\frac{315\,Aa^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1920\,Ba^2\right.}{16128\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16128*(315*A*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*b^3*x^5
+ 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x^2 + 315*A*a^3*b*x
 - 256*B*a^4)*sqrt(b*x^2 + a))/b^2, 1/8064*(315*A*a^4*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) + (896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6
 + 2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b
*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 21.0506, size = 442, normalized size = 2.95

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23A\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Ab^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}\\ \frac{\sqrt{a}x^4}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(384*sqrt(1
 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*A*sqrt(
a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + A*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewis
e((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + 2*B*a*b*Piecewi
se((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*
b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0))
, (sqrt(a)*x**6/6, True)) + B*b**2*Piecewise((-16*a**4*sqrt(a + b*x**2)/(31
5*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*
x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9
, Ne(b, 0)), (sqrt(a)*x**8/8, True))
```

Giac [A] time = 1.23044, size = 173, normalized size = 1.15

$$\frac{5\,Aa^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}-\frac{1}{8064}\left(\frac{256\,Ba^4}{b^2}-\left(\frac{315\,Aa^3}{b}+2\left(\frac{64\,Ba^3}{b}+\left(1239\,Aa^2+4\left(240\,Ba^2+\left(357\,Aab+2\left(15\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*A*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/8064*(256*B*
a^4/b^2 - (315*A*a^3/b + 2*(64*B*a^3/b + (1239*A*a^2 + 4*(240*B*a^2 + (357*
A*a*b + 2*(152*B*a*b + 7*(8*B*b^2*x + 9*A*b^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(b*
x^2 + a)
```

3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=126

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/2))/(192*b
) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(a + b*x^2)^(7/2))/(5
6*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.0435616, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/2))/(192*b
) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(a + b*x^2)^(7/2))/(5
6*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(aB)\int\left(a+bx^2\right)^{5/2}\,dx}{8b}$$

$$=-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{\left(5a^2B\right)\int\left(a+bx^2\right)^{3/2}\,dx}{48b}$$

$$=-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{\left(5a^3B\right)\int\sqrt{a+bx^2}}{64b}$$

$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-$$

$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-$$

$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-$$

Mathematica [A] time = 0.516493, size = 112, normalized size = 0.89

$$\frac{\left(a+bx^2\right)^{7/2}\left(-\frac{7aBx\left(\left(a+bx^2\right)\left(33a^2+26abx^2+8b^2x^4\right)+\frac{15a^{7/2}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}x}\right)}{\left(a+bx^2\right)^4}+384A+336Bx\right)}{2688b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(384*A + 336*B*x - (7*a*B*x*((a + b*x^2)*(33*a^2 + 26*a*
b*x^2 + 8*b^2*x^4) + (15*a^(7/2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sq
rt[a]])/(Sqrt[b]*x)))/(a + b*x^2)^4))/(2688*b)
```

Maple [A] time = 0.006, size = 113, normalized size = 0.9

$$\frac{Bx}{8\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Bax}{48\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,a^2Bx}{192\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,a^3Bx}{128\,b}\sqrt{bx^2+a}-\frac{5\,Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{A}{7\,b}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/8*B*x*(b*x^2+a)^(7/2)/b-1/48*B/b*a*x*(b*x^2+a)^(5/2)-5/192*B/b*a^2*x*(b*x
^2+a)^(3/2)-5/128*B/b*a^3*x*(b*x^2+a)^(1/2)-5/128*B/b^(3/2)*a^4*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))+1/7*A/b*(b*x^2+a)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66048, size = 635, normalized size = 5.04

$$\left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(336\,Bb^4x^7+384\,Ab^4x^6+952\,Bab^3x^5+1152\,Aab^3x^4+826\,Ba^2b^2x^3\right.}{5376\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/5376*(105*B*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 + 8
26*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(b
*x^2 + a))/b^2, 1/2688*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) + (336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 +
826*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(
b*x^2 + a))/b^2]
```

Sympy [A] time = 19.8831, size = 354, normalized size = 2.81

$$Aa^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2} \\ \frac{\sqrt{a}x^6}{6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), Tr
ue)) + 2*A*a*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(
a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, T
rue)) + A*b**2*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*
sqrt(a + b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a
+ b*x**2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + 5*B*a**(7/2)*x/(128*b*sqr
t(1 + b*x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**
(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 +
 b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**
9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.22827, size = 154, normalized size = 1.22

$$\frac{5\,Ba^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{2688}\left(\frac{384\,Aa^3}{b}+\left(\frac{105\,Ba^3}{b}+2\left(576\,Aa^2+\left(413\,Ba^2+4\left(144\,Aab+\left(119\,Bab+6\left(7\,B\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/2688*(384*A*
a^3/b + (105*B*a^3/b + 2*(576*A*a^2 + (413*B*a^2 + 4*(144*A*a*b + (119*B*a*
b + 6*(7*B*b^2*x + 8*A*b^2)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 + (A*x*(a +
 b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3*A*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi [A] time = 0.0369465, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 + (A*x*(a +
 b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3*A*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (A+Bx)\left(a+bx^2\right)^{5/2} dx = \frac{B\left(a+bx^2\right)^{7/2}}{7b} + A\int \left(a+bx^2\right)^{5/2} dx$$

$$= \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{6}(5aA)\int \left(a+bx^2\right)^{3/2} dx$$

$$= \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{8}\left(5a^2A\right)\int \sqrt{a+bx^2}\, dx$$

$$= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{16}\left(5a^3A\right.$$

$$= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{16}\left(5a^3A\right.$$

$$= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{5a^3A\tan}{1}$$

Mathematica [A] time = 0.0805573, size = 108, normalized size = 1.01

$$\frac{\sqrt{a+bx^2}\left(3a^2bx(77A+48Bx)+48a^3B+2ab^2x^3(91A+72Bx)+8b^3x^5(7A+6Bx)\right)+105a^3A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+b\right.}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(77*A + 48
*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) + 105*a^3*A*Sqrt[b]*Log[b*x + Sqrt[b]*
```

```
Sqrt[a + b*x^2]])/(336*b)
```

Maple [A] time = 0.005, size = 85, normalized size = 0.8

$$\frac{B}{7b}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Ax}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,aAx}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2Ax}{16}\sqrt{bx^2+a}+\frac{5\,Aa^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/7*B*(b*x^2+a)^(7/2)/b+1/6*A*x*(b*x^2+a)^(5/2)+5/24*A*a*x*(b*x^2+a)^(3/2)+
5/16*A*a^2*x*(b*x^2+a)^(1/2)+5/16*A*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58966, size = 549, normalized size = 5.13

$$\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+182\,Aab^2x^3+144\,Ba^2bx^2+23\right.}{672\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*A*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B
```

```
*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x^2 + a))/b, -1/336*(105*A*a^
3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*B*b^3*x^6 + 56*A*b^3*x^
5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*x + 4
8*B*a^3)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 12.5701, size = 348, normalized size = 3.25

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{f}\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{o}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b*x**2/a))
+ 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*sqrt(a)*b**2*x**5/(24
*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + A*b
**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + 2*B*a*b*Piecewise((-2*a**2
*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a
+ b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b**2*Piecewise((8*a**3*
sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + a*x
**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*
x**6/6, True))
```

Giac [A] time = 1.21431, size = 136, normalized size = 1.27

$$-\frac{5\,Aa^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,Ba^3}{b}+\left(231\,Aa^2+2\left(72\,Ba^2+\left(91\,Aab+4\left(18\,Bab+\left(6\,Bb^2x+7\,Ab^2\right)x\right)x\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*B*a^
3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b + (6*B*b^2*x + 7*A
*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.19 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$\frac{1}{16}a^2\sqrt{a+bx^2}(16A+5Bx)-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{30}\left(a\right.$$

```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b*x^2)^(3/2
))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

Rubi [A] time = 0.158863, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$\frac{1}{16}a^2\sqrt{a+bx^2}(16A+5Bx)-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{30}\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```


```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b*x^2)^(3/2
))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```


Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} + \frac{\int \frac{(6aAb+5abBx)\left(a+bx^2\right)^{3/2}}{x}\,dx}{6b}$$

$$= \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} + \frac{\int \frac{\left(24a^2Ab^2+15a^2b^2Bx\right)\sqrt{a+bx^2}}{x}\,dx}{24b^2}$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} +$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} +$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} +$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} +$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} +$$

Mathematica [A] time = 0.328817, size = 139, normalized size = 1.05

$$\frac{1}{240}\sqrt{a+bx^2}\left(a^2(368A+165Bx)+2abx^2(88A+65Bx)+8b^2x^4(6A+5Bx)\right) - a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{5a^{7/2}B\sqrt{\frac{bx^2}{a}}}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(88*A + 65*B*x) + a^2
*(368*A + 165*B*x)))/240 + (5*a^(7/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b
]*x)/Sqrt[a]])/(16*Sqrt[b]*Sqrt[a + b*x^2]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]]
```

Maple [A] time = 0.005, size = 138, normalized size = 1.1

$$\frac{Bx}{6}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{5\,Bax}{24}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{5\,a^2Bx}{16}\sqrt{bx^2+a} + \frac{5\,Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} + \frac{A}{5}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{Aa}{3}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x,x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)+5/24*B*a*x*(b*x^2+a)^(3/2)+5/16*B*a^2*x*(b*x^2+a)^(
1/2)+5/16*B*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/5*A*(b*x^2+a)^(5/2)
+1/3*a*A*(b*x^2+a)^(3/2)-A*a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+a^
2*A*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74486, size = 1353, normalized size = 10.25

$$\left[\frac{75\,Ba^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+240\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab\right.}{480\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/480*(75*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
240*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(40
*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b
*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - 120*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A
*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, 1/480*(480*A*
sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 75*B*a^3*sqrt(b)*log(-2*b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*B*b^3*x^5 + 48*A*b^3*x^4 +
```

```
130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2
 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 24
0*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*
b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*
sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 24.4322, size = 323, normalized size = 2.45

$$-Aa^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+2Aab\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\cdots}{} \\ \frac{\sqrt{a}x^4}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

```
[Out] -A*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**3/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*A*a*b*Piecewise((sqrt(a)*
x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + A*b**2*Piecewise((-
2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*s
qrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*a**(5/2)*x*sqrt(1
 + b*x**2/a)/2 + 3*B*a**(5/2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x
**3/(48*sqrt(1 + b*x**2/a)) + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)
) + 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)
*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.18606, size = 169, normalized size = 1.28

$$\frac{2\,Aa^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{240}\left(368\,Aa^2+\left(165\,Ba^2+2\left(88\,Aab+\left(65\,Bab+4\left(5\cdots\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 5/16*B*a
^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/240*(368*A*a^2 + (165
*B*a^2 + 2*(88*A*a*b + (65*B*a*b + 4*(5*B*b^2*x + 6*A*b^2)*x)*x)*x)*x)*sqrt
```

```
(b*x^2 + a)
```

3.20 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=136

$$\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{8}a\sqrt{a+bx^2}(8aB+15Abx)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+$$

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a + b*x^2)^
(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15*a^2*A*Sqrt[b]*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]]
```

Rubi [A] time = 0.132421, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{8}a\sqrt{a+bx^2}(8aB+15Abx)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a + b*x^2)^
(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15*a^2*A*Sqrt[b]*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = -\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}-\frac{1}{2}\int \frac{(-2aB-10Abx)\left(a+bx^2\right)^{3/2}}{x}\,dx$$

$$= \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}-\frac{\int \frac{\left(-8a^2bB-30aAb^2x\right)\sqrt{a+bx^2}}{x}\,dx}{8b}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}-\int$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{1}{8}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{1}{8}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{15}{8}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{15}{8}$$

Mathematica [C] time = 0.222027, size = 117, normalized size = 0.86

$$-\frac{a^3A\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{a+bx^2}}+\frac{1}{15}B\sqrt{a+bx^2}\left(23a^2+11abx^2+3b^2x^4\right)-a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (B*Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]] - (a^3*A*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-
5/2, -1/2, 1/2, -((b*x^2)/a)])/(x*Sqrt[a + b*x^2])
```

Maple [A] time = 0.009, size = 158, normalized size = 1.2

$$\frac{B}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ba}{3}\left(bx^2+a\right)^{\frac{3}{2}}-Ba^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)+B\sqrt{bx^2+a}a^2-\frac{A}{ax}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Abx}{a}\left(bx^2+a\right)^{\frac{5}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^2,x)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)+1/3*B*a*(b*x^2+a)^(3/2)-B*a^(5/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)+B*(b*x^2+a)^(1/2)*a^2-A/a/x*(b*x^2+a)^(7/2)+A*b/a*x*(b*x
^2+a)^(5/2)+5/4*A*b*x*(b*x^2+a)^(3/2)+15/8*A*b*a*x*(b*x^2+a)^(1/2)+15/8*A*b
^(1/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70425, size = 1319, normalized size = 9.7

$$\left[\frac{225\,Aa^2\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+120\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(24\,Bb^2x^5+30\,Ab^2x^4+88\,Ba\right.}{240\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 120*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x -
 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(225*A*a^2*sqrt(-b)*x*arctan(sqrt(-b
```

```
)*x/sqrt(b*x^2 + a)) - 60*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^
2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/240*(240*B*sqrt(-a)*a^2*
x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x
^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(2
25*A*a^2*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*B*sqrt(-a)*a^2
*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a
*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x]
```

Sympy [A] time = 10.135, size = 318, normalized size = 2.34

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} + Aa^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}} - \frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{3A\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{15Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8} + \frac{Ab^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*
A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**2*x**3/(8*sqrt(1 + b
*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8 + A*b**3*x**5/(4*s
qrt(a)*sqrt(1 + b*x**2/a)) - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3
/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) +
 2*B*a*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b),
True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt
(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4,
True))
```

Giac [A] time = 1.21472, size = 203, normalized size = 1.49

$$\frac{2\,Ba^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8}\,Aa^2\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{120}\left(184\,Ba^2 + \left(135\,Aab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*A*a
^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^3*sqrt(b)/((sqrt(
b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*(184*B*a^2 + (135*A*a*b + 2*(44*B*a*
b + 3*(4*B*b^2*x + 5*A*b^2)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.21 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=141

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-2Abx)}{12x}+$$

```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a + b*x^2)^
(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2) + (15*a^2*Sqrt[b]*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5*a^(3/2)*A*b*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.117082, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-2Abx)}{12x}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```


```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a + b*x^2)^
(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2) + (15*a^2*Sqrt[b]*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5*a^(3/2)*A*b*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/2
```


Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = -\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} - \frac{5}{16}\int \frac{(-4aB-8Abx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{5}{32}\int \frac{(16aAb+24abBx)\sqrt{a+b}}{x}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{5\int \frac{32a}{}}{}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{1}{2}\left(5a^2\right.$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{1}{4}\left(5a^2\right.$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{15}{8}a^2\sqrt{}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{15}{8}a^2\sqrt{}$$

Mathematica [C] time = 0.0259002, size = 92, normalized size = 0.65

$$\frac{Ab\left(a+bx^2\right)^{7/2}{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^2} - \frac{a^2B\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] -((a^2*B*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/
(x*Sqrt[1 + (b*x^2)/a])) + (A*b*(a + b*x^2)^(7/2)*Hypergeometric2F1[2, 7/2,
 9/2, 1 + (b*x^2)/a])/(7*a^2)
```

Maple [A] time = 0.007, size = 181, normalized size = 1.3

$$-\frac{A}{2\,a x^{2}}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{A b}{2\,a}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{5\,A b}{6}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{5\,A b}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)+\frac{5\,A a b}{2}\sqrt{b x^{2}+a}-\frac{B}{a x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(7/2)/a/x^2+1/2*A*b/a*(b*x^2+a)^(5/2)+5/6*A*b*(b*x^2+a)^(3
/2)-5/2*A*b*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+5/2*A*b*a*(b*x^2+
a)^(1/2)-B/a/x*(b*x^2+a)^(7/2)+B*b/a*x*(b*x^2+a)^(5/2)+5/4*B*b*x*(b*x^2+a)^
(3/2)+15/8*B*b*a*x*(b*x^2+a)^(1/2)+15/8*B*b^(1/2)*a^2*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70185, size = 1316, normalized size = 9.33

$$\left[\frac{45\,B a^{2}\sqrt{b}x^{2}\log\left(-2\,b x^{2}-2\,\sqrt{b x^{2}+a}\sqrt{b}x-a\right)+60\,A a^{\frac{3}{2}}b x^{2}\log\left(-\frac{b x^{2}-2\,\sqrt{b x^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(6\,B b^{2}x^{5}+8\,A b^{2}x^{4}+27\,B a b\right.}{48\,x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/48*(45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 60*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x -
```

```
12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 30*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x
^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/48*(120*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*
x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45
*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*
b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] time = 11.3184, size = 279, normalized size = 1.98

$$-\frac{5Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} - \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} + Ab^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right) - \frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} + Ba^{\frac{3}{2}}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a +
 b*x**2)**(3/2)/(3*b), True)) - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3
/2)*b*x*sqrt(1 + b*x**2/a) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*
sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*
x/sqrt(a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.18631, size = 296, normalized size = 2.1

$$\frac{5\,Aa^2b\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8}\,Ba^2\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{1}{24}\left(56\,Aab+\left(27\,Bab+2\left(3\,Bb^2x+4\,Ab^2\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 5*A*a^2*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*B
*a^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/24*(56*A*a*b + (27*
B*a*b + 2*(3*B*b^2*x + 4*A*b^2)*x)*x)*sqrt(b*x^2 + a) + ((sqrt(b)*x - sqrt(
b*x^2 + a))^3*A*a^2*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) + (
sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b - 2*B*a^4*sqrt(b))/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^2
```

3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (a*(16*A +
9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.0772985, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {833, 780, 217, 206}

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (a*(16*A +
9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(8*b^(5/2))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^3\sqrt{a+bx^2}}{4b}+\frac{\int \frac{x^2(-3aB+4Abx)}{\sqrt{a+bx^2}}\,dx}{4b}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}+\frac{\int \frac{x(-8aAb-9abBx)}{\sqrt{a+bx^2}}\,dx}{12b^2}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{\left(3a^2B\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{\left(3a^2B\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0436885, size = 76, normalized size = 0.73

$$
\frac{9a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\sqrt{b}\sqrt{a+bx^2}\left(-16aA-9aBx+8Abx^2+6bBx^3\right)}{24b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) + 9*a^
2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(24*b^(5/2))
```

Maple [A] time = 0.007, size = 96, normalized size = 0.9

$$\frac{x^3B}{4\,b}\sqrt{bx^2+a}-\frac{3\,Bax}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,Ba^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{Ax^2}{3\,b}\sqrt{bx^2+a}-\frac{2\,Aa}{3\,b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^3*(b*x^2+a)^(1/2)/b-3/8*B/b^2*a*x*(b*x^2+a)^(1/2)+3/8*B/b^(5/2)*a^2
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/3*A*x^2*(b*x^2+a)^(1/2)/b-2/3*A*a/b^2*(b*x
^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58804, size = 385, normalized size = 3.7

$$\left[\frac{9\,Ba^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6\,Bb^2x^3+8\,Ab^2x^2-9\,Babx-16\,Aab\right)\sqrt{bx^2+a}}{48\,b^3},-\frac{9\,Ba^2\sqrt{-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3, -1
/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8
*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 5.28368, size = 150, normalized size = 1.44

$$A\left(\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}\right)-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b),
Ne(b, 0)), (x**4/(4*sqrt(a)), True)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x*
*2/a)) - B*sqrt(a)*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x
/sqrt(a))/(8*b**(5/2)) + B*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.23076, size = 100, normalized size = 0.96

$$\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3Bx}{b}+\frac{4A}{b}\right)x-\frac{9Ba}{b^2}\right)x-\frac{16Aa}{b^2}\right)-\frac{3Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x^2 + a)*((2*(3*B*x/b + 4*A/b)*x - 9*B*a/b^2)*x - 16*A*a/b^2) -
 3/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=81

$$-\frac{\sqrt{a+bx^2}(4aB-3Abx)}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2])/(6*b^2)
 - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.0421102, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {833, 780, 217, 206}

$$-\frac{\sqrt{a+bx^2}(4aB-3Abx)}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2])/(6*b^2)
 - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^2\sqrt{a+bx^2}}{3b}+\frac{\int \frac{x(-2aB+3Abx)}{\sqrt{a+bx^2}}\,dx}{3b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{(aA)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.035785, size = 64, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}(bx(3A+2Bx)-4aB)-3aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*a*B + b*x*(3*A + 2*B*x)) - 3*a*A*Sqrt[b]*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(6*b^2)
```

Maple [A] time = 0.005, size = 75, normalized size = 0.9

$$\frac{Bx^2}{3\,b}\sqrt{bx^2+a}-\frac{2\,Ba}{3\,b^2}\sqrt{bx^2+a}+\frac{Ax}{2\,b}\sqrt{bx^2+a}-\frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*B*x^2*(b*x^2+a)^(1/2)/b-2/3*B*a/b^2*(b*x^2+a)^(1/2)+1/2*A*x/b*(b*x^2+a)
^(1/2)-1/2*A*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57618, size = 320, normalized size = 3.95

$$\left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,Bbx^2+3\,Abx-4\,Ba\right)\sqrt{bx^2+a}}{12\,b^2},\ \frac{3\,Aa\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(2\,Bbx^2\right.}{6\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a))/b^2, 1/6*(3*A*a*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 +
a))/b^2]
```

Sympy [A] time = 3.52792, size = 94, normalized size = 1.16

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+B\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + B*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)
/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A] time = 1.21206, size = 82, normalized size = 1.01

$$\frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Bx}{b}+\frac{3\,A}{b}\right)x-\frac{4\,Ba}{b^2}\right)+\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*((2*B*x/b + 3*A/b)*x - 4*B*a/b^2) + 1/2*A*a*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{a+bx^2}(2A+Bx)}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.0230157, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {780, 217, 206}

$$\frac{\sqrt{a+bx^2}(2A+Bx)}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{(aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.0321117, size = 57, normalized size = 1.02

$$\frac{\sqrt{b}\sqrt{a+bx^2}(2A+Bx) - aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*(2*A + B*x)*Sqrt[a + b*x^2] - a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

Maple [A] time = 0.004, size = 55, normalized size = 1.

$$\frac{Bx}{2\,b}\sqrt{bx^2+a} - \frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} + \frac{A}{b}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*x/b*(b*x^2+a)^(1/2)-1/2*B*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+A/b
*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79326, size = 275, normalized size = 4.91

$$\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\,(Bbx + 2\,Ab)\sqrt{bx^2+a}}{4\,b^2}, \frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) + (Bbx + 2\,Ab)\sqrt{bx^2+a}}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b*
x + 2*A*b)*sqrt(b*x^2 + a))/b^2, 1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 3.2929, size = 70, normalized size = 1.25

$$A\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b = 0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right) + \frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b} - \frac{Ba\,\text{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + B*s
qrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2
))
```

Giac [A] time = 1.18342, size = 68, normalized size = 1.21

$$\frac{1}{2}\,\sqrt{bx^2+a}\left(\frac{Bx}{b}+\frac{2\,A}{b}\right)+\frac{Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*(B*x/b + 2*A/b) + 1/2*B*a*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0149365, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {641, 217, 206}

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx &= \frac{B\sqrt{a+bx^2}}{b}+A\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{B\sqrt{a+bx^2}}{b}+A\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{B\sqrt{a+bx^2}}{b}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0294159, size = 46, normalized size = 1.07

$$\frac{A\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] time = 0.004, size = 37, normalized size = 0.9

$$\frac{B}{b}\sqrt{bx^2+a}+A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] B*(b*x^2+a)^(1/2)/b+A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77601, size = 223, normalized size = 5.19

$$\left[\frac{A\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}B}{2\,b},-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}B}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*
x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x^2
 + a)*B)/b]
```

Sympy [B] time = 1.01588, size = 102, normalized size = 2.37

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+B\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sq
rt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x
*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + B*Piecewise((x**2/(2*sqrt(a)),
 Eq(b, 0)), (sqrt(a + b*x**2)/b, True))
```

Giac [A] time = 1.18092, size = 53, normalized size = 1.23

$$-\frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0397549, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {844, 217, 206, 266, 63, 208}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx &= A\int \frac{1}{x\sqrt{a+bx^2}}\,dx + B\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0137493, size = 53, normalized size = 1.

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.006, size = 52, normalized size = 1.

$$B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5845, size = 683, normalized size = 12.89

$$\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\,ab},-\frac{2\,Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}b\log\left(-\frac{b}{}\right.}{2\,ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A*sqrt(
a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), -1/2*(2*B*
a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*log(-(b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), 1/2*(2*A*sqrt(-a)*b*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a))/(a*b), -(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqr
t(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a*b)]
```

Sympy [A] time = 2.40735, size = 99, normalized size = 1.87

$$-\frac{A\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((sqrt(-a/b)*asin(x*sqrt
(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a),
 (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a
 < 0)))
```

Giac [A] time = 1.21745, size = 78, normalized size = 1.47

$$\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0325682, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {807, 266, 63, 208}

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{ax}+B\int \frac{1}{x\sqrt{a+bx^2}}\,dx \\
&= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right) \\
&= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b} \\
&= -\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0146143, size = 47, normalized size = 1.

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.007, size = 49, normalized size = 1.

$$-B\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{A}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-A*(b*x^2+a)^(1/2)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55569, size = 240, normalized size = 5.11

$$\left[\frac{B\sqrt{a}x\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}A}{2ax}, \frac{B\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}A}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*s
qrt(b*x^2 + a)*A)/(a*x), (B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - s
qrt(b*x^2 + a)*A)/(a*x)]
```

Sympy [A] time = 2.14908, size = 41, normalized size = 0.87

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}-\frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Giac [A] time = 1.18573, size = 88, normalized size = 1.87

$$\frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\,A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.28 $\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=72

$$\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] time = 0.0531302, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {835, 807, 266, 63, 208}

$$\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{\int \frac{-2aB+Abx}{x^2\sqrt{a+bx^2}}\,dx}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.101697, size = 63, normalized size = 0.88

$$\frac{\sqrt{a+bx^2}\left(\frac{Ab\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}-\frac{a(A+2Bx)}{x^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((a*(A + 2*B*x))/x^2) + (A*b*ArcTanh[Sqrt[1 + (b*x^2)/a]
])/Sqrt[1 + (b*x^2)/a]))/(2*a^2)
```

Maple [A] time = 0.008, size = 68, normalized size = 0.9

$$-\frac{A}{2\,ax^2}\sqrt{bx^2+a}+\frac{Ab}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{B}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*(b*x^2+a)^(1/2)/a/x^2+1/2*A*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1
/2))/x)-B*(b*x^2+a)^(1/2)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56395, size = 306, normalized size = 4.25

$$\left[\frac{A\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2(2Bax+Aa)\sqrt{bx^2+a}}{4a^2x^2}, -\frac{A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2Bax+Aa)\sqrt{bx^2+a}}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(A*sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2), -1/2*(A*sqrt(-a)*b*x^2*arcta
n(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2)]
```

Sympy [A] time = 3.15571, size = 66, normalized size = 0.92

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [B] time = 1.26266, size = 197, normalized size = 2.74

$$-\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt(
b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(
b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^2*a)
```

3.29 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=81

$$\frac{\sqrt{a+bx^2}(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a + b*x^2])/(2
*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] time = 0.0434866, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {819, 780, 217, 206}

$$\frac{\sqrt{a+bx^2}(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a + b*x^2])/(2
*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{\int \frac{x(2aA+3aBx)}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0522584, size = 72, normalized size = 0.89

$$\frac{a(4A+3Bx)+bx^2(2A+Bx)}{2b^2\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (b*x^2*(2*A + B*x) + a*(4*A + 3*B*x))/(2*b^2*Sqrt[a + b*x^2]) - (3*a*B*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Maple [A] time = 0.008, size = 93, normalized size = 1.2

$$\frac{x^3B}{2b}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,Bax}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{Ax^2}{b}\frac{1}{\sqrt{bx^2+a}}+2\,\frac{Aa}{b^2\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*B*x^3/b/(b*x^2+a)^(1/2)+3/2*B/b^2*a*x/(b*x^2+a)^(1/2)-3/2*B/b^(5/2)*a*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))+A*x^2/b/(b*x^2+a)^(1/2)+2*A*a/b^2/(b*x^2+a)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59985, size = 448, normalized size = 5.53

$$\left[\frac{3\left(Babx^2+Ba^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(Bb^2x^3+2\,Ab^2x^2+3\,Babx+4\,Aab\right)\sqrt{bx^2+a}}{4\left(b^4x^2+ab^3\right)},\frac{3\left(Babx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*x^2 + a
))/(b^4*x^2 + a*b^3), 1/2*(3*(B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*
```

```
x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 7.67201, size = 117, normalized size = 1.44

$$A\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a
)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 +
 b*x**2/a)))
```

Giac [A] time = 1.21729, size = 95, normalized size = 1.17

$$\frac{\left(\left(\frac{Bx}{b}+\frac{2\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{4\,Aa}{b^2}}{2\,\sqrt{bx^2+a}}+\frac{3\,Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((B*x/b + 2*A/b)*x + 3*B*a/b^2)*x + 4*A*a/b^2)/sqrt(b*x^2 + a) + 3/2*B
*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.30 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2 + (A*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] time = 0.0363449, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {819, 641, 217, 206}

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2 + (A*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{\int \frac{aA+2aBx}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0342738, size = 67, normalized size = 1.02

$$
\frac{A\sqrt{b}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+2aB+bx(Bx-A)}{b^2\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*B + b*x*(-A + B*x) + A*Sqrt[b]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.006, size = 72, normalized size = 1.1

$$\frac{Bx^2}{b}\frac{1}{\sqrt{bx^2+a}}+2\frac{Ba}{b^2\sqrt{bx^2+a}}-\frac{Ax}{b}\frac{1}{\sqrt{bx^2+a}}+A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] B*x^2/b/(b*x^2+a)^(1/2)+2*B*a/b^2/(b*x^2+a)^(1/2)-A*x/b/(b*x^2+a)^(1/2)+A/b
^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59525, size = 369, normalized size = 5.59

$$\left[\frac{\left(Abx^2+Aa\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(Bbx^2-Abx+2\,Ba\right)\sqrt{bx^2+a}}{2\left(b^3x^2+ab^2\right)},-\frac{\left(Abx^2+Aa\right)\sqrt{-b}\arctan\left(\cdots\right.}{b\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - A*b*x + 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((A*b
*x^2 + A*a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - A*b*x
+ 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2)]
```

Sympy [A] time = 5.85696, size = 83, normalized size = 1.26

$$A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True))
```

Giac [A] time = 1.22896, size = 78, normalized size = 1.18

$$\frac{\left(\frac{Bx}{b}-\frac{A}{b}\right)x+\frac{2\,Ba}{b^2}}{\sqrt{bx^2+a}}-\frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((B*x/b - A/b)*x + 2*B*a/b^2)/sqrt(b*x^2 + a) - A*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(3/2)
```

3.31 $\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

```
[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/b^(3/2)
```

Rubi [A] time = 0.0201247, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {778, 217, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/b^(3/2)
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$

$$= -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$

$$= -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0536328, size = 64, normalized size = 1.33

$$\frac{\sqrt{a}B\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{b}(A+Bx)}{b^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*(A + B*x)) + Sqrt[a]*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/S
qrt[a]])/(b^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 54, normalized size = 1.1

$$-\frac{Bx}{b}\frac{1}{\sqrt{bx^2+a}} + B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} - \frac{A}{b}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -B*x/b/(b*x^2+a)^(1/2)+B/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-A/b/(b*x^2+a
)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58117, size = 336, normalized size = 7.

$$\left[\frac{\left(Bbx^2 + Ba\right)\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) - 2\,(Bbx + Ab)\sqrt{bx^2 + a}}{2\left(b^3x^2 + ab^2\right)}, -\frac{\left(Bbx^2 + Ba\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) + (Bb}{b^3x^2 + ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^2 + B*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) - 2*(B*b*x + A*b)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((B*b*x^2 + B*a)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*x + A*b)*sqrt(b*x^2 + a)
)/(b^3*x^2 + a*b^2)]
```

Sympy [A] time = 4.69385, size = 66, normalized size = 1.38

$$A\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1 + \frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
 + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

<u>Giac</u> [A] time = 1.19577, size = 65, normalized size = 1.35

$$-\frac{\frac{Bx}{b}+\frac{A}{b}}{\sqrt{bx^2+a}}-\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*x/b + A/b)/sqrt(b*x^2 + a) - B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(3/2)
```

3.32 $\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=28

$$-\frac{aB - Abx}{ab\sqrt{a + bx^2}}$$

```
[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rubi [A] time = 0.006717, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {637}

$$-\frac{aB - Abx}{ab\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{A + Bx}{\left(a + bx^2\right)^{3/2}} \, dx = -\frac{aB - Abx}{ab\sqrt{a + bx^2}}$$

Mathematica [A] time = 0.014167, size = 27, normalized size = 0.96

$$\frac{Abx - aB}{ab\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.001, size = 26, normalized size = 0.9

$$\frac{Abx - Ba}{ab}\frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

Maxima [A] time = 0.992055, size = 42, normalized size = 1.5

$$\frac{Ax}{\sqrt{bx^2 + a}a} - \frac{B}{\sqrt{bx^2 + a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

Fricas [A] time = 1.60548, size = 69, normalized size = 2.46

$$\frac{(Abx - Ba)\sqrt{bx^2 + a}}{ab^2x^2 + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

Sympy [A] time = 3.63549, size = 46, normalized size = 1.64

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(3/2)), True))
```

Giac [A] time = 1.20417, size = 31, normalized size = 1.11

$$\frac{\frac{Ax}{a}-\frac{B}{b}}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (A*x/a - B/b)/sqrt(b*x^2 + a)
```

3.33 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

Rubi [A] time = 0.0380149, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {823, 12, 266, 63, 208}

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{\int \frac{aAb}{x\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0272287, size = 47, normalized size = 1.

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

Maple [A] time = 0.007, size = 60, normalized size = 1.3

$$\frac{Bx}{a}\frac{1}{\sqrt{bx^2+a}}+\frac{A}{a}\frac{1}{\sqrt{bx^2+a}}-A\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(3/2),x)
```

```
[Out] B*x/a/(b*x^2+a)^(1/2)+A/a/(b*x^2+a)^(1/2)-A/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^
2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84966, size = 338, normalized size = 7.19

$$\left[\frac{\left(Abx^2+Aa\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(Bax+Aa\right)\sqrt{bx^2+a}}{2\left(a^2bx^2+a^3\right)}, \frac{\left(Abx^2+Aa\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(Bax+Aa\right)\sqrt{}}{a^2bx^2+a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^2 + a^3), ((A*b*x^2 + A*
a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (B*a*x + A*a)*sqrt(b*x^2 + a
))/(a^2*b*x^2 + a^3)]
```

Sympy [B] time = 5.90854, size = 206, normalized size = 4.38

$$A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*
x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/
2) + 2*a**(7/2)*b*x**2)) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.189, size = 80, normalized size = 1.7

$$\frac{\frac{Bx}{a}+\frac{A}{a}}{\sqrt{bx^2+a}}+\frac{2A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*x/a + A/a)/sqrt(b*x^2 + a) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*a)
```

3.34 $\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) - (B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0570752, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {823, 807, 266, 63, 208}

$$-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) - (B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{\int \frac{-2aAb-abBx}{x^2\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0376964, size = 72, normalized size = 1.03

$$-\frac{a(A-Bx)+\sqrt{a}Bx\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+2Abx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((2*A*b*x^2 + a*(A - B*x) + Sqrt[a]*B*x*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(a^2*x*Sqrt[a + b*x^2]))
```

Maple [A] time = 0.008, size = 80, normalized size = 1.1

$$\frac{B}{a}\frac{1}{\sqrt{bx^2+a}}-B\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{A}{ax}\frac{1}{\sqrt{bx^2+a}}-2\,\frac{Abx}{a^2\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] B/a/(b*x^2+a)^(1/2)-B/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-A/a/x/(
b*x^2+a)^(1/2)-2*A*b/a^2*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88477, size = 381, normalized size = 5.44

$$\left[\frac{\left(Bbx^3+Bax\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(2Abx^2-Bax+Aa\right)\sqrt{bx^2+a}}{2\left(a^2bx^3+a^3x\right)}, \frac{\left(Bbx^3+Bax\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(2A\right.}{a^2bx^3+a^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^3 + B*a*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(2*A*b*x^2 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)
, ((B*b*x^3 + B*a*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*A*b*x^2
 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)]
```

Sympy [B] time = 8.08795, size = 235, normalized size = 3.36

$$A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) +
 a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 +
b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)
/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
```

Giac [A] time = 1.22262, size = 130, normalized size = 1.86

$$-\frac{\frac{Abx}{a^2}-\frac{B}{a}}{\sqrt{bx^2+a}}+\frac{2B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{2A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(A*b*x/a^2 - B/a)/sqrt(b*x^2 + a) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a)
```

3.35 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}+\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*x^2) - (2*
B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^
(5/2))
```

Rubi [A] time = 0.0783209, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {823, 835, 807, 266, 63, 208}

$$-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}+\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*x^2) - (2*
B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^
(5/2))
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{\int \frac{-3aAb-2abBx}{x^3\sqrt{a+bx^2}}\,dx}{a^2b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} + \frac{\int \frac{4a^2bB-3aAb^2x}{x^2\sqrt{a+bx^2}}\,dx}{2a^3b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3A)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} + \frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.15961, size = 75, normalized size = 0.79

$$\frac{3Ab\sqrt{\frac{bx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right) - \frac{a(A+2Bx)}{x^2} - b(3A+4Bx)}{2a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-((a*(A + 2*B*x))/x^2) - b*(3*A + 4*B*x) + 3*A*b*Sqrt[1 + (b*x^2)/a]*ArcTa
nh[Sqrt[1 + (b*x^2)/a]])/(2*a^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 101, normalized size = 1.1

$$-\frac{A}{2\,ax^2}\frac{1}{\sqrt{bx^2+a}} - \frac{3\,Ab}{2\,a^2}\frac{1}{\sqrt{bx^2+a}} + \frac{3\,Ab}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}} - \frac{B}{ax}\frac{1}{\sqrt{bx^2+a}} - 2\,\frac{bBx}{a^2\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(1/2)-3/2*A*b/a^2/(b*x^2+a)^(1/2)+3/2*A*b/a^(5/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-B/a/x/(b*x^2+a)^(1/2)-2*B*b/a^2*x/(b*x^
2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87817, size = 474, normalized size = 4.99

$$\left[\frac{3\left(Ab^2x^4 + Aabx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right) - 2\left(4Babx^3 + 3Aabx^2 + 2Ba^2x + Aa^2\right)\sqrt{bx^2+a}}{4\left(a^3bx^4 + a^4x^2\right)}, -\frac{3\left(Ab^2x^4 + Aabx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) - 2*(4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x + A*a^2)*sqrt(b
*x^2 + a))/(a^3*b*x^4 + a^4*x^2), -1/2*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x +
 A*a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [A] time = 8.60718, size = 124, normalized size = 1.31

$$A\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + B\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(-1/(a*sq
rt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

<u>Giac</u> [B] time = 1.25495, size = 231, normalized size = 2.43

$$-\frac{\frac{Bbx}{a^2}+\frac{Ab}{a^2}}{\sqrt{bx^2+a}}-\frac{3\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*x/a^2 + A*b/a^2)/sqrt(b*x^2 + a) - 3*A*b*arctan(-(sqrt(b)*x - sqrt(b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b +
 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 +
a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

3.36 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] -(x^2*(A + B*x))/(3*b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*Sqrt[a + b*
x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi [A] time = 0.042459, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {819, 778, 217, 206}

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(A + B*x))/(3*b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*Sqrt[a + b*
x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
```

```
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{x(2aA+3aBx)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0764119, size = 69, normalized size = 0.87

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{a(2A+3Bx)+bx^2(3A+4Bx)}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*(2*A + 3*B*x) + b*x^2*(3*A + 4*B*x))/(3*b^2*(a + b*x^2)^(3/2)) + (B*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Maple [A] time = 0.008, size = 91, normalized size = 1.2

$$-\frac{x^3B}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Bx}{b^2}\frac{1}{\sqrt{bx^2+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}-\frac{Ax^2}{b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{2\,Aa}{3\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*B*x^3/b/(b*x^2+a)^(3/2)-B/b^2*x/(b*x^2+a)^(1/2)+B/b^(5/2)*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))-A*x^2/b/(b*x^2+a)^(3/2)-2/3*A*a/b^2/(b*x^2+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7397, size = 536, normalized size = 6.78

$$\left[\frac{3\left(Bb^2x^4+2\,Babx^2+Ba^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(4\,Bb^2x^3+3\,Ab^2x^2+3\,Babx+2\,Aab\right)\sqrt{bx^2+a}}{6\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) - 2*(4*B*b^2*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b
```

```
)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3), -1/3*(3*(B*b^2*x^4 +
2*B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (4*B*b^2
*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b
^4*x^2 + a^2*b^3)]
```

Sympy [A] time = 12.5789, size = 400, normalized size = 5.06

$$A\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a}{3a^{\frac{39}{2}}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/
a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a
**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1
 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b
*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.41643, size = 95, normalized size = 1.2

$$-\frac{\left(\left(\frac{4\,Bx}{b} + \frac{3\,A}{b}\right)x + \frac{3\,Ba}{b^2}\right)x + \frac{2\,Aa}{b^2}}{3\left(bx^2+a\right)^{\frac{3}{2}}} - \frac{B\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((4*B*x/b + 3*A/b)*x + 3*B*a/b^2)*x + 2*A*a/b^2)/(b*x^2 + a)^(3/2) -
B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.37 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

```
[Out] -(x^2*(a*B - A*b*x))/(3*a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^
2])
```

Rubi [A] time = 0.0217641, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {805, 261}

$$-\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(a*B - A*b*x))/(3*a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^
2])
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}} + \frac{(2B)\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}$$
$$= -\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0169559, size = 44, normalized size = 0.83

$$\frac{-2a^2B-3abBx^2+Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 41, normalized size = 0.8

$$\frac{Ax^3b^2-3\,Bx^2ab-2\,Ba^2}{3\,ab^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(A*b^2*x^3-3*B*a*b*x^2-2*B*a^2)/(b*x^2+a)^(3/2)/a/b^2
```

Maxima [A] time = 1.01277, size = 95, normalized size = 1.79

$$-\frac{Bx^2}{\left(bx^2+a\right)^{\frac{3}{2}}b} - \frac{Ax}{3\left(bx^2+a\right)^{\frac{3}{2}}b} + \frac{Ax}{3\sqrt{bx^2+a}\,ab} - \frac{2\,Ba}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqr
t(b*x^2 + a)*a*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

Fricas [A] time = 1.54911, size = 128, normalized size = 2.42

$$\frac{\left(Ab^2x^3 - 3\,Babx^2 - 2\,Ba^2\right)\sqrt{bx^2 + a}}{3\left(ab^4x^4 + 2\,a^2b^3x^2 + a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^2*x^3 - 3*B*a*b*x^2 - 2*B*a^2)*sqrt(b*x^2 + a)/(a*b^4*x^4 + 2*a^2*
b^3*x^2 + a^3*b^2)
```

Sympy [B] time = 10.6081, size = 141, normalized size = 2.66

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*
x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
), Ne(b, 0)), (x**4/(4*a**(5/2)), True))
```

Giac [A] time = 1.18286, size = 49, normalized size = 0.92

$$\frac{\left(\frac{Ax}{a} - \frac{3\,B}{b}\right)x^2 - \frac{2\,Ba}{b^2}}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((A*x/a - 3*B/b)*x^2 - 2*B*a/b^2)/(b*x^2 + a)^(3/2)
```

3.38 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(A + B*x)/(3*b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0144857, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {778, 191}

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(A + B*x)/(3*b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}} + \frac{B\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}$$
$$= -\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}} + \frac{Bx}{3ab\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0171072, size = 32, normalized size = 0.68

$$\frac{bBx^3 - aA}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 29, normalized size = 0.6

$$-\frac{-bBx^3 + Aa}{3\,ab}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-B*b*x^3+A*a)/(b*x^2+a)^(3/2)/a/b
```

Maxima [A] time = 0.991677, size = 69, normalized size = 1.47

$$-\frac{Bx}{3\left(bx^2+a\right)^{\frac{3}{2}}b} + \frac{Bx}{3\sqrt{bx^2+a}\,ab} - \frac{A}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

Fricas [A] time = 1.54832, size = 99, normalized size = 2.11

$$\frac{\left(Bbx^3 - Aa\right)\sqrt{bx^2 + a}}{3\left(ab^3x^4 + 2\,a^2b^2x^2 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^3 - A*a)*sqrt(b*x^2 + a)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [A] time = 9.03744, size = 95, normalized size = 2.02

$$A\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a)
 + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.16797, size = 35, normalized size = 0.74

$$\frac{\frac{Bx^3}{a} - \frac{A}{b}}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3/a - A/b)/(b*x^2 + a)^(3/2)
```

3.39 $\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}} - \frac{aB - Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(a*B - A*b*x)/(3*a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0104064, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {639, 191}

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}} - \frac{aB - Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*B - A*b*x)/(3*a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}} + \frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}$$
$$= -\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}} + \frac{2Ax}{3a^2\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0199786, size = 43, normalized size = 0.84

$$\frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 40, normalized size = 0.8

$$\frac{2\,Ab^2x^3+3\,Axab-Ba^2}{3\,a^2b}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

Maxima [A] time = 1.0094, size = 65, normalized size = 1.27

$$\frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\left(bx^2+a\right)^{\frac{3}{2}}a} - \frac{B}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x
^2 + a)^(3/2)*b)
```

Fricas [A] time = 1.5894, size = 126, normalized size = 2.47

$$\frac{\left(2\,Ab^2x^3 + 3\,Aabx - Ba^2\right)\sqrt{bx^2 + a}}{3\left(a^2b^3x^4 + 2\,a^3b^2x^2 + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3*
b^2*x^2 + a^4*b)
```

Sympy [B] time = 8.9493, size = 146, normalized size = 2.86

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a
 + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Giac [A] time = 1.22283, size = 50, normalized size = 0.98

$$\frac{\left(\frac{2\,Abx^2}{a^2} + \frac{3\,A}{a}\right)x - \frac{B}{b}}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

3.40 $\int \frac{A+Bx}{x(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0634318, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {823, 12, 266, 63, 208}

$$\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-3aAb-2abBx}{x\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}$$

$$= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{\int \frac{3a^2Ab^2}{x\sqrt{a+bx^2}}\,dx}{3a^4b^2}$$

$$= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}$$

$$= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^2}$$

$$= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^2b}$$

$$= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.0553323, size = 69, normalized size = 0.91

$$\frac{a(4A+3Bx)+bx^2(3A+2Bx)}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (b*x^2*(3*A + 2*B*x) + a*(4*A + 3*B*x))/(3*a^2*(a + b*x^2)^(3/2)) - (A*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Maple [A] time = 0.007, size = 92, normalized size = 1.2

$$\frac{Bx}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}} + \frac{2\,Bx}{3\,a^2}\frac{1}{\sqrt{bx^2+a}} + \frac{A}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}} + \frac{A}{a^2}\frac{1}{\sqrt{bx^2+a}} - A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*B*x/a/(b*x^2+a)^(3/2)+2/3*B/a^2*x/(b*x^2+a)^(1/2)+1/3*A/a/(b*x^2+a)^(3/
2)+A/a^2/(b*x^2+a)^(1/2)-A/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69197, size = 537, normalized size = 7.07

$$\left[\frac{3\left(Ab^2x^4 + 2\,Aabx^2 + Aa^2\right)\sqrt{a}\log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\sqrt{a} + 2\,a}{x^2}\right) + 2\left(2\,Babx^3 + 3\,Aabx^2 + 3\,Ba^2x + 4\,Aa^2\right)\sqrt{bx^2 + a}}{6\left(a^3b^2x^4 + 2\,a^4bx^2 + a^5\right)}, \frac{3\left(Ab^2x^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A
*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5), 1/3*(3*(A*b^2*x^4
 + 2*A*a*b*x^2 + A*a^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*
b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 +
2*a^4*b*x^2 + a^5)]
```

Sympy [B] time = 22.9917, size = 840, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15
/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2)
+ 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6
*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**
2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13
/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log
(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6
*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**
3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(1
9/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2
)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)) + B*(3*a*x/(3*a
**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x*
*3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.19623, size = 111, normalized size = 1.46

$$\frac{\left(\left(\frac{2\,Bbx}{a^2}+\frac{3\,Ab}{a^2}\right)x+\frac{3\,B}{a}\right)x+\frac{4\,A}{a}}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((2*B*b*x/a^2 + 3*A*b/a^2)*x + 3*B/a)*x + 4*A/a)/(b*x^2 + a)^(3/2) + 2
*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

3.41 $\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=104

$$\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt[a + b*x^2
]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(5/2)
```

Rubi [A] time = 0.0880568, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {823, 807, 266, 63, 208}

$$\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```


```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt[a + b*x^2
]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(5/2)
```


Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-4aAb-3abBx}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} + \frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x^2\sqrt{a+bx^2}}\,dx}{3a^4b^2}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} + \frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^2}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^2b}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} - \frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.0548632, size = 95, normalized size = 0.91

$$\frac{a^2(4Bx-3A)+3abx^2(Bx-4A)-3\sqrt{a}Bx\left(a+bx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 + 3*a*b*x^2*(-4*A + B*x) + a^2*(-3*A + 4*B*x) - 3*Sqrt[a]*B*x
*(a + b*x^2)^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(3*a^3*x*(a + b*x^2)^(
3/2))
```

Maple [A] time = 0.008, size = 112, normalized size = 1.1

$$\frac{B}{3a}\left(bx^2+a\right)^{-\frac{3}{2}} + \frac{B}{a^2}\frac{1}{\sqrt{bx^2+a}} - B\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}} - \frac{A}{ax}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{4Abx}{3a^2}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{8Abx}{3a^3}\frac{}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*B/a/(b*x^2+a)^(3/2)+B/a^2/(b*x^2+a)^(1/2)-B/a^(5/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)-A/a/x/(b*x^2+a)^(3/2)-4/3*A*b/a^2*x/(b*x^2+a)^(3/2)-8/3*
A*b/a^3*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68784, size = 589, normalized size = 5.66

$$\left[\frac{3\left(Bb^2x^5 + 2\,Babx^3 + Ba^2x\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) - 2\left(8\,Ab^2x^4 - 3\,Babx^3 + 12\,Aabx^2 - 4\,Ba^2x + 3\,Aa^2\right)\sqrt{bx^2 +}{6\left(a^3b^2x^5 + 2\,a^4bx^3 + a^5x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2
- 4*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
, 1/3*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) - (8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x + 3*A*a
^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]
```

Sympy [B] time = 20.0836, size = 910, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(8*a**7*sqrt(1
+ b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6
*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sqrt(1
+ b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*a**(19/2)
 + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) +
9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/
2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt(1 + b*x**2/
a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a
**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**5*
b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqrt(1 + b*x**2/a
) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(
13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b**3*x
**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**
(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

Giac [A] time = 1.20275, size = 161, normalized size = 1.55

$$-\frac{\left(\left(\frac{5\,Ab^2x}{a^3}-\frac{3\,Bb}{a^2}\right)x+\frac{6\,Ab}{a^2}\right)x-\frac{4\,B}{a}}{3\,\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*A*b^2*x/a^3 - 3*B*b/a^2)*x + 6*A*b/a^2)*x - 4*B/a)/(b*x^2 + a)^(3
/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) +
2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

3.42 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}+\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*Sqrt[a + b
*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[a + b*x^2])/(3*a^3*x
) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.11944, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {823, 835, 807, 266, 63, 208}

$$\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}+\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*Sqrt[a + b
*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[a + b*x^2])/(3*a^3*x
) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-5aAb-4abBx}{x^3\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} + \frac{\int \frac{15a^2Ab^2+8a^2b^2Bx}{x^3\sqrt{a+bx^2}}\,dx}{3a^4b^2}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{\int \frac{-16a^3b^2B+15a^2Ab^3x}{x^2\sqrt{a+bx^2}}\,dx}{6a^5b^2}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^3}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x\right.}{4a^3}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5A)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{2a^3}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} + \frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.148715, size = 106, normalized size = 0.82

$$\frac{-4a^2b(5A+6Bx) - \frac{3a^3(A+2Bx)}{x^2} - ab^2x^2(15A+16Bx) + \frac{15Ab\left(a+bx^2\right)^2\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}}{6a^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((-3*a^3*(A + 2*B*x))/x^2 - 4*a^2*b*(5*A + 6*B*x) - a*b^2*x^2*(15*A + 16*B*
x) + (15*A*b*(a + b*x^2)^2*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a
])/(6*a^4*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.008, size = 134, normalized size = 1.

$$-\frac{A}{2\,a x^{2}}\left(b x^{2}+a\right)^{-\frac{3}{2}}-\frac{5\,A b}{6\,a^{2}}\left(b x^{2}+a\right)^{-\frac{3}{2}}-\frac{5\,A b}{2\,a^{3}}\frac{1}{\sqrt{b x^{2}+a}}+\frac{5\,A b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{7}{2}}-\frac{B}{a x}\left(b x^{2}+a\right)^{-\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(3/2)-5/6*A*b/a^2/(b*x^2+a)^(3/2)-5/2*A*b/a^3/(b*x^2
+a)^(1/2)+5/2*A*b/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-B/a/x/(b*x^
2+a)^(3/2)-4/3*B*b/a^2*x/(b*x^2+a)^(3/2)-8/3*B*b/a^3*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69019, size = 683, normalized size = 5.29

$$\left[\frac{15\left(A b^{3} x^{6}+2\,A a b^{2} x^{4}+A a^{2} b x^{2}\right)\sqrt{a}\log\left(-\frac{b x^{2}+2\,\sqrt{b x^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)-2\left(16\,B a b^{2} x^{5}+15\,A a b^{2} x^{4}+24\,B a^{2} b x^{3}+20\,A a^{2} b x^{2}+\right.}{12\left(a^{4} b^{2} x^{6}+2\,a^{5} b x^{4}+a^{6} x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*B*a*b^2*x^5 + 15*A*a*b^2*x^4 +
 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqrt(b*x^2 + a))/(a
```

```
^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 +
A*a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (16*B*a*b^2*x^5 +
15*A*a*b^2*x^4 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqr
t(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

Sympy [B] time = 35.2767, size = 1034, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a
**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

Giac [A] time = 1.24502, size = 266, normalized size = 2.06

$$-\frac{\left(\left(\frac{5\,Bb^2x}{a^3}+\frac{6\,Ab^2}{a^3}\right)x+\frac{6\,Bb}{a^2}\right)x+\frac{7\,Ab}{a^2}}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{\ }}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*B*b^2*x/a^3 + 6*A*b^2/a^3)*x + 6*B*b/a^2)*x + 7*A*b/a^2)/(b*x^2 +
 a)^(3/2) - 5*A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)
*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/
(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^3)
```

3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rubi [A] time = 0.007999, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0145519, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

Maple [A] time = 0.007, size = 29, normalized size = 1.1

$$\frac{x}{2}\sqrt{-x^2+1}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.85078, size = 38, normalized size = 1.41

$$\frac{1}{2}\sqrt{-x^2+1}x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A] time = 1.4484, size = 82, normalized size = 3.04

$$\frac{1}{2}\sqrt{-x^2+1}(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.205776, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

Giac [A] time = 1.18072, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-x^2+1}(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rubi [A] time = 0.0170198, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx \\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.006537, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

Maple [A] time = 0.003, size = 29, normalized size = 1.1

$$\frac{x}{2}\sqrt{-x^2+1}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.52885, size = 38, normalized size = 1.41

$$\frac{1}{2}\sqrt{-x^2+1}x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A] time = 1.52475, size = 82, normalized size = 3.04

$$\frac{1}{2}\sqrt{-x^2+1}(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.21362, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

Giac [A] time = 1.23446, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-x^2+1}(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.45 $\int \frac{3+x^2}{-3+x^2} \, dx$

Optimal. Leaf size=17

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rubi [A] time = 0.0057999, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {388, 207}

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{3+x^2}{-3+x^2}\,dx = x + 6\int \frac{1}{-3+x^2}\,dx$$
$$= x - 2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.0074821, size = 33, normalized size = 1.94

$$x + \sqrt{3}\log\left(\sqrt{3} - x\right) - \sqrt{3}\log\left(x + \sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x + Sqrt[3]*Log[Sqrt[3] - x] - Sqrt[3]*Log[Sqrt[3] + x]
```

Maple [A] time = 0.002, size = 15, normalized size = 0.9

$$x - 2\,\mathrm{Artanh}\left(1/3\,x\sqrt{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2-3),x)
```

```
[Out] x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.45037, size = 30, normalized size = 1.76

$$\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + x
```

Fricas [A] time = 1.41783, size = 70, normalized size = 4.12

$$\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="fricas")
```

```
[Out] sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3)) + x
```

Sympy [A] time = 0.082351, size = 27, normalized size = 1.59

$$x+\sqrt{3}\log\left(x-\sqrt{3}\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2-3),x)
```

```
[Out] x + sqrt(3)*log(x - sqrt(3)) - sqrt(3)*log(x + sqrt(3))
```

Giac [B] time = 1.13001, size = 41, normalized size = 2.41

$$\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}\right|}{\left|2\,x+2\,\sqrt{3}\right|}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="giac")
```

```
[Out] sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + x
```

3.46 $\int \frac{-1+x^2}{1+x^2} \, dx$

Optimal. Leaf size=6

$$x - 2\tan^{-1}(x)$$

```
[Out] x - 2*ArcTan[x]
```

Rubi [A] time = 0.0034794, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {388, 203}

$$x - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{1+x^2} \, dx &= x - 2\int \frac{1}{1+x^2} \, dx \\ &= x - 2\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0036768, size = 6, normalized size = 1.

$$x - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Maple [A] time = 0.001, size = 7, normalized size = 1.2

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1),x)
```

```
[Out] x-2*arctan(x)
```

Maxima [A] time = 1.60279, size = 8, normalized size = 1.33

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="maxima")
```

```
[Out] x - 2*arctan(x)
```

Fricas [A] time = 1.44712, size = 23, normalized size = 3.83

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="fricas")
```

```
[Out] x - 2*arctan(x)
```

Sympy [A] time = 0.079618, size = 5, normalized size = 0.83

$$x - 2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1),x)
```

```
[Out] x - 2*atan(x)
```

Giac [A] time = 1.15304, size = 8, normalized size = 1.33

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="giac")
```

```
[Out] x - 2*arctan(x)
```

3.47 $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{16\sqrt{a+bx^2}(Ab-8aC)}{35ab^5}-\frac{x^7(aB-x}{7ab\left(a\right.}$$

```
[Out] -(x^7*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^5*(7*a*B - (A*b
 - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a*B - 6*(A*b - 8*a*C)
*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a*B - 8*(A*b - 8*a*C)*x))/(35*a
*b^4*Sqrt[a + b*x^2]) - (16*(A*b - 8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (B*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.324291, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1804, 819, 641, 217, 206}

$$-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{16\sqrt{a+bx^2}(Ab-8aC)}{35ab^5}-\frac{x^7(aB-x}{7ab\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^7*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^5*(7*a*B - (A*b
 - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a*B - 6*(A*b - 8*a*C)
*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a*B - 8*(A*b - 8*a*C)*x))/(35*a
*b^4*Sqrt[a + b*x^2]) - (16*(A*b - 8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (B*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^6(-7aB+(Ab-8aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x^4\left(-35a^2B+6a(Ab-8aC)x\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{x^2\left(-105a^3B \cdots\right.}{(a \cdots}}{10 \cdots}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(\cdots}{35ab^4\sqrt{\cdots}}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(\cdots}{35ab^4\sqrt{\cdots}}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(\cdots}{35ab^4\sqrt{\cdots}}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(\cdots}{35ab^4\sqrt{\cdots}}$$

<u>Mathematica</u> [A] time = 0.432563, size = 165, normalized size = 0.77

$$\frac{14a^2b^2x^2(5x(24Cx-5B)-12A)-3a^3b(16A+7x(5B-64Cx))+384a^4C+14ab^3x^4(x(60Cx-29B)-15A)+105\sqrt{a}\sqrt{b}\cdots}{105b^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2*(-12*A +
5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B + 60*C*x)) + b^4*x^6*
(-105*A + x*(-176*B + 105*C*x)) + 105*Sqrt[a]*Sqrt[b]*B*(a + b*x^2)^3*Sqrt[
1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(105*b^5*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.043, size = 265, normalized size = 1.2

$$\frac{Cx^8}{b}\left(bx^2+a\right)^{-\frac{7}{2}}+8\,\frac{aCx^6}{b^2\left(bx^2+a\right)^{7/2}}+16\,\frac{a^2Cx^4}{b^3\left(bx^2+a\right)^{7/2}}+\frac{64\,Ca^3x^2}{5\,b^4}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{128\,Ca^4}{35\,b^5}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Bx^7}{7\,b}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] C*x^8/b/(b*x^2+a)^(7/2)+8*C/b^2*a*x^6/(b*x^2+a)^(7/2)+16*C/b^3*a^2*x^4/(b*x
^2+a)^(7/2)+64/5*C/b^4*a^3*x^2/(b*x^2+a)^(7/2)+128/35*C/b^5*a^4/(b*x^2+a)^(
7/2)-1/7*B*x^7/b/(b*x^2+a)^(7/2)-1/5*B/b^2*x^5/(b*x^2+a)^(5/2)-1/3*B/b^3*x^
3/(b*x^2+a)^(3/2)-B/b^4*x/(b*x^2+a)^(1/2)+B/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))-A*x^6/b/(b*x^2+a)^(7/2)-2*A/b^2*a*x^4/(b*x^2+a)^(7/2)-8/5*A/b^3*a^2*
x^2/(b*x^2+a)^(7/2)-16/35*A/b^4*a^3/(b*x^2+a)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81342, size = 1160, normalized size = 5.45

$$\left[\frac{105\left(Bb^4x^8+4\,Bab^3x^6+6\,Ba^2b^2x^4+4\,Ba^3bx^2+Ba^4\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(105\,Cb^4x^8-176\,Bb^4\right.}{210\left(b^9\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 +
B*a^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(105*C*b
^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b
^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2
 - A*a*b^3)*x^4 + 168*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^
8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(B*
b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 + B*a^4)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (105*C*b^4*x^8 - 176*B*b^4*x^7 - 406*
B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 105*B*a^3*b
*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^
3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x
^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21738, size = 275, normalized size = 1.29

$$\frac{\left(\left(\left(\left(\left(\left(\left(\frac{105\,Cx}{b}-\frac{176\,B}{b}\right)x+\frac{105\left(8\,Ca^4b^7-Aa^3b^8\right)}{a^3b^9}\right)x-\frac{406\,Ba}{b^2}\right)x+\frac{210\left(8\,Ca^5b^6-Aa^4b^7\right)}{a^3b^9}\right)x-\frac{350\,Ba^2}{b^3}\right)x+\frac{168\left(8\,Ca^6b^5-Aa^5b^6\right)}{a^3b^9}\right)x-\frac{105\,Ba^3}{b^4}\right)\cdots}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b^8)/(a^3*b
^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)/(a^3*b^9))*x - 350*
B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/(a^3*b^9))*x - 105*B*a^3/b^4)*
x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a^3*b^9))/(b*x^2 + a)^(7/2) - B*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.48 $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=150

$$-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] -(x^6*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(6*B + 7*C*x)
)/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(105*b^3*(a + b*x^2)^(
3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^2]) + (C*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.165252, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1804, 819, 778, 217, 206}

$$-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^6*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(6*B + 7*C*x)
)/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(105*b^3*(a + b*x^2)^(
3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^2]) + (C*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/b^(9/2)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^5(-6aB-7aCx)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x^3\left(-24a^2B-35a^2Cx\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{x\left(-48a^3B-105a^3Cx\right)}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^3b^3}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\int \frac{}{\sqrt{a}}}{b}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\,\mathrm{Subs}}{}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\tanh}{}$$

Mathematica [A] time = 0.268011, size = 147, normalized size = 0.98

$$\frac{\sqrt{a}C\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}} - \frac{14a^2b^2x^4(15B+29Cx)+14a^3bx^2(12B+25Cx)+3a^4(16B+35Cx)+ab^3x^6(105B+176C}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(-15*A*b^4*x^7 + 14*a^3*b*x^2*(12*B + 25*C*x) + 14*a^2*b^2*x^4*(15*B + 29*
C*x) + 3*a^4*(16*B + 35*C*x) + a*b^3*x^6*(105*B + 176*C*x))/(105*a*b^4*(a +
 b*x^2)^(7/2)) + (Sqrt[a]*C*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]
])/(b^(9/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.012, size = 277, normalized size = 1.9

$$-\frac{Cx^7}{7b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Cx^5}{5b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Cx^3}{3b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Cx}{b^4}\frac{1}{\sqrt{bx^2+a}}+C\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}-\frac{Bx^6}{b}\left(bx^2+a\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*C*x^7/b/(b*x^2+a)^(7/2)-1/5*C/b^2*x^5/(b*x^2+a)^(5/2)-1/3*C/b^3*x^3/(b
*x^2+a)^(3/2)-C/b^4*x/(b*x^2+a)^(1/2)+C/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))-B*x^6/b/(b*x^2+a)^(7/2)-2*B/b^2*a*x^4/(b*x^2+a)^(7/2)-8/5*B/b^3*a^2*x^2/
(b*x^2+a)^(7/2)-16/35*B/b^4*a^3/(b*x^2+a)^(7/2)-1/2*A*x^5/b/(b*x^2+a)^(7/2)
-5/8*A/b^2*a*x^3/(b*x^2+a)^(7/2)-15/56*A/b^3*a^2*x/(b*x^2+a)^(7/2)+3/56*A/b
^3*a*x/(b*x^2+a)^(5/2)+1/14*A/b^3*x/(b*x^2+a)^(3/2)+1/7*A/b^3/a*x/(b*x^2+a)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74462, size = 1044, normalized size = 6.96

$$\left[\frac{105\left(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(105\,Bab^4x^6+406\,C\right.}{210\left(ab^9x^8+4\,a^2b^8x^6+6\,a^3b^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x^
2 + C*a^5)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(105
```

```
*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3 +
168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*a^4
*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*b^6
*x^2 + a^5*b^5), -1/105*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x
^4 + 4*C*a^4*b*x^2 + C*a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (
105*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3
 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*
a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*
b^6*x^2 + a^5*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22477, size = 186, normalized size = 1.24

$$-\frac{\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{\left(176\,Ca^3b^7-15\,Aa^2b^8\right)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba^3}{b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{C\log\left(\left|-\sqrt{b}x+\right.\right.}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b^8)) + 406
*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B*a^2/b^3)*x + 105*C*
a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2) - C*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(9/2)
```

3.49 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{x^4(6aC+Ab-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(x^5*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(A*b + 6*a*C
- 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*a*C))/(105*b^4*(a +
b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.166223, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1804, 805, 266, 43}

$$-\frac{x^4(6aC+Ab-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^5*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(A*b + 6*a*C
- 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*a*C))/(105*b^4*(a +
b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*Sqrt[a + b*x^2])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
```

```
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^4(-5aB-(Ab+6aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4(Ab+6aC))\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab^2}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(2(Ab+6aC))\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx,x,x^2\right)}{35ab^2}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(2(Ab+6aC))\,\mathrm{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx,x,x^2\right)}{35ab^2}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4(Ab+6aC)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{4(Ab+6aC)}{35ab^4\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0792782, size = 89, normalized size = 0.67

$$\frac{-14a^2b^2x^2\left(2A+15Cx^2\right)-8a^3b\left(A+21Cx^2\right)-48a^4C-35ab^3x^4\left(A+3Cx^2\right)+15b^4Bx^7}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b^2*x^2*(2*
A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.005, size = 95, normalized size = 0.7

$$-\frac{-15\,Bx^{7}b^{4}+105\,Cx^{6}ab^{3}+35\,Aab^{3}x^{4}+210\,Ca^{2}b^{2}x^{4}+28\,Aa^{2}b^{2}x^{2}+168\,Ca^{3}bx^{2}+8\,Aa^{3}b+48\,Ca^{4}}{105\,ab^{4}}\left(bx^{2}+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-15*B*b^4*x^7+105*C*a*b^3*x^6+35*A*a*b^3*x^4+210*C*a^2*b^2*x^4+28*A
*a^2*b^2*x^2+168*C*a^3*b*x^2+8*A*a^3*b+48*C*a^4)/(b*x^2+a)^(7/2)/a/b^4
```

Maxima [B] time = 1.08059, size = 324, normalized size = 2.45

$$-\frac{Cx^{6}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{Bx^{5}}{2\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{2\,Cax^{4}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{Ax^{4}}{3\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{5\,Bax^{3}}{8\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{8\,Ca^{2}x^{2}}{5\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}-\frac{4\,Aax^{2}}{15\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 2*C*a*x^4/
((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/((b
*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 4/15*A*a*x^2
/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt
(b*x^2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b
*x^2 + a)^(7/2)*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*a^2/((
b*x^2 + a)^(7/2)*b^3)
```

Fricas [A] time = 1.6037, size = 290, normalized size = 2.2

$$\frac{\left(15\,Bb^{4}x^{7}-105\,Cab^{3}x^{6}-48\,Ca^{4}-8\,Aa^{3}b-35\left(6\,Ca^{2}b^{2}+Aab^{3}\right)x^{4}-28\left(6\,Ca^{3}b+Aa^{2}b^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{105\left(ab^{8}x^{8}+4\,a^{2}b^{7}x^{6}+6\,a^{3}b^{6}x^{4}+4\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35*(6*C*a^2*
b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a*b^8
*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20047, size = 151, normalized size = 1.14

$$\frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7\,C}{b}\right)x^{2}-\frac{7\left(6\,Ca^{4}b^{2}+Aa^{3}b^{3}\right)}{a^{3}b^{4}}\right)x^{2}-\frac{28\left(6\,Ca^{5}b+Aa^{4}b^{2}\right)}{a^{3}b^{4}}\right)x^{2}-\frac{8\left(6\,Ca^{6}+Aa^{5}b\right)}{a^{3}b^{4}}}{105\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a^3*b^4))*x
^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C*a^6 + A*a^5*b)/(a^3
*b^4))/(b*x^2 + a)^(7/2)
```

3.50 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=149

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(x^4*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^2*(4*a*B + (2*A
*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B + 3*(2*A*b + 5*a*C)*x
)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5*a*C)*x)/(35*a^2*b^3*Sqrt[a +
b*x^2])
```

Rubi [A] time = 0.181302, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1804, 819, 778, 191}

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^4*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^2*(4*a*B + (2*A
*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B + 3*(2*A*b + 5*a*C)*x
)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5*a*C)*x)/(35*a^2*b^3*Sqrt[a +
b*x^2])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^3(-4aB-(2Ab+5aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x\left(-8a^2B-3a(2Ab+5aC)x\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2Ab+5aC}{35}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2Ab+5a}{35a^2b^3\sqrt{a+}}$$

Mathematica [A] time = 0.0819922, size = 78, normalized size = 0.52

$$\frac{-35a^2b^2Bx^4-28a^3bBx^2-8a^4B+3ab^3x^5\left(7A+5Cx^2\right)+6Ab^4x^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 6*A*b^4*x^7 + 3*a*b^3*x^5*(
7*A + 5*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 76, normalized size = 0.5

$$\frac{6\,Ab^4x^7+15\,Cab^3x^7+21\,Ax^5ab^3-35\,Bx^4a^2b^2-28\,Ba^3x^2b-8\,a^4B}{105\,a^2b^3}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(6*A*b^4*x^7+15*C*a*b^3*x^7+21*A*a*b^3*x^5-35*B*a^2*b^2*x^4-28*B*a^3*
b*x^2-8*B*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

Maxima [A] time = 1.02215, size = 342, normalized size = 2.3

$$-\frac{Cx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Cax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,Bax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{Cx}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{Cx}{7\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*C*
a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*B*a*
x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(s
qrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/
((b*x^2 + a)^(7/2)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqr
t(b*x^2 + a)*a^2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b
*x^2 + a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

Fricas [A] time = 1.63217, size = 254, normalized size = 1.7

$$\frac{\left(21\,Aab^3x^5 - 35\,Ba^2b^2x^4 + 3\left(5\,Cab^3 + 2\,Ab^4\right)x^7 - 28\,Ba^3bx^2 - 8\,Ba^4\right)\sqrt{bx^2 + a}}{105\left(a^2b^7x^8 + 4\,a^3b^6x^6 + 6\,a^4b^5x^4 + 4\,a^5b^4x^2 + a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4)*x^7 - 28
*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*a^
4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19, size = 109, normalized size = 0.73

$$\frac{\left(\left(3\,x\left(\frac{7\,A}{a} + \frac{\left(5\,Ca^2b^3 + 2\,Aab^4\right)x^2}{a^3b^3}\right) - \frac{35\,B}{b}\right)x^2 - \frac{28\,Ba}{b^2}\right)x^2 - \frac{8\,Ba^2}{b^3}}{105\left(bx^2 + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) - 35*B/b)*x^
2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

3.51 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(x^3*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x*(3*a*B + (3*A*b
 + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b + 4*a*C) - 3*b*B*x)/
(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a^2*b^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.151264, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1804, 819, 639, 191}

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```


```
[Out] -(x^3*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x*(3*a*B + (3*A*b
 + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b + 4*a*C) - 3*b*B*x)/
(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a^2*b^2*Sqrt[a + b*x^2])
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2(-3aB-(3Ab+4aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{-3a^2B-2a(3Ab+4aC)x}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2B)\int \frac{1}{\left(a+bx^2\right)}}{35ab^2}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{2Bx}{35a^2b^2\sqrt{a+b}}$$

Mathematica [A] time = 0.089708, size = 84, normalized size = 0.6

$$\frac{-7a^2b^2x^2\left(3A+5Cx^2\right)-2a^3b\left(3A+14Cx^2\right)-8a^4C+21ab^3Bx^5+6b^4Bx^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5*C*x^2) -
2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.005, size = 85, normalized size = 0.6

$$-\frac{-6\,Bx^7b^4-21\,Bx^5ab^3+35\,Cx^4a^2b^2+21\,Aa^2b^2x^2+28\,Ca^3bx^2+6\,Aa^3b+8\,Ca^4}{105\,a^2b^3}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6*B*b^4*x^7-21*B*a*b^3*x^5+35*C*a^2*b^2*x^4+21*A*a^2*b^2*x^2+28*C*
a^3*b*x^2+6*A*a^3*b+8*C*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

Maxima [A] time = 0.988154, size = 242, normalized size = 1.74

$$-\frac{Cx^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,Cax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,Bx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Bx}{35\sqrt{bx^2+a}a^2b^2}+\frac{Bx}{35\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*C
*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C*a^2
/((b*x^2 + a)^(7/2)*b^3) - 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

Fricas [A] time = 1.6596, size = 271, normalized size = 1.95

$$\frac{\left(6\,Bb^4x^7+21\,Bab^3x^5-35\,Ca^2b^2x^4-8\,Ca^4-6\,Aa^3b-7\left(4\,Ca^3b+3\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4 - 6*A*a^3*
b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b
^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

Sympy [A] time = 142.489, size = 660, normalized size = 4.75

$$A\left(\begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} & \text{f} \\ \frac{x^4}{4a^{\frac{9}{2}}} & \text{o} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(
a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x*
*2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
+ b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2
)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + B*(7*a*x**5/(35*a**(11/2)*sqrt(
1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*
x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x
**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a))) + C*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 3
15*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105
*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2)
+ 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) +
105*b**6*x**6*sqrt(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x*
*2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2
) + 105*b**6*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

Giac [A] time = 1.21818, size = 128, normalized size = 0.92

$$\frac{\left(\left(3\left(\frac{2Bbx^2}{a^2}+\frac{7B}{a}\right)x-\frac{35C}{b}\right)x^2-\frac{7\left(4Ca^4b+3Aa^3b^2\right)}{a^3b^3}\right)x^2-\frac{2\left(4Ca^5+3Aa^4b\right)}{a^3b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b + 3*A*a^3
*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

3.52 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(x^2*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (2*a*B + (4*A*b +
3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*C)*x)/(105*a^2*b^2*(
a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(105*a^3*b^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.13103, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1804, 778, 192, 191}

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^2*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (2*a*B + (4*A*b +
3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*C)*x)/(105*a^2*b^2*(
a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(105*a^3*b^2*Sqrt[a + b*x^2])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
```

```
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x(-2aB-(4Ab+3aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab^2}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}} + \frac{(2(4Ab+3aC))\int}{105a^2b^2}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}} + \frac{2(4Ab+3aC)x}{105a^3b^2\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0945074, size = 87, normalized size = 0.63

$$\frac{7a^2b^2x^3\left(5A+3Cx^2\right)-21a^3bBx^2-6a^4B+2ab^3x^5\left(14A+3Cx^2\right)+8Ab^4x^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3*C*x^2) +
2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 88, normalized size = 0.6

$$\frac{8\,Ab^4x^7+6\,Cab^3x^7+28\,Aab^3x^5+21\,Ca^2b^2x^5+35\,Ax^3b^2a^2-21\,Bx^2ba^3-6\,Ba^4}{105\,b^2a^3}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(8*A*b^4*x^7+6*C*a*b^3*x^7+28*A*a*b^3*x^5+21*C*a^2*b^2*x^5+35*A*a^2*b
^2*x^3-21*B*a^3*b*x^2-6*B*a^4)/(b*x^2+a)^(7/2)/b^2/a^3
```

Maxima [A] time = 1.03386, size = 266, normalized size = 1.91

$$-\frac{Cx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,Cx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2}+\frac{Cx}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{}{7\,(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*
C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x
/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/(
(b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^
2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*x^2 +
 a)^(7/2)*b^2)
```

Fricas [A] time = 1.67192, size = 277, normalized size = 1.99

$$\frac{\left(35\,Aa^2b^2x^3+2\left(3\,Cab^3+4\,Ab^4\right)x^7-21\,Ba^3bx^2+7\left(3\,Ca^2b^2+4\,Aab^3\right)x^5-6\,Ba^4\right)\sqrt{bx^2+a}}{105\left(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*b*x^2 + 7*
(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a
^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

Sympy [B] time = 110.567, size = 904, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 1
05*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*
b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105
*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b*
*5*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + C*(7*a*x
**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*
b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*
a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.18506, size = 127, normalized size = 0.91

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,Cab^4+4\,Ab^5\right)x^2}{a^3b^3}+\frac{7\left(3\,Ca^2b^3+4\,Aab^4\right)}{a^3b^3}\right)+\frac{35\,A}{a}\right)x-\frac{21\,B}{b}\right)x^2-\frac{6\,Ba}{b^2}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*b^3 + 4*A*
a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2)/(b*x^2 + a)^(7/2)
```

3.53 $\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=119

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(x*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2*a*C - b*B
*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2)) +
(8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0876955, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1804, 639, 192, 191}

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```


```
[Out] -(x*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2*a*C - b*B
*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2)) +
(8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
```

```
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{-aB-(5Ab+2aC)x}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4B)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}+\frac{(8B)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^2b}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}+\frac{8Bx}{105a^3b\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0580431, size = 75, normalized size = 0.63

$$
\frac{-3a^3b\left(5A+7Cx^2\right)+35a^2b^2Bx^3-6a^4C+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*C + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^5 + 8*b^4*B*x^7 - 3*a^3*b*(5*A
+ 7*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.005, size = 73, normalized size = 0.6

$$-\frac{-8\,Bx^7b^4-28\,Bx^5ab^3-35\,Bx^3a^2b^2+21\,Cx^2a^3b+15\,Aa^3b+6\,Ca^4}{105\,a^3b^2}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-8*B*b^4*x^7-28*B*a*b^3*x^5-35*B*a^2*b^2*x^3+21*C*a^3*b*x^2+15*A*a^
3*b+6*C*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

Maxima [A] time = 1.17386, size = 166, normalized size = 1.39

$$-\frac{Cx^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,Bx}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}-\frac{2\,Ca}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{}{7\,(bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*
x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/
((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*A/((b*x^2
+ a)^(7/2)*b)
```

Fricas [A] time = 1.69919, size = 250, normalized size = 2.1

$$\frac{\left(8\,Bb^4x^7+28\,Bab^3x^5+35\,Ba^2b^2x^3-21\,Ca^3bx^2-6\,Ca^4-15\,Aa^3b\right)\sqrt{bx^2+a}}{105\left(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3*b*x^2 - 6
*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5*b
^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

Sympy [A] time = 71.2865, size = 796, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x
**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(9/2)), True)) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1
+ b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*
x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*
a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqr
t(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b
**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19
/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(
15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**
2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*
a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 63
0*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 +
 b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))) + C*Piecewise((-2
*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 1
05*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2
/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105
*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)), Ne(b, 0)),
(x**4/(4*a**(9/2)), True))
```

Giac [A] time = 1.22822, size = 111, normalized size = 0.93

$$\frac{\left(\left(4\left(\frac{2\,Bb^2x^2}{a^3}+\frac{7\,Bb}{a^2}\right)x^2+\frac{35\,B}{a}\right)x-\frac{21\,C}{b}\right)x^2-\frac{3\left(2\,Ca^4b+5\,Aa^3b^2\right)}{a^3b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b)*x^2 - 3*
(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

3.54 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}} dx$

Optimal. Leaf size=127

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\left(a+bx^2\right)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(a*B - (A*b - a*C)*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*x)/(35*a^
2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b*(a + b*x^2)^(3/2))
+ (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0686267, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\left(a+bx^2\right)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] -(a*B - (A*b - a*C)*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*x)/(35*a^
2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b*(a + b*x^2)^(3/2))
+ (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x^2])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{-6A-\frac{aC}{b}}{\left(a+bx^2\right)^{7/2}}\,dx}{7a}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{(4(6Ab+aC))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{(8(6Ab+aC))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^3b}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0626063, size = 92, normalized size = 0.72

$$\frac{14a^2b^2x^3\left(15A+2Cx^2\right)+35a^3bx\left(3A+Cx^2\right)-15a^4B+8ab^3x^5\left(21A+Cx^2\right)+48Ab^4x^7}{105a^4b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A +
C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.003, size = 96, normalized size = 0.8

$$\frac{48\,Ab^4x^7+8\,Cab^3x^7+168\,Aab^3x^5+28\,Ca^2b^2x^5+210\,Aa^2b^2x^3+35\,Ca^3bx^3+105\,Axa^3b-15\,Ba^4}{105\,a^4b}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^5+210*A*a^
2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2+a)^(7/2)/a^4/b
```

Maxima [A] time = 1.02492, size = 207, normalized size = 1.63

$$\frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4}+\frac{8\,Ax}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{Cx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,C}{105\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*x^
2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^(7/2
)*b)
```

Fricas [A] time = 1.62715, size = 289, normalized size = 2.28

$$\frac{\left(8\left(Cab^3+6\,Ab^4\right)x^7+105\,Aa^3bx+28\left(Ca^2b^2+6\,Aab^3\right)x^5-15\,Ba^4+35\left(Ca^3b+6\,Aa^2b^2\right)x^3\right)\sqrt{bx^2+a}}{105\left(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*b^
5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

Sympy [B] time = 111.682, size = 1880, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
```

```
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqr
t(a + b*x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(
19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a*
*(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x
**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(1
05*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) +
 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x
**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**
2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)))
```

Giac [A] time = 1.19645, size = 151, normalized size = 1.19

$$\frac{\left(\left(4\,x^2\left(\frac{2\left(Cab^5+6\,Ab^6\right)x^2}{a^4b^3}+\frac{7\left(Ca^2b^4+6\,Aab^5\right)}{a^4b^3}\right)+\frac{35\left(Ca^3b^3+6\,Aa^2b^4\right)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a*
b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x
- 15*B/b)/(b*x^2 + a)^(7/2)
```

3.55 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=138

$$\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35*a^2*(a +
b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*A + 16*B*
x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rubi [A] time = 0.162066, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35*a^2*(a +
b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*A + 16*B*
x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-6Bx}{x\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35aAb+24abBx}{x\left(a+bx^2\right)^{5/2}}\,dx}{35a^3b}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105a^2Ab^2-48a^2b^2Bx}{x\left(a+bx^2\right)^{3/2}}\,dx}{105a^5b^2}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{\int \frac{105a^3Ab^3}{x\sqrt{a+bx^2}}\,dx}{105a^7b^3}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^4}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} \ldots\right)}{2a^4}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} \ldots\right)}{a^4b}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Mathematica [A] time = 0.156086, size = 120, normalized size = 0.87

$$\frac{14a^2b^2x^2(29A+15Bx)+a^3b(176A+105Bx)-15a^4C+14ab^3x^4(25A+12Bx)+3b^4x^6(35A+16Bx)}{105a^4b\left(a+bx^2\right)^{7/2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A + 15*B*x)
+ 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/(105*a^4*b*(a + b*x^
```

```
2)^(7/2)) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Maple [A] time = 0.009, size = 169, normalized size = 1.2

$$-\frac{C}{7b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{Bx}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{6\,Bx}{35\,a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{8\,Bx}{35\,a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{16\,Bx}{35\,a^4}\frac{1}{\sqrt{bx^2+a}}+\frac{A}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{A}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*C/b/(b*x^2+a)^(7/2)+1/7*B*x/a/(b*x^2+a)^(7/2)+6/35*B/a^2*x/(b*x^2+a)^(
5/2)+8/35*B/a^3*x/(b*x^2+a)^(3/2)+16/35*B/a^4*x/(b*x^2+a)^(1/2)+1/7*A/a/(b*
x^2+a)^(7/2)+1/5*A/a^2/(b*x^2+a)^(5/2)+1/3*A/a^3/(b*x^2+a)^(3/2)+A/a^4/(b*x
^2+a)^(1/2)-A/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85621, size = 1040, normalized size = 7.54

$$\left[\frac{105\left(Ab^5x^8+4\,Aab^4x^6+6\,Aa^2b^3x^4+4\,Aa^3b^2x^2+Aa^4b\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(48\,Bab^4x^7+105\,Aab^4x^6+\right.}{210\left(a^5b^5x^8+4\,a^6b^4x^6+6\,a^7b^3x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2
+ A*a^4*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 +
 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a
^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8
*b^2*x^2 + a^9*b), 1/105*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4
+ 4*A*a^3*b^2*x^2 + A*a^4*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4
8*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 + 2
10*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a^4
*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b
^2*x^2 + a^9*b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2207, size = 205, normalized size = 1.49

$$\frac{\left(\left(\left(\left(3\left(\left(\frac{16\,Bb^3x}{a^4}+\frac{35\,Ab^3}{a^4}\right)x+\frac{56\,Bb^2}{a^3}\right)x+\frac{350\,Ab^2}{a^3}\right)x+\frac{210\,Bb}{a^2}\right)x+\frac{406\,Ab}{a^2}\right)x+\frac{105\,B}{a}\right)x-\frac{15\,Ca^{14}b^2-176\,Aa^{13}b^3}{a^{14}b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\cdots}{\cdots}\right)}{\sqrt{-a}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x + 350*A*b
^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*x - (15*C*a^14*b^2 -
 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2) + 2*A*arctan(-(sqrt(b)*x - s
qrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4)
```

3.56 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=188

$$\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{B-x\left(\frac{Ab}{a}\right.}{7a\left(a+bx\right.}$$

```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/a - 6*C)*x
)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a - 8*C)*x)/(105*a^3*(a
+ b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)*x)/(35*a^4*Sqrt[a + b*x^2]) -
 (A*Sqrt[a + b*x^2])/(a^5*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rubi [A] time = 0.381314, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1805, 807, 266, 63, 208}

$$\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{B-x\left(\frac{Ab}{a}\right.}{7a\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```


```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/a - 6*C)*x
)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a - 8*C)*x)/(105*a^3*(a
+ b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)*x)/(35*a^4*Sqrt[a + b*x^2]) -
 (A*Sqrt[a + b*x^2])/(a^5*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-7Bx+6\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-4\left(\frac{13Ab}{a}-6C\right)x^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105A-105Bx+6\left(\frac{29Ab}{a}-8C\right)x^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{105a^3}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}} + \int 1$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{}}{}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{}}{}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{}}{}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{}}{}$$

Mathematica [A] time = 0.15615, size = 158, normalized size = 0.84

$$\frac{14a^2b^2x^4(x(25B+12Cx)-120A)+14a^3bx^2(x(29B+15Cx)-60A)+a^4(x(176B+105Cx)-105A)+3ab^3x^6(x(35B+}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 14*a^3*b*x^
2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x*(35*B + 16*C*x)) +
a^4*(-105*A + x*(176*B + 105*C*x)) - 105*Sqrt[a]*B*x*(a + b*x^2)^(7/2)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.009, size = 240, normalized size = 1.3

$$\frac{Cx}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{6\,Cx}{35\,a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{8\,Cx}{35\,a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{16\,Cx}{35\,a^4}\frac{1}{\sqrt{bx^2+a}}+\frac{B}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{B}{5\,a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{B}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*C*x/a/(b*x^2+a)^(7/2)+6/35*C/a^2*x/(b*x^2+a)^(5/2)+8/35*C/a^3*x/(b*x^2+
a)^(3/2)+16/35*C/a^4*x/(b*x^2+a)^(1/2)+1/7*B/a/(b*x^2+a)^(7/2)+1/5*B/a^2/(b
*x^2+a)^(5/2)+1/3*B/a^3/(b*x^2+a)^(3/2)+B/a^4/(b*x^2+a)^(1/2)-B/a^(9/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-A/a/x/(b*x^2+a)^(7/2)-8/7*A*b/a^2*x/(b*x
^2+a)^(7/2)-48/35*A*b/a^3*x/(b*x^2+a)^(5/2)-64/35*A*b/a^4*x/(b*x^2+a)^(3/2)
-128/35*A*b/a^5*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88288, size = 1172, normalized size = 6.23

$$\left[\frac{105\left(Bb^4x^9+4\,Bab^3x^7+6\,Ba^2b^2x^5+4\,Ba^3bx^3+Ba^4x\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(105\,Bab^3x^7+350\,Ba^2b^2x^5+48\right.}{210\left(a^5b^4x^9+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 +
B*a^4*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(1
05*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3
*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C
*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(
a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1/105*(
105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B*a^4*x)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (105*B*a*b^3*x^7 + 350*B*a^2*b
^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*
A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 +
105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7
+ 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17356, size = 323, normalized size = 1.72

$$\frac{\left(\left(\left(\left(3\left(x\left(\frac{35\,Bb^3}{a^4}+\frac{\left(16\,Ca^{20}b^6-93\,Aa^{19}b^7\right)x}{a^{24}b^3}\right)+\frac{28\left(2\,Ca^{21}b^5-11\,Aa^{20}b^6\right)}{a^{24}b^3}\right)x+\frac{350\,Bb^2}{a^3}\right)x+\frac{210\left(Ca^{22}b^4-5\,Aa^{21}b^5\right)}{a^{24}b^3}\right)x+\frac{406\,Bb}{a^2}\right)x+\frac{105\left(Ca^{23}b\right.}{a^2}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x/(a^24*b^3
)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x + 350*B*b^2/a^3)*x + 2
10*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x + 406*B*b/a^2)*x + 105*(C*a^23
*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x + 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*arcta
n(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b)/(((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

3.57 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}+\frac{(9Ab-2aC)\tanh^{-1}}{2a^{11/2}}$$

```
[Out] -(a*((A*b)/a - C) + b*B*x)/(7*a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b - a*C) + 1
3*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C) + 87*b*B*x)/(105*a^
4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93*b*B*x)/(35*a^5*Sqrt[a + b*x^2
]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2) - (B*Sqrt[a + b*x^2])/(a^5*x) + ((9*A*
b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rubi [A] time = 0.48031, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1805, 1807, 807, 266, 63, 208}

$$-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}+\frac{(9Ab-2aC)\tanh^{-1}}{2a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```



```
[Out] -(a*((A*b)/a - C) + b*B*x)/(7*a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b - a*C) + 1
3*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C) + 87*b*B*x)/(105*a^
4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93*b*B*x)/(35*a^5*Sqrt[a + b*x^2
]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2) - (B*Sqrt[a + b*x^2])/(a^5*x) + ((9*A*
b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-7Bx+7\left(\frac{Ab}{a}-C\right)x^2+\frac{6bBx^3}{a}}{x^3\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-35\left(\frac{2Ab}{a}-C\right)x^2-\frac{52bBx^3}{a}}{x^3\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105A-105Bx+105\left(\frac{3Ab}{a}-\right.}{x^3\left(a+bx^2\right)^{3/}}}{105a^3}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bB}{35a^5\sqrt{a+bx^2}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bB}{35a^5\sqrt{a+bx^2}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bB}{35a^5\sqrt{a+bx^2}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bB}{35a^5\sqrt{a+bx^2}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bB}{35a^5\sqrt{a+bx^2}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bB}{35a^5\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.377404, size = 178, normalized size = 0.81

$$\frac{42a^2b^3x^4(x(5Cx-64B)-75A)+14a^3b^2x^2(10x(5Cx-24B)-261A)-4a^4b(396A+7x(60B-29Cx))+\frac{a^5\left(-105A-210Bx\right.}{x^2}}{210a^6\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3*a*b^4*x^6*(315*A + 256*B*x) + (a^5*(-105*A - 210*B*x + 352*C*x^2))/x^2
- 4*a^4*b*(396*A + 7*x*(60*B - 29*C*x)) + 42*a^2*b^3*x^4*(-75*A + x*(-64*B
+ 5*C*x)) + 14*a^3*b^2*x^2*(-261*A + 10*x*(-24*B + 5*C*x)) + (105*(9*A*b -
2*a*C)*(a + b*x^2)^4*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a])/(21
0*a^6*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.011, size = 288, normalized size = 1.3

$$\frac{C}{7\,a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{C}{5\,a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{C}{3\,a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{C}{a^4}\frac{1}{\sqrt{bx^2+a}}-C\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{9}{2}}-\frac{A}{2\,ax^2}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*C/a/(b*x^2+a)^(7/2)+1/5*C/a^2/(b*x^2+a)^(5/2)+1/3*C/a^3/(b*x^2+a)^(3/2)
+C/a^4/(b*x^2+a)^(1/2)-C/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*
A/a/x^2/(b*x^2+a)^(7/2)-9/14*A*b/a^2/(b*x^2+a)^(7/2)-9/10*A*b/a^3/(b*x^2+a)
^(5/2)-3/2*A*b/a^4/(b*x^2+a)^(3/2)-9/2*A*b/a^5/(b*x^2+a)^(1/2)+9/2*A*b/a^(1
1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-B/a/x/(b*x^2+a)^(7/2)-8/7*B*b/a^
2*x/(b*x^2+a)^(7/2)-48/35*B*b/a^3*x/(b*x^2+a)^(5/2)-64/35*B*b/a^4*x/(b*x^2+
a)^(3/2)-128/35*B*b/a^5*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12306, size = 1527, normalized size = 6.97

$$\left[-\frac{105\left(\left(2\,Cab^4-9\,Ab^5\right)x^{10}+4\left(2\,Ca^2b^3-9\,Aab^4\right)x^8+6\left(2\,Ca^3b^2-9\,Aa^2b^3\right)x^6+4\left(2\,Ca^4b-9\,Aa^3b^2\right)x^4+\left(2\,Ca^5-9\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8
+ 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*
C*a^5 - 9*A*a^4*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(768*B*a*b^4*x^9 + 2688*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5 + 1
680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 350*(2*
C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b^2)*x^
4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b
^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), 1/210*(105*((2*C*a*b^4 -
9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 - 9*A*a^2*
b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b)*x^2)*sqr
t(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (768*B*a*b^4*x^9 + 2688*B*a^2*b^3*
x^7 + 3360*B*a^3*b^2*x^5 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)
*x^8 + 210*B*a^5*x - 350*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*
(2*C*a^4*b - 9*A*a^3*b^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 +
 a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20897, size = 439, normalized size = 2.

$$-\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^4x}{a^5}-\frac{35\left(Ca^{24}b^6-4\,Aa^{23}b^7\right)}{a^{28}b^3}\right)x+\frac{308\,Bb^3}{a^4}\right)x-\frac{35\left(10\,Ca^{25}b^5-39\,Aa^{24}b^6\right)}{a^{28}b^3}\right)x+\frac{1050\,Bb^2}{a^3}\right)x-\frac{14\left(29\,Ca^{26}b^4-108\,Aa^{25}b^5\right)}{a^{28}b^3}\right)x+\frac{420\,B}{a^2}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(a^28*b^3))
*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b^6)/(a^28*b^3))*x +
1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A*a^25*b^5)/(a^28*b^3))*x + 420
*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 291*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a)^(7
/2) + (2*C*a - 9*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt
(-a)*a^5) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(
b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

3.58 $\int \frac{A(cx)^m}{a+bx^2}\,dx$

Optimal. Leaf size=45

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m))
```

Rubi [A] time = 0.0166907, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 364}

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = A\int \frac{(cx)^m}{a+bx^2}\,dx$$
$$= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}$$

Mathematica [A] time = 0.0085597, size = 43, normalized size = 0.96

$$\frac{Ax(cx)^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/
(a*(1 + m))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{A\,(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A*(c*x)^m/(b*x^2+a),x)
```

```
[Out] int(A*(c*x)^m/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$A\int \frac{(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] A*integrate((c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m A}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*A/(b*x^2 + a), x)
```

Sympy [C] time = 1.51525, size = 97, normalized size = 2.16

$$A\left(\frac{c^m m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)**m/(b*x**2+a),x)
```

```
[Out] A*(c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m A}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*A/(b*x^2 + a), x)
```

3.59 $\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx$

Optimal. Leaf size=91

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rubi [A] time = 0.0427198, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {808, 364}

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = A\int \frac{(cx)^m}{a+bx^2}\,dx + \frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}$$
$$= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)} + \frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

Mathematica [A] time = 0.0290382, size = 82, normalized size = 0.9

$$\frac{x(cx)^m\left(A(m+2)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+B(m+1)x\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;-\frac{bx^2}{a}\right)\right)}{a(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -((b*x^2)/a)
] + A*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a
*(1 + m)*(2 + m))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{(Bx+A)\,(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)\,(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx+A)(cx)^m}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] time = 4.73584, size = 192, normalized size = 2.11

$$\frac{Ac^m mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m mx^2x^m\Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}\right.}{4a\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**m*m*x**2
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*ga
mma(m/2 + 2)) + B*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx+A)(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.60 $\int \frac{(cx)^m \left(A+Cx^2\right)}{a+bx^2} \, dx$

Optimal. Leaf size=76

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m))
```

Rubi [A] time = 0.0386468, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {459, 364}

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m\left(A+Cx^2\right)}{a+bx^2}\,dx = \frac{C(cx)^{1+m}}{bc(1+m)} - \frac{(-Ab(1+m)+aC(1+m))\int \frac{(cx)^m}{a+bx^2}\,dx}{b(1+m)}$$
$$= \frac{C(cx)^{1+m}}{bc(1+m)} + \frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}$$

Mathematica [A] time = 0.0537702, size = 56, normalized size = 0.74

$$\frac{x(cx)^m\left((Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aC\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(a*C + (A*b - a*C)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+A\right)(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+A\right)(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + A\right)(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] time = 5.37861, size = 204, normalized size = 2.68

$$\frac{Ac^m mxx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Ac^m xx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Cc^m mx^3x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right)\Gamma\left(\frac{m}{2}\right.}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(C*x**2+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + C*c**m*m*x**3
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*
a*gamma(m/2 + 5/2)) + 3*C*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a,
 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + A\right)(cx)^m}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

3.61 $\int \frac{(cx)^m\left(A+Bx+Cx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=121

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m)) + (B*(c*x)^(2 +
 m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m
))
```

Rubi [A] time = 0.122474, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1802, 808, 364}

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m)) + (B*(c*x)^(2 +
 m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m
))
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m\left(A+Bx+Cx^2\right)}{a+bx^2}\,dx &= \int \left(\frac{C(cx)^m}{b}+\frac{(cx)^m(Ab-aC+bBx)}{b\left(a+bx^2\right)}\right)dx\\ &= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{\int \frac{(cx)^m(Ab-aC+bBx)}{a+bx^2}\,dx}{b}\\ &= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}+\frac{(Ab-aC)\int \frac{(cx)^m}{a+bx^2}\,dx}{b}\\ &= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}+\frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}\end{aligned}$$

Mathematica [A] time = 0.0745928, size = 99, normalized size = 0.82

$$\frac{x(cx)^m\left((m+2)(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+bB(m+1)x\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;-\frac{bx^2}{a}\right)+aC(m+2)\right)}{ab(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(a*C*(2 + m) + b*B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m
/2, -((b*x^2)/a)] + (A*b - a*C)*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*(2 + m))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+Bx+A\right)(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Cx^2 + Bx + A)(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] time = 6.41407, size = 298, normalized size = 2.46

$$\frac{Ac^m mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m mx^2x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}\right.}{4a\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**m*m*x**2
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*ga
mma(m/2 + 2)) + B*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2)) + C*c**m*m*x**3*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
 + 3*C*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamm
a(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.62 $\int x^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=65

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Rubi [A] time = 0.0742998, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) dx &= \int \left(aAx^3+aBx^4+(Ab+aC)x^5+(bB+aD)x^6+bCx^7+bDx^8\right) dx\\ &= \frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9\end{aligned}$$

Mathematica [A] time = 0.0152672, size = 65, normalized size = 1.

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Maple [A] time = 0.002, size = 54, normalized size = 0.8

$$\frac{aAx^4}{4} + \frac{aBx^5}{5} + \frac{(Ab + aC)\,x^6}{6} + \frac{(Bb + aD)\,x^7}{7} + \frac{bCx^8}{8} + \frac{bDx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

Maxima [A] time = 1.06304, size = 72, normalized size = 1.11

$$\frac{1}{9}\,Dbx^9 + \frac{1}{8}\,Cbx^8 + \frac{1}{7}\,(Da + Bb)x^7 + \frac{1}{5}\,Bax^5 + \frac{1}{6}\,(Ca + Ab)x^6 + \frac{1}{4}\,Aax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
A*b)*x^6 + 1/4*A*a*x^4
```

Fricas [A] time = 1.23511, size = 150, normalized size = 2.31

$$\frac{1}{9}x^9bD + \frac{1}{8}x^8bC + \frac{1}{7}x^7aD + \frac{1}{7}x^7bB + \frac{1}{6}x^6aC + \frac{1}{6}x^6bA + \frac{1}{5}x^5aB + \frac{1}{4}x^4aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b*D + 1/8*x^8*b*C + 1/7*x^7*a*D + 1/7*x^7*b*B + 1/6*x^6*a*C + 1/6*x
^6*b*A + 1/5*x^5*a*B + 1/4*x^4*a*A
```

Sympy [A] time = 0.06462, size = 60, normalized size = 0.92

$$\frac{Aax^4}{4} + \frac{Bax^5}{5} + \frac{Cbx^8}{8} + \frac{Dbx^9}{9} + x^7\left(\frac{Bb}{7} + \frac{Da}{7}\right) + x^6\left(\frac{Ab}{6} + \frac{Ca}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 + D*a/7) +
x**6*(A*b/6 + C*a/6)
```

Giac [A] time = 1.56146, size = 77, normalized size = 1.18

$$\frac{1}{9}\,Dbx^9 + \frac{1}{8}\,Cbx^8 + \frac{1}{7}\,Dax^7 + \frac{1}{7}\,Bbx^7 + \frac{1}{6}\,Cax^6 + \frac{1}{6}\,Abx^6 + \frac{1}{5}\,Bax^5 + \frac{1}{4}\,Aax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x^6 + 1/6*A
*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

3.63 $\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Rubi [A] time = 0.0764878, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx&=\int\left(aAx^2+aBx^3+(Ab+aC)x^4+(bB+aD)x^5+bCx^6+bDx^7\right)\,dx\\&=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8\end{aligned}$$

Mathematica [A] time = 0.0155026, size = 65, normalized size = 1.

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Maple [A] time = 0.002, size = 54, normalized size = 0.8

$$\frac{aAx^3}{3} + \frac{aBx^4}{4} + \frac{(Ab + aC)\,x^5}{5} + \frac{(Bb + aD)\,x^6}{6} + \frac{bCx^7}{7} + \frac{bDx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

Maxima [A] time = 1.05344, size = 72, normalized size = 1.11

$$\frac{1}{8}\,Dbx^8 + \frac{1}{7}\,Cbx^7 + \frac{1}{6}\,(Da + Bb)x^6 + \frac{1}{4}\,Bax^4 + \frac{1}{5}\,(Ca + Ab)x^5 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A] time = 1.23049, size = 150, normalized size = 2.31

$$\frac{1}{8}x^8bD + \frac{1}{7}x^7bC + \frac{1}{6}x^6aD + \frac{1}{6}x^6bB + \frac{1}{5}x^5aC + \frac{1}{5}x^5bA + \frac{1}{4}x^4aB + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b*D + 1/7*x^7*b*C + 1/6*x^6*a*D + 1/6*x^6*b*B + 1/5*x^5*a*C + 1/5*x
^5*b*A + 1/4*x^4*a*B + 1/3*x^3*a*A
```

Sympy [A] time = 0.06503, size = 60, normalized size = 0.92

$$\frac{Aax^3}{3} + \frac{Bax^4}{4} + \frac{Cbx^7}{7} + \frac{Dbx^8}{8} + x^6\left(\frac{Bb}{6} + \frac{Da}{6}\right) + x^5\left(\frac{Ab}{5} + \frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 + D*a/6) +
x**5*(A*b/5 + C*a/5)
```

Giac [A] time = 1.53007, size = 77, normalized size = 1.18

$$\frac{1}{8}Dbx^8 + \frac{1}{7}Cbx^7 + \frac{1}{6}Dax^6 + \frac{1}{6}Bbx^6 + \frac{1}{5}Cax^5 + \frac{1}{5}Abx^5 + \frac{1}{4}Bax^4 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*A
*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

3.64 $\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Rubi [A] time = 0.0609981, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1802}

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(aAx+aBx^2+(Ab+aC)x^3+(bB+aD)x^4+bCx^5+bDx^6\right)\,dx\\ &= \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7\end{aligned}$$

Mathematica [A] time = 0.0088981, size = 65, normalized size = 1.

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Maple [A] time = 0., size = 54, normalized size = 0.8

$$\frac{aAx^2}{2} + \frac{aBx^3}{3} + \frac{(Ab + aC)\,x^4}{4} + \frac{(Bb + aD)\,x^5}{5} + \frac{bCx^6}{6} + \frac{bDx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

Maxima [A] time = 1.22424, size = 72, normalized size = 1.11

$$\frac{1}{7}\,Dbx^7 + \frac{1}{6}\,Cbx^6 + \frac{1}{5}\,(Da + Bb)x^5 + \frac{1}{3}\,Bax^3 + \frac{1}{4}\,(Ca + Ab)x^4 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A] time = 1.22168, size = 150, normalized size = 2.31

$$\frac{1}{7}x^7bD + \frac{1}{6}x^6bC + \frac{1}{5}x^5aD + \frac{1}{5}x^5bB + \frac{1}{4}x^4aC + \frac{1}{4}x^4bA + \frac{1}{3}x^3aB + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*D + 1/6*x^6*b*C + 1/5*x^5*a*D + 1/5*x^5*b*B + 1/4*x^4*a*C + 1/4*x
^4*b*A + 1/3*x^3*a*B + 1/2*x^2*a*A
```

Sympy [A] time = 0.064319, size = 60, normalized size = 0.92

$$\frac{Aax^2}{2} + \frac{Bax^3}{3} + \frac{Cbx^6}{6} + \frac{Dbx^7}{7} + x^5\left(\frac{Bb}{5} + \frac{Da}{5}\right) + x^4\left(\frac{Ab}{4} + \frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 + D*a/5) +
x**4*(A*b/4 + C*a/4)
```

Giac [A] time = 1.53279, size = 77, normalized size = 1.18

$$\frac{1}{7}\,Dbx^7 + \frac{1}{6}\,Cbx^6 + \frac{1}{5}\,Dax^5 + \frac{1}{5}\,Bbx^5 + \frac{1}{4}\,Cax^4 + \frac{1}{4}\,Abx^4 + \frac{1}{3}\,Bax^3 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x^4 + 1/4*A
*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

3.65 $\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Rubi [A] time = 0.0411648, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1810}

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(aA+aBx+(Ab+aC)x^2+(bB+aD)x^3+bCx^4+bDx^5\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6\end{aligned}$$

Mathematica [A] time = 0.0089797, size = 60, normalized size = 1.

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Maple [A] time = 0.001, size = 51, normalized size = 0.9

$$aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+aC)\,x^{3}}{3}+\frac{(Bb+aD)\,x^{4}}{4}+\frac{bC\,x^{5}}{5}+\frac{bD\,x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

Maxima [A] time = 1.03893, size = 68, normalized size = 1.13

$$\frac{1}{6}\,Dbx^{6}+\frac{1}{5}\,Cbx^{5}+\frac{1}{4}\,(Da+Bb)x^{4}+\frac{1}{2}\,Bax^{2}+\frac{1}{3}\,(Ca+Ab)x^{3}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
A*b)*x^3 + A*a*x
```

Fricas [A] time = 1.28853, size = 142, normalized size = 2.37

$$\frac{1}{6}x^{6}bD+\frac{1}{5}x^{5}bC+\frac{1}{4}x^{4}aD+\frac{1}{4}x^{4}bB+\frac{1}{3}x^{3}aC+\frac{1}{3}x^{3}bA+\frac{1}{2}x^{2}aB+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b*D + 1/5*x^5*b*C + 1/4*x^4*a*D + 1/4*x^4*b*B + 1/3*x^3*a*C + 1/3*x
^3*b*A + 1/2*x^2*a*B + x*a*A
```

Sympy [A] time = 0.062735, size = 56, normalized size = 0.93

$$Aax + \frac{Bax^2}{2} + \frac{Cbx^5}{5} + \frac{Dbx^6}{6} + x^4\left(\frac{Bb}{4} + \frac{Da}{4}\right) + x^3\left(\frac{Ab}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3*
(A*b/3 + C*a/3)
```

Giac [A] time = 1.15165, size = 73, normalized size = 1.22

$$\frac{1}{6}Dbx^6 + \frac{1}{5}Cbx^5 + \frac{1}{4}Dax^4 + \frac{1}{4}Bbx^4 + \frac{1}{3}Cax^3 + \frac{1}{3}Abx^3 + \frac{1}{2}Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*A
*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

3.66 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx$

Optimal. Leaf size=56

$$\frac{1}{2}x^2(aC + Ab) + aA\log(x) + \frac{1}{3}x^3(aD + bB) + aBx + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5$$

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Rubi [A] time = 0.0400955, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{2}x^2(aC + Ab) + aA\log(x) + \frac{1}{3}x^3(aD + bB) + aBx + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a + bx^2)(A + Bx + Cx^2 + Dx^3)}{x} \, dx &= \int \left(aB + \frac{aA}{x} + (Ab + aC)x + (bB + aD)x^2 + bCx^3 + bDx^4\right) dx \\ &= aBx + \frac{1}{2}(Ab + aC)x^2 + \frac{1}{3}(bB + aD)x^3 + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5 + aA\log(x)\end{aligned}$$

Mathematica [A] time = 0.0147159, size = 56, normalized size = 1.

$$\frac{1}{2}x^2(aC + Ab) + aA\log(x) + \frac{1}{3}x^3(aD + bB) + aBx + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Maple [A] time = 0.004, size = 53, normalized size = 1.

$$\frac{bDx^5}{5} + \frac{bCx^4}{4} + \frac{bBx^3}{3} + \frac{Dx^3a}{3} + \frac{Ax^2b}{2} + \frac{Cx^2a}{2} + Bax + aA\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/5*b*D*x^5+1/4*b*C*x^4+1/3*b*B*x^3+1/3*D*x^3*a+1/2*A*x^2*b+1/2*C*x^2*a+B*a
*x+a*A*ln(x)
```

Maxima [A] time = 1.0298, size = 65, normalized size = 1.16

$$\frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}(Da + Bb)x^3 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.285238, size = 54, normalized size = 0.96

$$Aa\log(x) + Bax + \frac{Cbx^4}{4} + \frac{Dbx^5}{5} + x^3\left(\frac{Bb}{3} + \frac{Da}{3}\right) + x^2\left(\frac{Ab}{2} + \frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/3) + x**2*
(A*b/2 + C*a/2)
```

Giac [A] time = 1.16539, size = 72, normalized size = 1.29

$$\frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}Dax^3 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*A
*b*x^2 + B*a*x + A*a*log(abs(x))
```

3.67 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} \, dx$

Optimal. Leaf size=54

$$x(aC + Ab) - \frac{aA}{x} + \frac{1}{2}x^2(aD + bB) + aB\log(x) + \frac{1}{3}bCx^3 + \frac{1}{4}bDx^4$$

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Rubi [A] time = 0.0481677, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$x(aC + Ab) - \frac{aA}{x} + \frac{1}{2}x^2(aD + bB) + aB\log(x) + \frac{1}{3}bCx^3 + \frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+(bB+aD)x+bCx^2+bDx^3\right)dx\\ &= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)\end{aligned}$$

Mathematica [A] time = 0.0326805, size = 54, normalized size = 1.

$$x(aC + Ab) - \frac{aA}{x} + \frac{1}{2}x^2(aD + bB) + aB\log(x) + \frac{1}{3}bCx^3 + \frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Maple [A] time = 0.007, size = 50, normalized size = 0.9

$$\frac{bDx^4}{4} + \frac{bCx^3}{3} + \frac{Bx^2b}{2} + \frac{Dx^2a}{2} + Abx + aCx + aB\ln(x) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/4*b*D*x^4+1/3*b*C*x^3+1/2*B*x^2*b+1/2*D*x^2*a+A*b*x+a*C*x+a*B*ln(x)-a*A/x
```

Maxima [A] time = 1.21773, size = 65, normalized size = 1.2

$$\frac{1}{4}Dbx^4 + \frac{1}{3}Cbx^3 + \frac{1}{2}(Da + Bb)x^2 + Ba\log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C*a + A*b)*
x - A*a/x
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.302393, size = 49, normalized size = 0.91

$$-\frac{Aa}{x} + Ba\log(x) + \frac{Cbx^3}{3} + \frac{Dbx^4}{4} + x^2\left(\frac{Bb}{2} + \frac{Da}{2}\right) + x(Ab + Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a/2) + x*(A
*b + C*a)
```

Giac [A] time = 1.17958, size = 68, normalized size = 1.26

$$\frac{1}{4}Dbx^4 + \frac{1}{3}Cbx^3 + \frac{1}{2}Dax^2 + \frac{1}{2}Bbx^2 + Cax + Abx + Ba\log(|x|) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*a
*log(abs(x)) - A*a/x
```

3.68 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx$

Optimal. Leaf size=54

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*b
 + a*C)*Log[x]
```

Rubi [A] time = 0.048458, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*b
 + a*C)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx &= \int \left(bB\left(1+\frac{aD}{bB}\right)+\frac{aA}{x^3}+\frac{aB}{x^2}+\frac{Ab+aC}{x}+bCx+bDx^2\right) dx\\ &= -\frac{aA}{2x^2}-\frac{aB}{x}+(bB+aD)x+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3+(Ab+aC)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0261517, size = 51, normalized size = 0.94

$$\log(x)(aC+Ab)-\frac{a\left(A+2Bx-2Dx^3\right)}{2x^2}+\frac{1}{6}bx\left(6B+3Cx+2Dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] (b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (A*b
+ a*C)*Log[x]
```

Maple [A] time = 0.005, size = 48, normalized size = 0.9

$$\frac{bDx^3}{3}+\frac{bCx^2}{2}+bBx+aDx+A\ln(x)\,b+C\ln(x)\,a-\frac{Aa}{2\,x^2}-\frac{Ba}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/3*b*D*x^3+1/2*b*C*x^2+b*B*x+a*D*x+A*ln(x)*b+C*ln(x)*a-1/2*a*A/x^2-a*B/x
```

Maxima [A] time = 1.06238, size = 65, normalized size = 1.2

$$\frac{1}{3}\,Dbx^3+\frac{1}{2}\,Cbx^2+(Da+Bb)x+(Ca+Ab)\log(x)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) - 1/2*(2*B*a
*x + A*a)/x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.415662, size = 49, normalized size = 0.91

$$\frac{Cbx^2}{2} + \frac{Dbx^3}{3} + x(Bb + Da) + (Ab + Ca)\log(x) - \frac{Aa + 2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) - (A*a + 2*B*a
*x)/(2*x**2)
```

Giac [A] time = 1.15191, size = 65, normalized size = 1.2

$$\frac{1}{3}Dbx^3 + \frac{1}{2}Cbx^2 + Dax + Bbx + (Ca + Ab)\log(|x|) - \frac{2Bax + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*(
2*B*a*x + A*a)/x^2
```

3.69 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4} dx$

Optimal. Leaf size=54

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*B
 + a*D)*Log[x]
```

Rubi [A] time = 0.0487032, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*B
 + a*D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4} dx &= \int \left(bC+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB+aD}{x}+bDx\right) dx\\ &= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0174944, size = 55, normalized size = 1.02

$$\frac{-aC-Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*x^2)/2 + (
b*B + a*D)*Log[x]
```

Maple [A] time = 0.005, size = 51, normalized size = 0.9

$$\frac{bDx^2}{2} + bCx + B\ln(x)\,b + D\ln(x)\,a - \frac{Aa}{3x^3} - \frac{Ba}{2x^2} - \frac{Ab}{x} - \frac{aC}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/2*b*D*x^2+b*C*x+B*ln(x)*b+D*ln(x)*a-1/3*a*A/x^3-1/2*a*B/x^2-1/x*A*b-1/x*a
*C
```

Maxima [A] time = 1.00688, size = 66, normalized size = 1.22

$$\frac{1}{2}Dbx^2 + Cbx + (Da + Bb)\log(x) - \frac{3Bax + 6(Ca + Ab)x^2 + 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^2
 + 2*A*a)/x^3
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.80509, size = 53, normalized size = 0.98

$$Cbx + \frac{Dbx^2}{2} + (Bb + Da)\log(x) - \frac{2Aa + 3Bax + x^2(6Ab + 6Ca)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) - (2*A*a + 3*B*a*x + x**2*(6*A*b +
6*C*a))/(6*x**3)
```

Giac [A] time = 1.19355, size = 68, normalized size = 1.26

$$\frac{1}{2}Dbx^2 + Cbx + (Da + Bb)\log(|x|) - \frac{3Bax + 6(Ca + Ab)x^2 + 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*b
)*x^2 + 2*A*a)/x^3
```

3.70 $\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=109

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{1}$$

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)*
x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)/1
0 + (b^2*D*x^11)/11
```

Rubi [A] time = 0.124172, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)*
x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)/1
0 + (b^2*D*x^11)/11
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(a^2Ax^3+a^2Bx^4+a(2Ab+aC)x^5+a(2bB+aD)x^6+b(Ab+2aC)x^7+\right.$$

$$=\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7+\frac{1}{8}b(Ab+2aC)$$

Mathematica [A] time = 0.0370082, size = 98, normalized size = 0.9

$$a^2\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{1}{42}x^6(7C+6Dx)\right)+\frac{1}{252}abx^6(84A+x(72B+7x(9C+8Dx)))+\frac{b^2x^8\left(495A+4x\left(110B+99Cx+9\right.\right.}{3960}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(495*A + 4*
x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A + x*(72*B + 7*x*(9*C
+ 8*D*x))))/252
```

Maple [A] time = 0.001, size = 102, normalized size = 0.9

$$\frac{b^2Dx^{11}}{11}+\frac{b^2Cx^{10}}{10}+\frac{\left(b^2B+2\,abD\right)x^9}{9}+\frac{\left(Ab^2+2\,abC\right)x^8}{8}+\frac{\left(2\,Bba+a^2D\right)x^7}{7}+\frac{\left(2\,Aab+a^2C\right)x^6}{6}+\frac{a^2Bx^5}{5}+\frac{a^2Ax^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^2+2*C*a*b)
*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/5*a^2*B*x^5+1/4*a^2*
A*x^4
```

Maxima [A] time = 1.1289, size = 136, normalized size = 1.25

$$\frac{1}{11}Db^2x^{11}+\frac{1}{10}Cb^2x^{10}+\frac{1}{9}\left(2\,Dab+Bb^2\right)x^9+\frac{1}{8}\left(2\,Cab+Ab^2\right)x^8+\frac{1}{5}Ba^2x^5+\frac{1}{7}\left(Da^2+2\,Bab\right)x^7+\frac{1}{4}Aa^2x^4+\frac{1}{6}\left(Ca\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a*
b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^4
+ 1/6*(C*a^2 + 2*A*a*b)*x^6
```

Fricas [A] time = 1.22682, size = 263, normalized size = 2.41

$$\frac{1}{11}x^{11}b^2D + \frac{1}{10}x^{10}b^2C + \frac{2}{9}x^9baD + \frac{1}{9}x^9b^2B + \frac{1}{4}x^8baC + \frac{1}{8}x^8b^2A + \frac{1}{7}x^7a^2D + \frac{2}{7}x^7baB + \frac{1}{6}x^6a^2C + \frac{1}{3}x^6baA + \frac{1}{5}x^5a^2B +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^2*D + 1/10*x^10*b^2*C + 2/9*x^9*b*a*D + 1/9*x^9*b^2*B + 1/4*x^8
*b*a*C + 1/8*x^8*b^2*A + 1/7*x^7*a^2*D + 2/7*x^7*b*a*B + 1/6*x^6*a^2*C + 1/
3*x^6*b*a*A + 1/5*x^5*a^2*B + 1/4*x^4*a^2*A
```

Sympy [A] time = 0.076776, size = 110, normalized size = 1.01

$$\frac{Aa^2x^4}{4} + \frac{Ba^2x^5}{5} + \frac{Cb^2x^{10}}{10} + \frac{Db^2x^{11}}{11} + x^9\left(\frac{Bb^2}{9} + \frac{2Dab}{9}\right) + x^8\left(\frac{Ab^2}{8} + \frac{Cab}{4}\right) + x^7\left(\frac{2Bab}{7} + \frac{Da^2}{7}\right) + x^6\left(\frac{Aab}{3} + \frac{Ca^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11 + x**9*(B
*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7*(2*B*a*b/7 + D*a**2
/7) + x**6*(A*a*b/3 + C*a**2/6)
```

Giac [A] time = 1.12518, size = 142, normalized size = 1.3

$$\frac{1}{11}Db^2x^{11} + \frac{1}{10}Cb^2x^{10} + \frac{2}{9}Dabx^9 + \frac{1}{9}Bb^2x^9 + \frac{1}{4}Cabx^8 + \frac{1}{8}Ab^2x^8 + \frac{1}{7}Da^2x^7 + \frac{2}{7}Babx^7 + \frac{1}{6}Ca^2x^6 + \frac{1}{3}Aabx^6 + \frac{1}{5}Ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9 + 1/4*C*a
*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7 + 1/6*C*a^2*x^6 + 1/
3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

3.71 $\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal. Leaf size=109

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^1$$

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)*
x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/9
+ (b^2*D*x^10)/10
```

Rubi [A] time = 0.111717, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^1$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)*
x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/9
+ (b^2*D*x^10)/10
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)dx=\int\left(a^2Ax^2+a^2Bx^3+a(2Ab+aC)x^4+a(2bB+aD)x^5+b(Ab+2aC)x^6\right.$$

$$=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6+\frac{1}{7}b(Ab+2a$$

Mathematica [A] time = 0.0477629, size = 92, normalized size = 0.84

$$\frac{42a^2x^3(20A + x(15B + 2x(6C + 5Dx))) + 6abx^5(168A + 5x(28B + 3x(8C + 7Dx))) + b^2x^7(360A + 7x(45B + 4x(10C + 9\ldots}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(168*A + 5*x*
(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*
x))))/2520
```

Maple [A] time = 0.001, size = 102, normalized size = 0.9

$$\frac{b^2Dx^{10}}{10} + \frac{b^2Cx^9}{9} + \frac{(b^2B + 2\,abD)\,x^8}{8} + \frac{(Ab^2 + 2\,abC)\,x^7}{7} + \frac{(2\,Bba + a^2D)\,x^6}{6} + \frac{(2\,Aab + a^2C)\,x^5}{5} + \frac{a^2Bx^4}{4} + \frac{a^2Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^2*D*x^10+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+2*C*a*b)*x
^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*
x^3
```

Maxima [A] time = 0.97115, size = 136, normalized size = 1.25

$$\frac{1}{10}\,Db^2x^{10} + \frac{1}{9}\,Cb^2x^9 + \frac{1}{8}\left(2\,Dab + Bb^2\right)x^8 + \frac{1}{7}\left(2\,Cab + Ab^2\right)x^7 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{6}\left(Da^2 + 2\,Bab\right)x^6 + \frac{1}{3}\,Aa^2x^3 + \frac{1}{5}\left(Ca^2 + \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
+ A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3 +
1/5*(C*a^2 + 2*A*a*b)*x^5
```

Fricas [A] time = 1.25343, size = 261, normalized size = 2.39

$$\frac{1}{10}x^{10}b^2D + \frac{1}{9}x^9b^2C + \frac{1}{4}x^8baD + \frac{1}{8}x^8b^2B + \frac{2}{7}x^7baC + \frac{1}{7}x^7b^2A + \frac{1}{6}x^6a^2D + \frac{1}{3}x^6baB + \frac{1}{5}x^5a^2C + \frac{2}{5}x^5baA + \frac{1}{4}x^4a^2B +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2*D + 1/9*x^9*b^2*C + 1/4*x^8*b*a*D + 1/8*x^8*b^2*B + 2/7*x^7*b
*a*C + 1/7*x^7*b^2*A + 1/6*x^6*a^2*D + 1/3*x^6*b*a*B + 1/5*x^5*a^2*C + 2/5*
x^5*b*a*A + 1/4*x^4*a^2*B + 1/3*x^3*a^2*A
```

Sympy [A] time = 0.077921, size = 110, normalized size = 1.01

$$\frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Cb^2x^9}{9} + \frac{Db^2x^{10}}{10} + x^8\left(\frac{Bb^2}{8} + \frac{Dab}{4}\right) + x^7\left(\frac{Ab^2}{7} + \frac{2Cab}{7}\right) + x^6\left(\frac{Bab}{3} + \frac{Da^2}{6}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 + x**8*(B*b
**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(B*a*b/3 + D*a**2/6)
+ x**5*(2*A*a*b/5 + C*a**2/5)
```

Giac [A] time = 1.13061, size = 142, normalized size = 1.3

$$\frac{1}{10}Db^2x^{10} + \frac{1}{9}Cb^2x^9 + \frac{1}{4}Dabx^8 + \frac{1}{8}Bb^2x^8 + \frac{2}{7}Cabx^7 + \frac{1}{7}Ab^2x^7 + \frac{1}{6}Da^2x^6 + \frac{1}{3}Babx^6 + \frac{1}{5}Ca^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{4}Ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 + 2/7*C*a*b
*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*
A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

3.72 $\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 + (
b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3)/
(6*b)
```

Rubi [A] time = 0.0738811, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1582, 1810}

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 + (
b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3)/
(6*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\left(a+bx^2\right)^3}{6b}+\int\left(a+bx^2\right)^2\left(-Ax+x\left(A+Bx+Cx^2+Dx^3\right)\right)\,dx$$
$$=\frac{A\left(a+bx^2\right)^3}{6b}+\int\left(a^2Bx^2+a^2Cx^3+a(2bB+aD)x^4+2abCx^5+b(bB+2$$
$$=\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{1}{5}a(2bB+aD)x^5+\frac{1}{3}abCx^6+\frac{1}{7}b(bB+2aD)x^7+\frac{1}{8}b^2$$

Mathematica [A] time = 0.0383493, size = 92, normalized size = 0.88

$$\frac{42a^2x^2(30A+x(20B+3x(5C+4Dx)))+12abx^4(105A+2x(42B+5x(7C+6Dx)))+5b^2x^6(84A+x(72B+7x(9C+}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(105*A + 2*x
*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x
))))/2520
```

Maple [A] time = 0.001, size = 102, normalized size = 1.

$$\frac{b^2Dx^9}{9}+\frac{b^2Cx^8}{8}+\frac{\left(b^2B+2\,abD\right)x^7}{7}+\frac{\left(Ab^2+2\,abC\right)x^6}{6}+\frac{\left(2\,Bba+a^2D\right)x^5}{5}+\frac{\left(2\,Aab+a^2C\right)x^4}{4}+\frac{a^2Bx^3}{3}+\frac{a^2Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/9*b^2*D*x^9+1/8*b^2*C*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*C*a*b)*x^6
+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^
2
```

Maxima [A] time = 0.990046, size = 136, normalized size = 1.31

$$\frac{1}{9}\,Db^2x^9 + \frac{1}{8}\,Cb^2x^8 + \frac{1}{7}\left(2\,Dab + Bb^2\right)x^7 + \frac{1}{6}\left(2\,Cab + Ab^2\right)x^6 + \frac{1}{3}\,Ba^2x^3 + \frac{1}{5}\left(Da^2 + 2\,Bab\right)x^5 + \frac{1}{2}\,Aa^2x^2 + \frac{1}{4}\left(Ca^2 + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 + 1/
4*(C*a^2 + 2*A*a*b)*x^4
```

Fricas [A] time = 1.28963, size = 258, normalized size = 2.48

$$\frac{1}{9}x^9b^2D + \frac{1}{8}x^8b^2C + \frac{2}{7}x^7baD + \frac{1}{7}x^7b^2B + \frac{1}{3}x^6baC + \frac{1}{6}x^6b^2A + \frac{1}{5}x^5a^2D + \frac{2}{5}x^5baB + \frac{1}{4}x^4a^2C + \frac{1}{2}x^4baA + \frac{1}{3}x^3a^2B + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2*D + 1/8*x^8*b^2*C + 2/7*x^7*b*a*D + 1/7*x^7*b^2*B + 1/3*x^6*b*a
*C + 1/6*x^6*b^2*A + 1/5*x^5*a^2*D + 2/5*x^5*b*a*B + 1/4*x^4*a^2*C + 1/2*x^
4*b*a*A + 1/3*x^3*a^2*B + 1/2*x^2*a^2*A
```

Sympy [A] time = 0.078765, size = 110, normalized size = 1.06

$$\frac{Aa^2x^2}{2} + \frac{Ba^2x^3}{3} + \frac{Cb^2x^8}{8} + \frac{Db^2x^9}{9} + x^7\left(\frac{Bb^2}{7} + \frac{2Dab}{7}\right) + x^6\left(\frac{Ab^2}{6} + \frac{Cab}{3}\right) + x^5\left(\frac{2Bab}{5} + \frac{Da^2}{5}\right) + x^4\left(\frac{Aab}{2} + \frac{Ca^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x**7*(B*b**
2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*B*a*b/5 + D*a**2/5)
+ x**4*(A*a*b/2 + C*a**2/4)
```

Giac [A] time = 1.16452, size = 142, normalized size = 1.37

$$\frac{1}{9}Db^2x^9 + \frac{1}{8}Cb^2x^8 + \frac{2}{7}Dabx^7 + \frac{1}{7}Bb^2x^7 + \frac{1}{3}Cabx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{5}Da^2x^5 + \frac{2}{5}Babx^5 + \frac{1}{4}Ca^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{3}Ba^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1/3*C*a*b*x
^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*
a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

3.73 $\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=99

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/5
 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b)
```

Rubi [A] time = 0.0723119, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1810}

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/5
 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{B\left(a+bx^2\right)^3}{6b} + \int \left(a+bx^2\right)^2 \left(A+Cx^2+Dx^3\right) dx$$
$$= \frac{B\left(a+bx^2\right)^3}{6b} + \int \left(a^2A + a(2Ab+aC)x^2 + a^2Dx^3 + b(Ab+2aC)x^4 + 2abD$$
$$= a^2Ax + \frac{1}{3}a(2Ab+aC)x^3 + \frac{1}{4}a^2Dx^4 + \frac{1}{5}b(Ab+2aC)x^5 + \frac{1}{3}abDx^6 + \frac{1}{7}b^2Cx$$

Mathematica [A] time = 0.0357294, size = 88, normalized size = 0.89

$$\frac{1}{840}\left(70a^2x(12A+x(6B+x(4C+3Dx))) + 28abx^3(20A+x(15B+2x(6C+5Dx))) + b^2x^5(168A+5x(28B+3x(8C+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B +
 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/84
0
```

Maple [A] time = 0.002, size = 99, normalized size = 1.

$$\frac{b^2Dx^8}{8} + \frac{b^2Cx^7}{7} + \frac{\left(b^2B + 2\,abD\right)x^6}{6} + \frac{\left(Ab^2 + 2\,abC\right)x^5}{5} + \frac{\left(2\,Bba + a^2D\right)x^4}{4} + \frac{\left(2\,Aab + a^2C\right)x^3}{3} + \frac{Bx^2a^2}{2} + a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^5
+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*x^2*a^2+a^2*A*x
```

Maxima [A] time = 1.05777, size = 132, normalized size = 1.33

$$\frac{1}{8}Db^2x^8 + \frac{1}{7}Cb^2x^7 + \frac{1}{6}\left(2\,Dab + Bb^2\right)x^6 + \frac{1}{5}\left(2\,Cab + Ab^2\right)x^5 + \frac{1}{2}Ba^2x^2 + \frac{1}{4}\left(Da^2 + 2\,Bab\right)x^4 + Aa^2x + \frac{1}{3}\left(Ca^2 + 2\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C*a
^2 + 2*A*a*b)*x^3
```

Fricas [A] time = 1.26653, size = 250, normalized size = 2.53

$$\frac{1}{8}x^8b^2D + \frac{1}{7}x^7b^2C + \frac{1}{3}x^6baD + \frac{1}{6}x^6b^2B + \frac{2}{5}x^5baC + \frac{1}{5}x^5b^2A + \frac{1}{4}x^4a^2D + \frac{1}{2}x^4baB + \frac{1}{3}x^3a^2C + \frac{2}{3}x^3baA + \frac{1}{2}x^2a^2B + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2*D + 1/7*x^7*b^2*C + 1/3*x^6*b*a*D + 1/6*x^6*b^2*B + 2/5*x^5*b*a
*C + 1/5*x^5*b^2*A + 1/4*x^4*a^2*D + 1/2*x^4*b*a*B + 1/3*x^3*a^2*C + 2/3*x^
3*b*a*A + 1/2*x^2*a^2*B + x*a^2*A
```

Sympy [A] time = 0.07495, size = 107, normalized size = 1.08

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Cb^2x^7}{7} + \frac{Db^2x^8}{8} + x^6\left(\frac{Bb^2}{6} + \frac{Dab}{3}\right) + x^5\left(\frac{Ab^2}{5} + \frac{2Cab}{5}\right) + x^4\left(\frac{Bab}{2} + \frac{Da^2}{4}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6 +
 D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**3*
(2*A*a*b/3 + C*a**2/3)
```

Giac [A] time = 1.15732, size = 138, normalized size = 1.39

$$\frac{1}{8}Db^2x^8 + \frac{1}{7}Cb^2x^7 + \frac{1}{3}Dabx^6 + \frac{1}{6}Bb^2x^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{4}Da^2x^4 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*x
^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*A*
a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

3.74 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$

Optimal. Leaf size=92

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B + 2
*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rubi [A] time = 0.0685905, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1802}

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B + 2
*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{C(a+bx^2)^3}{6b} + \int \frac{(a+bx^2)^2(A+Bx+Dx^3)}{x}\,dx$$
$$= \frac{C(a+bx^2)^3}{6b} + \int \left(a^2B + \frac{a^2A}{x} + 2aAbx + a(2bB+aD)x^2 + Ab^2x^3 + b(bB\right.$$
$$= a^2Bx + aAbx^2 + \frac{1}{3}a(2bB+aD)x^3 + \frac{1}{4}Ab^2x^4 + \frac{1}{5}b(bB+2aD)x^5 + \frac{1}{7}b^2Dx^7$$

Mathematica [A] time = 0.0434073, size = 88, normalized size = 0.96

$$\frac{1}{420}x\left(70a^2(6B+x(3C+2Dx)) + 14abx(30A+x(20B+3x(5C+4Dx))) + b^2x^3(105A+2x(42B+5x(7C+6Dx)))\right) +$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3*x*(5*C +
4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)))))/420 + a^2*A*L
og[x]
```

Maple [A] time = 0.003, size = 100, normalized size = 1.1

$$\frac{b^2Dx^7}{7} + \frac{Cb^2x^6}{6} + \frac{Bx^5b^2}{5} + \frac{2\,Dx^5ab}{5} + \frac{Ab^2x^4}{4} + \frac{Cx^4ab}{2} + \frac{2\,Bx^3ab}{3} + \frac{Dx^3a^2}{3} + aAbx^2 + \frac{Cx^2a^2}{2} + a^2Bx + a^2A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/7*b^2*D*x^7+1/6*C*b^2*x^6+1/5*B*x^5*b^2+2/5*D*x^5*a*b+1/4*A*b^2*x^4+1/2*C
*x^4*a*b+2/3*B*x^3*a*b+1/3*D*x^3*a^2+a*A*b*x^2+1/2*C*x^2*a^2+a^2*B*x+a^2*A*
ln(x)
```

Maxima [A] time = 0.97097, size = 130, normalized size = 1.41

$$\frac{1}{7}Db^2x^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{5}\left(2\,Dab + Bb^2\right)x^5 + \frac{1}{4}\left(2\,Cab + Ab^2\right)x^4 + Ba^2x + \frac{1}{3}\left(Da^2 + 2\,Bab\right)x^3 + Aa^2\log(x) + \frac{1}{2}\left(Ca^2 +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.36586, size = 104, normalized size = 1.13

$$Aa^2\log(x) + Ba^2x + \frac{Cb^2x^6}{6} + \frac{Db^2x^7}{7} + x^5\left(\frac{Bb^2}{5} + \frac{2Dab}{5}\right) + x^4\left(\frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^3\left(\frac{2Bab}{3} + \frac{Da^2}{3}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(B*b**2/5 +
 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b/3 + D*a**2/3) + x**
2*(A*a*b + C*a**2/2)
```

Giac [A] time = 1.16297, size = 135, normalized size = 1.47

$$\frac{1}{7}Db^2x^7 + \frac{1}{6}Cb^2x^6 + \frac{2}{5}Dabx^5 + \frac{1}{5}Bb^2x^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{3}Da^2x^3 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1/2*C*a*b*x
^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*b*
x^2 + B*a^2*x + A*a^2*log(abs(x))
```

3.75 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (b
^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

Rubi [A] time = 0.080234, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1628}

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (b
^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} dx = \frac{D(a+bx^2)^3}{6b} + \int \frac{(a+bx^2)^2 (A+Bx+Cx^2)}{x^2} dx$$
$$= \frac{D(a+bx^2)^3}{6b} + \int \left(a(2Ab+aC) + \frac{a^2A}{x^2} + \frac{a^2B}{x} + 2abBx + b(Ab+2aC)x^2 +\right.$$
$$= -\frac{a^2A}{x} + a(2Ab+aC)x + abBx^2 + \frac{1}{3}b(Ab+2aC)x^3 + \frac{1}{4}b^2Bx^4 + \frac{1}{5}b^2Cx^5 +$$

Mathematica [A] time = 0.0503056, size = 88, normalized size = 0.98

$$a^2\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+a^2B\log(x)+\frac{1}{6}abx(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^2x^3(20A+x(15B+2x(6C+5Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))
/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*B*Log[x]
```

Maple [A] time = 0.006, size = 98, normalized size = 1.1

$$\frac{Db^2x^6}{6}+\frac{b^2Cx^5}{5}+\frac{b^2Bx^4}{4}+\frac{Dx^4ab}{2}+\frac{Ax^3b^2}{3}+\frac{2\,Cx^3ab}{3}+Bx^2ab+\frac{Dx^2a^2}{2}+2\,Aabx+a^2Cx+a^2B\ln(x)-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/6*D*b^2*x^6+1/5*b^2*C*x^5+1/4*b^2*B*x^4+1/2*D*x^4*a*b+1/3*A*x^3*b^2+2/3*C
*x^3*a*b+B*x^2*a*b+1/2*D*x^2*a^2+2*A*a*b*x+a^2*C*x+a^2*B*ln(x)-a^2*A/x
```

Maxima [A] time = 0.994366, size = 130, normalized size = 1.44

$$\frac{1}{6}Db^2x^6+\frac{1}{5}Cb^2x^5+\frac{1}{4}\left(2\,Dab+Bb^2\right)x^4+\frac{1}{3}\left(2\,Cab+Ab^2\right)x^3+Ba^2\log(x)+\frac{1}{2}\left(Da^2+2\,Bab\right)x^2-\frac{Aa^2}{x}+\left(Ca^2+2\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(2*C*a*b +
A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 - A*a^2/x + (C*a^2 +
2*A*a*b)*x
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.37863, size = 99, normalized size = 1.1

$$-\frac{Aa^2}{x} + Ba^2\log(x) + \frac{Cb^2x^5}{5} + \frac{Db^2x^6}{6} + x^4\left(\frac{Bb^2}{4} + \frac{Dab}{2}\right) + x^3\left(\frac{Ab^2}{3} + \frac{2Cab}{3}\right) + x^2\left(Bab + \frac{Da^2}{2}\right) + x\left(2Aab + Ca^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*(B*b**2/4
+ D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b + D*a**2/2) + x*(2*A
*a*b + C*a**2)
```

Giac [A] time = 1.22434, size = 132, normalized size = 1.47

$$\frac{1}{6}Db^2x^6 + \frac{1}{5}Cb^2x^5 + \frac{1}{2}Dabx^4 + \frac{1}{4}Bb^2x^4 + \frac{2}{3}Cabx^3 + \frac{1}{3}Ab^2x^3 + \frac{1}{2}Da^2x^2 + Babx^2 + Ca^2x + 2\,Aabx + Ba^2\log(|x|) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2/3*C*a*b*x
^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*a^
2*log(abs(x)) - A*a^2/x
```

3.76 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
+ (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*C)
*Log[x]
```

Rubi [A] time = 0.0863914, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
+ (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*C)
*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^3}\,dx = \int \left(a(2bB+aD)+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+b(bB+2aL\right.$$

$$=-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2+\frac{1}{3}b(bB+2aD)x^3+\frac{1}{4}b^2C$$

Mathematica [A] time = 0.0384175, size = 87, normalized size = 0.89

$$-\frac{a^2\left(A+2Bx-2Dx^3\right)}{2x^2}+a\log(x)(aC+2Ab)+\frac{1}{3}abx(6B+x(3C+2Dx))+\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^2*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (a*b*x*(6*B + x*(3*C + 2*D*x)))/3 +
(b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*(2*A*b + a*C)*Log[x]
```

Maple [A] time = 0.006, size = 97, normalized size = 1.

$$\frac{b^2Dx^5}{5}+\frac{b^2Cx^4}{4}+\frac{Bx^3b^2}{3}+\frac{2\,Dx^3ab}{3}+\frac{Ax^2b^2}{2}+Cx^2ab+2\,Bxab+a^2Dx+2\,A\ln(x)\,ab+C\ln(x)\,a^2-\frac{Aa^2}{2\,x^2}-\frac{Ba^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*B*x^3*b^2+2/3*D*x^3*a*b+1/2*A*x^2*b^2+C*x^2
*a*b+2*B*x*a*b+a^2*D*x+2*A*ln(x)*a*b+C*ln(x)*a^2-1/2*a^2*A/x^2-a^2*B/x
```

Maxima [A] time = 0.988491, size = 130, normalized size = 1.33

$$\frac{1}{5}\,Db^2x^5+\frac{1}{4}\,Cb^2x^4+\frac{1}{3}\left(2\,Dab+Bb^2\right)x^3+\frac{1}{2}\left(2\,Cab+Ab^2\right)x^2+\left(Da^2+2\,Bab\right)x+\left(Ca^2+2\,Aab\right)\log(x)-\frac{2\,Ba^2x+A}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(2*C*a*b +
A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^2*
x + A*a^2)/x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.511069, size = 99, normalized size = 1.01

$$\frac{Cb^2x^4}{4} + \frac{Db^2x^5}{5} + a(2Ab + Ca)\log(x) + x^3\left(\frac{Bb^2}{3} + \frac{2Dab}{3}\right) + x^2\left(\frac{Ab^2}{2} + Cab\right) + x\left(2Bab + Da^2\right) - \frac{Aa^2 + 2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*b**2/3 + 2
*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a**2) - (A*a**2 + 2*B*
a**2*x)/(2*x**2)
```

Giac [A] time = 1.16058, size = 131, normalized size = 1.34

$$\frac{1}{5}Db^2x^5 + \frac{1}{4}Cb^2x^4 + \frac{2}{3}Dabx^3 + \frac{1}{3}Bb^2x^3 + Cabx^2 + \frac{1}{2}Ab^2x^2 + Da^2x + 2Babx + \left(Ca^2 + 2Aab\right)\log(|x|) - \frac{2Ba^2x + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C*a*b*x^2 +
 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/2*
(2*B*a^2*x + A*a^2)/x^2
```

3.77 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)*
x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B + a*
D)*Log[x]
```

Rubi [A] time = 0.0857923, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)*
x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B + a*
D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx &= \int \left(b(Ab+2aC)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\frac{a(2bB+aD)}{x}+b(bB+2a\right.\\ &= -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+\frac{1}{2}b(bB+2aD)x^2+\frac{1}{3}b^2Cx^3\end{aligned}$$

Mathematica [A] time = 0.0472645, size = 83, normalized size = 0.85

$$-\frac{a^2(2A+3x(B+2Cx))}{6x^3}-\frac{2aAb}{x}+a\log(x)(aD+2bB)+abx(2C+Dx)+\frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] (-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) +
(b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*B + a*D)*Log[x]
```

Maple [A] time = 0.006, size = 97, normalized size = 1.

$$\frac{b^2Dx^4}{4}+\frac{b^2Cx^3}{3}+\frac{Bx^2b^2}{2}+Dx^2ab+Ab^2x+2\,abCx+2\,B\ln(x)\,ab+D\ln(x)\,a^2-\frac{Aa^2}{3\,x^3}-\frac{Ba^2}{2\,x^2}-2\,\frac{Aab}{x}-\frac{a^2C}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/4*b^2*D*x^4+1/3*b^2*C*x^3+1/2*B*x^2*b^2+D*x^2*a*b+A*b^2*x+2*a*b*C*x+2*B*l
n(x)*a*b+D*ln(x)*a^2-1/3*a^2*A/x^3-1/2*a^2*B/x^2-2*a/x*A*b-a^2/x*C
```

Maxima [A] time = 0.98409, size = 131, normalized size = 1.34

$$\frac{1}{4}\,Db^2x^4+\frac{1}{3}\,Cb^2x^3+\frac{1}{2}\left(2\,Dab+Bb^2\right)x^2+\left(2\,Cab+Ab^2\right)x+\left(Da^2+2\,Bab\right)\log(x)-\frac{3\,Ba^2x+2\,Aa^2+6\left(Ca^2+2\,Aab\right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*a*b + A*b^
2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A
*a*b)*x^2)/x^3
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.974943, size = 99, normalized size = 1.01

$$\frac{Cb^2x^3}{3} + \frac{Db^2x^4}{4} + a(2Bb + Da)\log(x) + x^2\left(\frac{Bb^2}{2} + Dab\right) + x\left(Ab^2 + 2Cab\right) - \frac{2Aa^2 + 3Ba^2x + x^2\left(12Aab + 6Ca^2\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*b**2/2 + D
*a*b) + x*(A*b**2 + 2*C*a*b) - (2*A*a**2 + 3*B*a**2*x + x**2*(12*A*a*b + 6*
C*a**2))/(6*x**3)
```

Giac [A] time = 1.16514, size = 131, normalized size = 1.34

$$\frac{1}{4}Db^2x^4 + \frac{1}{3}Cb^2x^3 + Dabx^2 + \frac{1}{2}Bb^2x^2 + 2Cabx + Ab^2x + \left(Da^2 + 2Bab\right)\log(|x|) - \frac{3Ba^2x + 2Aa^2 + 6\left(Ca^2 + 2Aab\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a*b*x + A*b
^2*x + (D*a^2 + 2*B*a*b)*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

3.78 $\int x^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=149

$$\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{5}a^3Bx^5+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+$$

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Rubi [A] time = 0.186161, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{5}a^3Bx^5+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx=\int\left(a^3Ax^3+a^3Bx^4+a^2(3Ab+aC)x^5+a^2(3bB+aD)x^6+3ab(Ab+aC)x^7\right.$$
$$=\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab(Ab+aC$$

Mathematica [A] time = 0.0271811, size = 149, normalized size = 1.

$$\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{5}a^3Bx^5+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Maple [A] time = 0.003, size = 150, normalized size = 1.

$$\frac{b^3Dx^{13}}{13}+\frac{b^3Cx^{12}}{12}+\frac{\left(b^3B+3\,ab^2D\right)x^{11}}{11}+\frac{\left(Ab^3+3\,ab^2C\right)x^{10}}{10}+\frac{\left(3\,ab^2B+3\,a^2bD\right)x^9}{9}+\frac{\left(3\,ab^2A+3\,a^2bC\right)x^8}{8}+\frac{\left(3\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/13*b^3*D*x^13+1/12*b^3*C*x^12+1/11*(B*b^3+3*D*a*b^2)*x^11+1/10*(A*b^3+3*C
*a*b^2)*x^10+1/9*(3*B*a*b^2+3*D*a^2*b)*x^9+1/8*(3*A*a*b^2+3*C*a^2*b)*x^8+1/
7*(3*B*a^2*b+D*a^3)*x^7+1/6*(3*A*a^2*b+C*a^3)*x^6+1/5*a^3*B*x^5+1/4*a^3*A*x
^4
```

Maxima [A] time = 1.01083, size = 196, normalized size = 1.32

$$\frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{1}{11}\left(3\,Dab^2+Bb^3\right)x^{11}+\frac{1}{10}\left(3\,Cab^2+Ab^3\right)x^{10}+\frac{1}{3}\left(Da^2b+Bab^2\right)x^9+\frac{1}{5}Ba^3x^5+\frac{3}{8}\left(Ca^2b+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(3
*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/8*
(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6
```

```
*(C*a^3 + 3*A*a^2*b)*x^6
```

Fricas [A] time = 1.37057, size = 382, normalized size = 2.56

$$\frac{1}{13}x^{13}b^3D + \frac{1}{12}x^{12}b^3C + \frac{3}{11}x^{11}b^2aD + \frac{1}{11}x^{11}b^3B + \frac{3}{10}x^{10}b^2aC + \frac{1}{10}x^{10}b^3A + \frac{1}{3}x^9ba^2D + \frac{1}{3}x^9b^2aB + \frac{3}{8}x^8ba^2C + \frac{3}{8}x^8b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^3*D + 1/12*x^12*b^3*C + 3/11*x^11*b^2*a*D + 1/11*x^11*b^3*B + 3
/10*x^10*b^2*a*C + 1/10*x^10*b^3*A + 1/3*x^9*b*a^2*D + 1/3*x^9*b^2*a*B + 3/
8*x^8*b*a^2*C + 3/8*x^8*b^2*a*A + 1/7*x^7*a^3*D + 3/7*x^7*b*a^2*B + 1/6*x^6
*a^3*C + 1/2*x^6*b*a^2*A + 1/5*x^5*a^3*B + 1/4*x^4*a^3*A
```

Sympy [A] time = 0.091382, size = 163, normalized size = 1.09

$$\frac{Aa^3x^4}{4} + \frac{Ba^3x^5}{5} + \frac{Cb^3x^{12}}{12} + \frac{Db^3x^{13}}{13} + x^{11}\left(\frac{Bb^3}{11} + \frac{3Dab^2}{11}\right) + x^{10}\left(\frac{Ab^3}{10} + \frac{3Cab^2}{10}\right) + x^9\left(\frac{Bab^2}{3} + \frac{Da^2b}{3}\right) + x^8\left(\frac{3Aab^2}{8} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13 + x**11*(
B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b**2/10) + x**9*(B*a*
b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3*C*a**2*b/8) + x**7*(3*B*a**2*
b/7 + D*a**3/7) + x**6*(A*a**2*b/2 + C*a**3/6)
```

Giac [A] time = 1.1554, size = 207, normalized size = 1.39

$$\frac{1}{13}Db^3x^{13} + \frac{1}{12}Cb^3x^{12} + \frac{3}{11}Dab^2x^{11} + \frac{1}{11}Bb^3x^{11} + \frac{3}{10}Cab^2x^{10} + \frac{1}{10}Ab^3x^{10} + \frac{1}{3}Da^2bx^9 + \frac{1}{3}Bab^2x^9 + \frac{3}{8}Ca^2bx^8 + \frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^3*x^11 + 3
/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 + 1/3*B*a*b^2*x^9 + 3/
8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x^7 + 3/7*B*a^2*b*x^7 + 1/6*C*a
^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^3*x^5 + 1/4*A*a^3*x^4
```

3.79 $\int x^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=149

$$\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{3}a^3Ax^3+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{4}a^3Bx^4+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+b$$

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B + a
*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2*(
A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b^3*
D*x^12)/12
```

Rubi [A] time = 0.141061, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{3}a^3Ax^3+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{4}a^3Bx^4+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+b$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B + a
*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2*(
A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b^3*
D*x^12)/12
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx=\int\left(a^3Ax^2+a^3Bx^3+a^2(3Ab+aC)x^4+a^2(3bB+aD)x^5+3ab(Ab+aC)x\right.$$

$$=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}ab(Ab+a$$

Mathematica [A] time = 0.0595595, size = 125, normalized size = 0.84

$$\frac{99a^2bx^5(168A+5x(28B+3x(8C+7Dx)))+462a^3x^3(20A+x(15B+2x(6C+5Dx)))+33ab^2x^7(360A+7x(45B+4x\ldots}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5*x*(28*B + 3*x*(8*C
+ 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/27720
```

Maple [A] time = 0.002, size = 150, normalized size = 1.

$$\frac{b^3Dx^{12}}{12}+\frac{b^3Cx^{11}}{11}+\frac{\left(b^3B+3\,ab^2D\right)x^{10}}{10}+\frac{\left(Ab^3+3\,ab^2C\right)x^9}{9}+\frac{\left(3\,ab^2B+3\,a^2bD\right)x^8}{8}+\frac{\left(3\,ab^2A+3\,a^2bC\right)x^7}{7}+\frac{\left(3\,a^2b\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/12*b^3*D*x^12+1/11*b^3*C*x^11+1/10*(B*b^3+3*D*a*b^2)*x^10+1/9*(A*b^3+3*C*
a*b^2)*x^9+1/8*(3*B*a*b^2+3*D*a^2*b)*x^8+1/7*(3*A*a*b^2+3*C*a^2*b)*x^7+1/6*
(3*B*a^2*b+D*a^3)*x^6+1/5*(3*A*a^2*b+C*a^3)*x^5+1/4*a^3*B*x^4+1/3*a^3*A*x^3
```

Maxima [A] time = 1.47662, size = 196, normalized size = 1.32

$$\frac{1}{12}\,Db^3x^{12}+\frac{1}{11}\,Cb^3x^{11}+\frac{1}{10}\left(3\,Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\left(3\,Cab^2+Ab^3\right)x^9+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}\,Ba^3x^4+\frac{3}{7}\left(Ca^2b+A\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3*
C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*(C
*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(
C*a^3 + 3*A*a^2*b)*x^5
```

Fricas [A] time = 1.31618, size = 377, normalized size = 2.53

$$\frac{1}{12}x^{12}b^3D + \frac{1}{11}x^{11}b^3C + \frac{3}{10}x^{10}b^2aD + \frac{1}{10}x^{10}b^3B + \frac{1}{3}x^9b^2aC + \frac{1}{9}x^9b^3A + \frac{3}{8}x^8ba^2D + \frac{3}{8}x^8b^2aB + \frac{3}{7}x^7ba^2C + \frac{3}{7}x^7b^2aA +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^3*D + 1/11*x^11*b^3*C + 3/10*x^10*b^2*a*D + 1/10*x^10*b^3*B + 1
/3*x^9*b^2*a*C + 1/9*x^9*b^3*A + 3/8*x^8*b*a^2*D + 3/8*x^8*b^2*a*B + 3/7*x^
7*b*a^2*C + 3/7*x^7*b^2*a*A + 1/6*x^6*a^3*D + 1/2*x^6*b*a^2*B + 1/5*x^5*a^3
*C + 3/5*x^5*b*a^2*A + 1/4*x^4*a^3*B + 1/3*x^3*a^3*A
```

Sympy [A] time = 0.086155, size = 165, normalized size = 1.11

$$\frac{Aa^3x^3}{3} + \frac{Ba^3x^4}{4} + \frac{Cb^3x^{11}}{11} + \frac{Db^3x^{12}}{12} + x^{10}\left(\frac{Bb^3}{10} + \frac{3Dab^2}{10}\right) + x^9\left(\frac{Ab^3}{9} + \frac{Cab^2}{3}\right) + x^8\left(\frac{3Bab^2}{8} + \frac{3Da^2b}{8}\right) + x^7\left(\frac{3Aab^2}{7} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12 + x**10*(
B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/3) + x**8*(3*B*a*b**
2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*C*a**2*b/7) + x**6*(B*a**2*b/2
 + D*a**3/6) + x**5*(3*A*a**2*b/5 + C*a**3/5)
```

Giac [A] time = 1.14337, size = 207, normalized size = 1.39

$$\frac{1}{12}Db^3x^{12} + \frac{1}{11}Cb^3x^{11} + \frac{3}{10}Dab^2x^{10} + \frac{1}{10}Bb^3x^{10} + \frac{1}{3}Cab^2x^9 + \frac{1}{9}Ab^3x^9 + \frac{3}{8}Da^2bx^8 + \frac{3}{8}Bab^2x^8 + \frac{3}{7}Ca^2bx^7 + \frac{3}{7}Aab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^3*x^10 + 1
/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*B*a*b^2*x^8 + 3/7*C*
```

```
a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 + 1/2*B*a^2*b*x^6 + 1/5*C*a^3*x
^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^4 + 1/3*A*a^3*x^3
```

3.80 $\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=138

$$\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+$$

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/2
 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9)/
9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

Rubi [A] time = 0.0947486, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1582, 1810}

$$\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/2
 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9)/
9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx = \frac{A\left(a+bx^2\right)^4}{8b}+\int\left(a+bx^2\right)^3\left(-Ax+x\left(A+Bx+Cx^2+Dx^3\right)\right)dx$$
$$= \frac{A\left(a+bx^2\right)^4}{8b}+\int\left(a^3Bx^2+a^3Cx^3+a^2(3bB+aD)x^4+3a^2bCx^5+3ab(bB$$
$$= \frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5+\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB+aD)x^7+\frac{3}{8}$$

Mathematica [A] time = 0.0499018, size = 124, normalized size = 0.9

$$\frac{198a^2bx^4(105A+2x(42B+5x(7C+6Dx)))+462a^3x^2(30A+x(20B+3x(5C+4Dx)))+165ab^2x^6(84A+x(72B+7x}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^2*(30*A +
x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A + 2*x*(42*B + 5*x*(7*C
 + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))))/27720
```

Maple [A] time = 0.001, size = 150, normalized size = 1.1

$$\frac{b^3Dx^{11}}{11}+\frac{b^3Cx^{10}}{10}+\frac{\left(b^3B+3\,ab^2D\right)x^9}{9}+\frac{\left(Ab^3+3\,ab^2C\right)x^8}{8}+\frac{\left(3\,ab^2B+3\,a^2bD\right)x^7}{7}+\frac{\left(3\,ab^2A+3\,a^2bC\right)x^6}{6}+\frac{\left(3\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^3*D*x^11+1/10*b^3*C*x^10+1/9*(B*b^3+3*D*a*b^2)*x^9+1/8*(A*b^3+3*C*a*
b^2)*x^8+1/7*(3*B*a*b^2+3*D*a^2*b)*x^7+1/6*(3*A*a*b^2+3*C*a^2*b)*x^6+1/5*(3
*B*a^2*b+D*a^3)*x^5+1/4*(3*A*a^2*b+C*a^3)*x^4+1/3*a^3*B*x^3+1/2*a^3*A*x^2
```

Maxima [A] time = 1.0011, size = 196, normalized size = 1.42

$$\frac{1}{11}\,Db^3x^{11} + \frac{1}{10}\,Cb^3x^{10} + \frac{1}{9}\left(3\,Dab^2 + Bb^3\right)x^9 + \frac{1}{8}\left(3\,Cab^2 + Ab^3\right)x^8 + \frac{3}{7}\left(Da^2b + Bab^2\right)x^7 + \frac{1}{3}\,Ba^3x^3 + \frac{1}{2}\left(Ca^2b + Aab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C*
a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C*a
^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*
a^3 + 3*A*a^2*b)*x^4
```

Fricas [A] time = 1.28047, size = 371, normalized size = 2.69

$$\frac{1}{11}x^{11}b^3D + \frac{1}{10}x^{10}b^3C + \frac{1}{3}x^9b^2aD + \frac{1}{9}x^9b^3B + \frac{3}{8}x^8b^2aC + \frac{1}{8}x^8b^3A + \frac{3}{7}x^7ba^2D + \frac{3}{7}x^7b^2aB + \frac{1}{2}x^6ba^2C + \frac{1}{2}x^6b^2aA + \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^3*D + 1/10*x^10*b^3*C + 1/3*x^9*b^2*a*D + 1/9*x^9*b^3*B + 3/8*x
^8*b^2*a*C + 1/8*x^8*b^3*A + 3/7*x^7*b*a^2*D + 3/7*x^7*b^2*a*B + 1/2*x^6*b*
a^2*C + 1/2*x^6*b^2*a*A + 1/5*x^5*a^3*D + 3/5*x^5*b*a^2*B + 1/4*x^4*a^3*C +
 3/4*x^4*b*a^2*A + 1/3*x^3*a^3*B + 1/2*x^2*a^3*A
```

Sympy [A] time = 0.084875, size = 163, normalized size = 1.18

$$\frac{Aa^3x^2}{2} + \frac{Ba^3x^3}{3} + \frac{Cb^3x^{10}}{10} + \frac{Db^3x^{11}}{11} + x^9\left(\frac{Bb^3}{9} + \frac{Dab^2}{3}\right) + x^8\left(\frac{Ab^3}{8} + \frac{3Cab^2}{8}\right) + x^7\left(\frac{3Bab^2}{7} + \frac{3Da^2b}{7}\right) + x^6\left(\frac{Aab^2}{2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11 + x**9*(B
*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8) + x**7*(3*B*a*b**2/7
 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*b/2) + x**5*(3*B*a**2*b/5 + D*
```

```
a**3/5) + x**4*(3*A*a**2*b/4 + C*a**3/4)
```

Giac [A] time = 1.2291, size = 207, normalized size = 1.5

$$\frac{1}{11} Db^3x^{11} + \frac{1}{10} Cb^3x^{10} + \frac{1}{3} Dab^2x^9 + \frac{1}{9} Bb^3x^9 + \frac{3}{8} Cab^2x^8 + \frac{1}{8} Ab^3x^8 + \frac{3}{7} Da^2bx^7 + \frac{3}{7} Bab^2x^7 + \frac{1}{2} Ca^2bx^6 + \frac{1}{2} Aab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/8*C
*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*b^2*x^7 + 1/2*C*a^2*
b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5*B*a^2*b*x^5 + 1/4*C*a^3*x^4 +
 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 + 1/2*A*a^3*x^2
```

3.81 $\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=133

$$\frac{1}{3}a^2x^3(aC+3Ab)+a^3Ax+\frac{1}{2}a^2bDx^6+\frac{1}{4}a^3Dx^4+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}$$

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x^
5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 + (b
^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Rubi [A] time = 0.0926051, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3(aC+3Ab)+a^3Ax+\frac{1}{2}a^2bDx^6+\frac{1}{4}a^3Dx^4+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x^
5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 + (b
^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{B\left(a+bx^2\right)^4}{8b}+\int \left(a+bx^2\right)^3 \left(A+Cx^2+Dx^3\right) dx$$
$$= \frac{B\left(a+bx^2\right)^4}{8b}+\int \left(a^3A+a^2(3Ab+aC)x^2+a^3Dx^3+3ab(Ab+aC)x^4+3a$$
$$= a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{1}{4}a^3Dx^4+\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2($$

Mathematica [A] time = 0.0473235, size = 121, normalized size = 0.91

$$\frac{126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+210a^3x(12A+x(6B+x(4C+3Dx)))+9ab^2x^5(168A+5x(28B+3x(8C+}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(15
*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*
x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

Maple [A] time = 0.003, size = 147, normalized size = 1.1

$$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+\frac{\left(b^3B+3\,ab^2D\right)x^8}{8}+\frac{\left(Ab^3+3\,ab^2C\right)x^7}{7}+\frac{\left(3\,ab^2B+3\,a^2bD\right)x^6}{6}+\frac{\left(3\,ab^2A+3\,a^2bC\right)x^5}{5}+\frac{\left(3\,a^2bB\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^3*D*x^10+1/9*b^3*C*x^9+1/8*(B*b^3+3*D*a*b^2)*x^8+1/7*(A*b^3+3*C*a*b^
2)*x^7+1/6*(3*B*a*b^2+3*D*a^2*b)*x^6+1/5*(3*A*a*b^2+3*C*a^2*b)*x^5+1/4*(3*B
*a^2*b+D*a^3)*x^4+1/3*(3*A*a^2*b+C*a^3)*x^3+1/2*B*x^2*a^3+a^3*A*x
```

Maxima [A] time = 1.01368, size = 192, normalized size = 1.44

$$\frac{1}{10}\,Db^3x^{10} + \frac{1}{9}\,Cb^3x^9 + \frac{1}{8}\left(3\,Dab^2 + Bb^3\right)x^8 + \frac{1}{7}\left(3\,Cab^2 + Ab^3\right)x^7 + \frac{1}{2}\left(Da^2b + Bab^2\right)x^6 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{5}\left(Ca^2b + Aab^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a*
b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a^2
*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*
A*a^2*b)*x^3
```

Fricas [A] time = 1.35735, size = 355, normalized size = 2.67

$$\frac{1}{10}x^{10}b^3D + \frac{1}{9}x^9b^3C + \frac{3}{8}x^8b^2aD + \frac{1}{8}x^8b^3B + \frac{3}{7}x^7b^2aC + \frac{1}{7}x^7b^3A + \frac{1}{2}x^6ba^2D + \frac{1}{2}x^6b^2aB + \frac{3}{5}x^5ba^2C + \frac{3}{5}x^5b^2aA + \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^3*D + 1/9*x^9*b^3*C + 3/8*x^8*b^2*a*D + 1/8*x^8*b^3*B + 3/7*x^7
*b^2*a*C + 1/7*x^7*b^3*A + 1/2*x^6*b*a^2*D + 1/2*x^6*b^2*a*B + 3/5*x^5*b*a^
2*C + 3/5*x^5*b^2*a*A + 1/4*x^4*a^3*D + 3/4*x^4*b*a^2*B + 1/3*x^3*a^3*C + x
^3*b*a^2*A + 1/2*x^2*a^3*B + x*a^3*A
```

Sympy [A] time = 0.085419, size = 158, normalized size = 1.19

$$Aa^3x + \frac{Ba^3x^2}{2} + \frac{Cb^3x^9}{9} + \frac{Db^3x^{10}}{10} + x^8\left(\frac{Bb^3}{8} + \frac{3Dab^2}{8}\right) + x^7\left(\frac{Ab^3}{7} + \frac{3Cab^2}{7}\right) + x^6\left(\frac{Bab^2}{2} + \frac{Da^2b}{2}\right) + x^5\left(\frac{3Aab^2}{5} + \frac{3Ca^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/8
 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*a*
*2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a**3/
```

```
4) + x**3*(A*a**2*b + C*a**3/3)
```

Giac [A] time = 1.12483, size = 201, normalized size = 1.51

$$\frac{1}{10}Db^3x^{10} + \frac{1}{9}Cb^3x^9 + \frac{3}{8}Dab^2x^8 + \frac{1}{8}Bb^3x^8 + \frac{3}{7}Cab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{1}{2}Da^2bx^6 + \frac{1}{2}Bab^2x^6 + \frac{3}{5}Ca^2bx^5 + \frac{3}{5}Aab^2x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*a
*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*b*
x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3 + A
*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

3.82 $\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=129

$$\frac{3}{2}a^2Abx^2 + a^3A\log(x) + \frac{1}{3}a^2x^3(aD+3bB) + a^3Bx + \frac{3}{4}aAb^2x^4 + \frac{1}{7}b^2x^7(3aD+bB) + \frac{3}{5}abx^5(aD+bB) + \frac{C\left(a+bx^2\right)^4}{8b} +$$

```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/4
 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7 +
(b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```

Rubi [A] time = 0.0904408, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1802}

$$\frac{3}{2}a^2Abx^2 + a^3A\log(x) + \frac{1}{3}a^2x^3(aD+3bB) + a^3Bx + \frac{3}{4}aAb^2x^4 + \frac{1}{7}b^2x^7(3aD+bB) + \frac{3}{5}abx^5(aD+bB) + \frac{C\left(a+bx^2\right)^4}{8b} +$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```


```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/4
 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7 +
(b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```


Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx = \frac{C\left(a+bx^2\right)^4}{8b}+\int \frac{\left(a+bx^2\right)^3\left(A+Bx+Dx^3\right)}{x}\,dx$$
$$= \frac{C\left(a+bx^2\right)^4}{8b}+\int \left(a^3B+\frac{a^3A}{x}+3a^2Abx+a^2(3bB+aD)x^2+3aAb^2x^3+3a$$
$$= a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2(3bB+aD)x^3+\frac{3}{4}aAb^2x^4+\frac{3}{5}ab(bB+aD)x^5+\frac{1}{6}Ab$$

Mathematica [A] time = 0.0606095, size = 121, normalized size = 0.94

$$\frac{x\left(126a^2bx(30A+x(20B+3x(5C+4Dx)))+420a^3(6B+x(3C+2Dx))+18ab^2x^3(105A+2x(42B+5x(7C+6Dx)))\right.}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B + 3*x*(5*
C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^
3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/2520 + a^3*A*Log[x]
```

Maple [A] time = 0.003, size = 148, normalized size = 1.2

$$\frac{b^3Dx^9}{9}+\frac{Cb^3x^8}{8}+\frac{Bx^7b^3}{7}+\frac{3\,Dx^7ab^2}{7}+\frac{Ax^6b^3}{6}+\frac{Cx^6ab^2}{2}+\frac{3\,Bx^5ab^2}{5}+\frac{3\,Dx^5a^2b}{5}+\frac{3\,Ax^4ab^2}{4}+\frac{3\,Cx^4a^2b}{4}+Bx^3a^2b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/9*b^3*D*x^9+1/8*C*b^3*x^8+1/7*B*x^7*b^3+3/7*D*x^7*a*b^2+1/6*A*x^6*b^3+1/2
*C*x^6*a*b^2+3/5*B*x^5*a*b^2+3/5*D*x^5*a^2*b+3/4*A*x^4*a*b^2+3/4*C*x^4*a^2*
b+B*x^3*a^2*b+1/3*D*x^3*a^3+3/2*A*x^2*a^2*b+1/2*C*x^2*a^3+a^3*B*x+a^3*A*ln(
x)
```

Maxima [A] time = 0.977444, size = 189, normalized size = 1.47

$$\frac{1}{9}Db^3x^9 + \frac{1}{8}Cb^3x^8 + \frac{1}{7}\left(3Dab^2 + Bb^3\right)x^7 + \frac{1}{6}\left(3Cab^2 + Ab^3\right)x^6 + \frac{3}{5}\left(Da^2b + Bab^2\right)x^5 + Ba^3x + \frac{3}{4}\left(Ca^2b + Aab^2\right)x^4 + A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.444424, size = 158, normalized size = 1.22

$$Aa^3\log(x) + Ba^3x + \frac{Cb^3x^8}{8} + \frac{Db^3x^9}{9} + x^7\left(\frac{Bb^3}{7} + \frac{3Dab^2}{7}\right) + x^6\left(\frac{Ab^3}{6} + \frac{Cab^2}{2}\right) + x^5\left(\frac{3Bab^2}{5} + \frac{3Da^2b}{5}\right) + x^4\left(\frac{3Aab^2}{4} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(B*b**3/7 +
 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3*B*a*b**2/5 + 3*D*a*
*2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4) + x**3*(B*a**2*b + D*a**3/3) +
 x**2*(3*A*a**2*b/2 + C*a**3/2)
```

Giac [A] time = 1.17059, size = 200, normalized size = 1.55

$$\frac{1}{9}Db^3x^9 + \frac{1}{8}Cb^3x^8 + \frac{3}{7}Dab^2x^7 + \frac{1}{7}Bb^3x^7 + \frac{1}{2}Cab^2x^6 + \frac{1}{6}Ab^3x^6 + \frac{3}{5}Da^2bx^5 + \frac{3}{5}Bab^2x^5 + \frac{3}{4}Ca^2bx^4 + \frac{3}{4}Aab^2x^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 + 1/2*C*a*b
^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*x^5 + 3/4*C*a^2*b*x^
4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b*x^3 + 1/2*C*a^3*x^2 + 3/2*A*a
^2*b*x^2 + B*a^3*x + A*a^3*log(abs(x))
```

3.83 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=124

$$a^2x(aC+3Ab)-\frac{a^3A}{x}+\frac{3}{2}a^2bBx^2+a^3B\log(x)+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3B$$

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x^
3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*C*
x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

Rubi [A] time = 0.10932, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1628}

$$a^2x(aC+3Ab)-\frac{a^3A}{x}+\frac{3}{2}a^2bBx^2+a^3B\log(x)+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3B$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x^
3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*C*
x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx = \frac{D\left(a+bx^2\right)^4}{8b}+\int \frac{\left(a+bx^2\right)^3\left(A+Bx+Cx^2\right)}{x^2}\,dx$$
$$= \frac{D\left(a+bx^2\right)^4}{8b}+\int \left(a^2(3Ab+aC)+\frac{a^3A}{x^2}+\frac{a^3B}{x}+3a^2bBx+3ab(Ab+aC)x$$
$$= -\frac{a^3A}{x}+a^2(3Ab+aC)x+\frac{3}{2}a^2bBx^2+ab(Ab+aC)x^3+\frac{3}{4}ab^2Bx^4+\frac{1}{5}b^2(Ab$$

Mathematica [A] time = 0.0800174, size = 123, normalized size = 0.99

$$\frac{1}{4}a^2bx(12A+x(6B+x(4C+3Dx)))+a^3\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+a^3B\log(x)+\frac{1}{20}ab^2x^3(20A+x(15B+2x(6C+5Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))
))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/20 + (b^3*x^5*(168
*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a^3*B*Log[x]
```

Maple [A] time = 0.006, size = 145, normalized size = 1.2

$$\frac{Db^3x^8}{8}+\frac{b^3Cx^7}{7}+\frac{Bx^6b^3}{6}+\frac{Dx^6ab^2}{2}+\frac{Ax^5b^3}{5}+\frac{3\,Cx^5ab^2}{5}+\frac{3\,Bx^4ab^2}{4}+\frac{3\,Dx^4a^2b}{4}+Ax^3ab^2+Cx^3a^2b+\frac{3\,Bx^2a^2b}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/8*D*b^3*x^8+1/7*b^3*C*x^7+1/6*B*x^6*b^3+1/2*D*x^6*a*b^2+1/5*A*x^5*b^3+3/5
*C*x^5*a*b^2+3/4*B*x^4*a*b^2+3/4*D*x^4*a^2*b+A*x^3*a*b^2+C*x^3*a^2*b+3/2*B*
x^2*a^2*b+1/2*D*x^2*a^3+3*A*a^2*b*x+a^3*C*x+a^3*B*ln(x)-a^3*A/x
```

Maxima [A] time = 1.0072, size = 188, normalized size = 1.52

$$\frac{1}{8}Db^3x^8 + \frac{1}{7}Cb^3x^7 + \frac{1}{6}\left(3Dab^2 + Bb^3\right)x^6 + \frac{1}{5}\left(3Cab^2 + Ab^3\right)x^5 + \frac{3}{4}\left(Da^2b + Bab^2\right)x^4 + Ba^3\log(x) + \left(Ca^2b + Aab^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5*(3*C*a*b^
2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*log(x) + (C*a^2*b + A*
a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b)*x^2 + (C*a^3 + 3*A*a^2*b)*x
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.456226, size = 150, normalized size = 1.21

$$-\frac{Aa^3}{x} + Ba^3\log(x) + \frac{Cb^3x^7}{7} + \frac{Db^3x^8}{8} + x^6\left(\frac{Bb^3}{6} + \frac{Dab^2}{2}\right) + x^5\left(\frac{Ab^3}{5} + \frac{3Cab^2}{5}\right) + x^4\left(\frac{3Bab^2}{4} + \frac{3Da^2b}{4}\right) + x^3\left(Aab^2 + C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*(B*b**3/6
+ D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(3*B*a*b**2/4 + 3*D*a
**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*(3*B*a**2*b/2 + D*a**3/2) + x*
(3*A*a**2*b + C*a**3)
```

Giac [A] time = 1.15394, size = 196, normalized size = 1.58

$$\frac{1}{8}Db^3x^8 + \frac{1}{7}Cb^3x^7 + \frac{1}{2}Dab^2x^6 + \frac{1}{6}Bb^3x^6 + \frac{3}{5}Cab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{4}Da^2bx^4 + \frac{3}{4}Bab^2x^4 + Ca^2bx^3 + Aab^2x^3 + \frac{1}{2}Da$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 + 3/5*C*a*b
^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*x^4 + C*a^2*b*x^3 +
A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2 + C*a^3*x + 3*A*a^2*b*x + B*a
^3*log(abs(x)) - A*a^3/x
```

3.84 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=135

$$a^2\log(x)(aC+3Ab)-\frac{a^3A}{2x^2}+a^2x(aD+3bB)-\frac{a^3B}{x}+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3($$

```
[Out] -(a^3*A)/(2*x^2) - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^2
)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D)*
x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

Rubi [A] time = 0.111828, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$a^2\log(x)(aC+3Ab)-\frac{a^3A}{2x^2}+a^2x(aD+3bB)-\frac{a^3B}{x}+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^2
)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D)*
x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx=\int \left(a^2(3bB+aD)+\frac{a^3A}{x^3}+\frac{a^3B}{x^2}+\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+3ab(bB\right.$$

$$=-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2(3bB+aD)x+\frac{3}{2}ab(Ab+aC)x^2+ab(bB+aD)x^3+\frac{1}{4}b^2(A$$

Mathematica [A] time = 0.060545, size = 124, normalized size = 0.92

$$a^2\log(x)(aC+3Ab)-\frac{a^3\left(A+2Bx-2Dx^3\right)}{2x^2}+\frac{1}{2}a^2bx(6B+x(3C+2Dx))+\frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^3*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (a^2*b*x*(6*B + x*(3*C + 2*D*x)))/2
+ (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20 + (b^3*x^4*(105*A +
2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3*A*b + a*C)*Log[x]
```

Maple [A] time = 0.007, size = 144, normalized size = 1.1

$$\frac{b^3Dx^7}{7}+\frac{b^3Cx^6}{6}+\frac{Bx^5b^3}{5}+\frac{3\,Dx^5ab^2}{5}+\frac{Ax^4b^3}{4}+\frac{3\,Cx^4ab^2}{4}+Bx^3ab^2+Dx^3a^2b+\frac{3\,Ax^2ab^2}{2}+\frac{3\,Cx^2a^2b}{2}+3\,Bxa^2b+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*B*x^5*b^3+3/5*D*x^5*a*b^2+1/4*A*x^4*b^3+3/4
*C*x^4*a*b^2+B*x^3*a*b^2+D*x^3*a^2*b+3/2*A*x^2*a*b^2+3/2*C*x^2*a^2*b+3*B*x*
a^2*b+a^3*D*x+3*A*ln(x)*a^2*b+C*ln(x)*a^3-1/2*a^3*A/x^2-a^3*B/x
```

Maxima [A] time = 1.02863, size = 188, normalized size = 1.39

$$\frac{1}{7}\,Db^3x^7+\frac{1}{6}\,Cb^3x^6+\frac{1}{5}\left(3\,Dab^2+Bb^3\right)x^5+\frac{1}{4}\left(3\,Cab^2+Ab^3\right)x^4+\left(Da^2b+Bab^2\right)x^3+\frac{3}{2}\left(Ca^2b+Aab^2\right)x^2+\left(Da^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4*(3*C*a*b^
2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b + A*a*b^2)*x^2 + (D
*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log(x) - 1/2*(2*B*a^3*x + A*a^3)/
x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.593057, size = 150, normalized size = 1.11

$$\frac{Cb^3x^6}{6} + \frac{Db^3x^7}{7} + a^2\left(3Ab + Ca\right)\log\left(x\right) + x^5\left(\frac{Bb^3}{5} + \frac{3Dab^2}{5}\right) + x^4\left(\frac{Ab^3}{4} + \frac{3Cab^2}{4}\right) + x^3\left(Bab^2 + Da^2b\right) + x^2\left(\frac{3Aab^2}{2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*(B*b**3/5
+ 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3*(B*a*b**2 + D*a**2*
b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(3*B*a**2*b + D*a**3) - (A*a**3
 + 2*B*a**3*x)/(2*x**2)
```

Giac [A] time = 1.14852, size = 194, normalized size = 1.44

$$\frac{1}{7}Db^3x^7 + \frac{1}{6}Cb^3x^6 + \frac{3}{5}Dab^2x^5 + \frac{1}{5}Bb^3x^5 + \frac{3}{4}Cab^2x^4 + \frac{1}{4}Ab^3x^4 + Da^2bx^3 + Bab^2x^3 + \frac{3}{2}Ca^2bx^2 + \frac{3}{2}Aab^2x^2 + Da^3x +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a*b
^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/2*C*a^2*b*x^2 + 3/2*
A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3 + 3*A*a^2*b)*log(abs(x)) - 1/2
*(2*B*a^3*x + A*a^3)/x^2
```

3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4} dx$

Optimal. Leaf size=139

$$-\frac{a^2(aC+3Ab)}{x}-\frac{a^3A}{3x^3}+a^2\log(x)(aD+3bB)-\frac{a^3B}{2x^2}+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}ab$$

```
[Out] -(a^3*A)/(3*x^3) - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b + a
*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B +
 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x]
```

Rubi [A] time = 0.113901, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2(aC+3Ab)}{x}-\frac{a^3A}{3x^3}+a^2\log(x)(aD+3bB)-\frac{a^3B}{2x^2}+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}ab$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^3*A)/(3*x^3) - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b + a
*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B +
 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4} dx = \int \left(3ab(Ab+aC)+\frac{a^3A}{x^4}+\frac{a^3B}{x^3}+\frac{a^2(3Ab+aC)}{x^2}+\frac{a^2(3bB+aD)}{x}+3ab(bB\right.$$

$$= -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2+\frac{1}{3}b^2(A$$

Mathematica [A] time = 0.0483706, size = 124, normalized size = 0.89

$$\frac{3a^2b\left(x^2(2C+Dx)-2A\right)}{2x}-\frac{a^3(2A+3x(B+2Cx))}{6x^3}+a^2\log(x)(aD+3bB)+\frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^3*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) + (3*a^2*b*(-2*A + x^2*(2*C + D*x)))
/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4 + (b^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*B + a*D)*Log[x]
```

Maple [A] time = 0.008, size = 146, normalized size = 1.1

$$\frac{b^3Dx^6}{6}+\frac{b^3Cx^5}{5}+\frac{Bx^4b^3}{4}+\frac{3\,Dx^4ab^2}{4}+\frac{Ax^3b^3}{3}+Cx^3ab^2+\frac{3\,Bx^2ab^2}{2}+\frac{3\,Dx^2a^2b}{2}+3\,Axab^2+3\,a^2bCx+3\,B\ln(x)\,a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*B*x^4*b^3+3/4*D*x^4*a*b^2+1/3*A*x^3*b^3+C*x
^3*a*b^2+3/2*B*x^2*a*b^2+3/2*D*x^2*a^2*b+3*A*x*a*b^2+3*a^2*b*C*x+3*B*ln(x)*
a^2*b+D*ln(x)*a^3-1/3*a^3*A/x^3-1/2*a^3*B/x^2-3*a^2/x*A*b-a^3/x*C
```

Maple [A] time = 1.00121, size = 192, normalized size = 1.38

$$\frac{1}{6}Db^3x^6+\frac{1}{5}Cb^3x^5+\frac{1}{4}\left(3\,Dab^2+Bb^3\right)x^4+\frac{1}{3}\left(3\,Cab^2+Ab^3\right)x^3+\frac{3}{2}\left(Da^2b+Bab^2\right)x^2+3\left(Ca^2b+Aab^2\right)x+\left(Da^3+3\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3*(3*C*a*b^
2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2*b + A*a*b^2)*x + (D
*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2*b)
*x^2)/x^3
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 1.06086, size = 153, normalized size = 1.1

$$\frac{Cb^3x^5}{5} + \frac{Db^3x^6}{6} + a^2(3Bb + Da)\log(x) + x^4\left(\frac{Bb^3}{4} + \frac{3Dab^2}{4}\right) + x^3\left(\frac{Ab^3}{3} + Cab^2\right) + x^2\left(\frac{3Bab^2}{2} + \frac{3Da^2b}{2}\right) + x\left(3Aab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*(B*b**3/4
+ 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*B*a*b**2/2 + 3*D*a**
2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) - (2*A*a**3 + 3*B*a**3*x + x**2*(18*A*
a**2*b + 6*C*a**3))/(6*x**3)
```

Giac [A] time = 1.16839, size = 197, normalized size = 1.42

$$\frac{1}{6}Db^3x^6 + \frac{1}{5}Cb^3x^5 + \frac{3}{4}Dab^2x^4 + \frac{1}{4}Bb^3x^4 + Cab^2x^3 + \frac{1}{3}Ab^3x^3 + \frac{3}{2}Da^2bx^2 + \frac{3}{2}Bab^2x^2 + 3Ca^2bx + 3Aab^2x + \left(Da^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 + C*a*b^2*x
^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2 + 3*C*a^2*b*x + 3*A*
a*b^2*x + (D*a^3 + 3*B*a^2*b)*log(abs(x)) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C
*a^3 + 3*A*a^2*b)*x^2)/x^3
```

3.86 $\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$

Optimal. Leaf size=151

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax(Ab-aC)}{b^3}+\frac{x^4(bB-aD)}{4b^2}-\frac{ax^2(bB-aD)}{2b^3}$$

```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b - a*C)*x^3)/
(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (D*x^6)/(6*b) + (a^(3
/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) + (a^2*(b*B - a*D)*Log
[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.144934, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax(Ab-aC)}{b^3}+\frac{x^4(bB-aD)}{4b^2}-\frac{ax^2(bB-aD)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```



```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b - a*C)*x^3)/
(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (D*x^6)/(6*b) + (a^(3
/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) + (a^2*(b*B - a*D)*Log
[a + b*x^2])/(2*b^4)
```


Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx=\int\left(-\frac{a(Ab-aC)}{b^3}-\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{b^2}+\frac{(bB-aD)x^3}{b^2}+\frac{Cx^4}{b}+\frac{Dx^5}{b}+\frac{a^2}{}\right.$$

$$=-\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\int\frac{a}{}$$

$$=-\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{\left(a^2\right.}{}$$

$$=-\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{a^{3/2}}{}$$

Mathematica [A] time = 0.0730102, size = 130, normalized size = 0.86

$$\frac{bx\left(30a^2(2C+Dx)-5ab(12A+x(6B+x(4C+3Dx)))+b^2x^2(20A+x(15B+2x(6C+5Dx)))\right)-60a^{3/2}\sqrt{b}(aC-Ab)}{60b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(30*a^2*(2*C + D*x) - 5*a*b*(12*A + x*(6*B + x*(4*C + 3*D*x))) + b^2*x
^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))) - 60*a^(3/2)*Sqrt[b]*(-(A*b) + a*
C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 30*a^2*(-(b*B) + a*D)*Log[a + b*x^2])/(60*
b^4)
```

Maple [A] time = 0.005, size = 176, normalized size = 1.2

$$\frac{Dx^6}{6b}+\frac{Cx^5}{5b}+\frac{Bx^4}{4b}-\frac{Dx^4a}{4b^2}+\frac{Ax^3}{3b}-\frac{Cx^3a}{3b^2}-\frac{Bax^2}{2b^2}+\frac{Dx^2a^2}{2b^3}-\frac{aAx}{b^2}+\frac{a^2Cx}{b^3}+\frac{a^2\ln\left(bx^2+a\right)B}{2b^3}-\frac{a^3\ln\left(bx^2+a\right)D}{2b^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/6*D*x^6/b+1/5*C*x^5/b+1/4/b*B*x^4-1/4/b^2*D*x^4*a+1/3/b*A*x^3-1/3/b^2*C*x
^3*a-1/2/b^2*B*x^2*a+1/2/b^3*D*x^2*a^2-1/b^2*A*a*x+1/b^3*a^2*C*x+1/2*a^2/b^
3*ln(b*x^2+a)*B-1/2*a^3/b^4*ln(b*x^2+a)*D+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))*A-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 1.11458, size = 308, normalized size = 2.04

$$\frac{Cx^5}{5b} + \frac{Dx^6}{6b} + \left(-\frac{a^2\left(-Bb+Da\right)}{2b^4} - \frac{\sqrt{-a^3b^9}\left(-Ab+Ca\right)}{2b^8}\right)\log\left(x + \frac{Ba^2b - Da^3 - 2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4} - \frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2 + Ca^2b}\right) + \left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**5/(5*b) + D*x**6/(6*b) + (-a**2*(-B*b + D*a)/(2*b**4) - sqrt(-a**3*b**
9)*(-A*b + C*a)/(2*b**8))*log(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b
+ D*a)/(2*b**4) - sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a
**2*b)) + (-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b
**8))*log(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) + sq
rt(-a**3*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) - x**4*(-B*b
 + D*a)/(4*b**2) - x**3*(-A*b + C*a)/(3*b**2) + x**2*(-B*a*b + D*a**2)/(2*b
**3) + x*(-A*a*b + C*a**2)/b**3
```

Giac [A] time = 1.19921, size = 217, normalized size = 1.44

$$-\frac{\left(Ca^3 - Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{\left(Da^3 - Ba^2b\right)\log\left(bx^2 + a\right)}{2\,b^4} + \frac{10\,Db^5x^6 + 12\,Cb^5x^5 - 15\,Dab^4x^4 + 15\,Bb^5x^4 - 20\,Cab^4x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(D*a^3 - B*a
^2*b)*log(b*x^2 + a)/b^4 + 1/60*(10*D*b^5*x^6 + 12*C*b^5*x^5 - 15*D*a*b^4*x
^4 + 15*B*b^5*x^4 - 20*C*a*b^4*x^3 + 20*A*b^5*x^3 + 30*D*a^2*b^3*x^2 - 30*B
*a*b^4*x^2 + 60*C*a^2*b^3*x - 60*A*a*b^4*x)/b^6
```

3.87 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=130

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^2(Ab-aC)}{2b^2}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^3(bB-aD)}{3b^2}-\frac{ax(bB-aD)}{b^3}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(3
*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.123792, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^2(Ab-aC)}{2b^2}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^3(bB-aD)}{3b^2}-\frac{ax(bB-aD)}{b^3}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(3
*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int \left(-\frac{a(bB-aD)}{b^3}+\frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{b^2}+\frac{Cx^3}{b}+\frac{Dx^4}{b}+\frac{a^2(bB-aD)-ab(A}{b^3\left(a+bx\right.}\right.$$

$$= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{\int \frac{a^2(bB-aD)-ab(Ab-aC)}{a+bx^2}}{b^3}$$

$$= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}-\frac{(a(Ab-aC))\int \frac{x}{a+bx^2}}{b^2}$$

$$= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{a^{3/2}(bB-aD)\tan^{-1}}{b^{7/2}}$$

Mathematica [A] time = 0.0903211, size = 114, normalized size = 0.88

$$\frac{x\left(60a^2D-10ab(6B+x(3C+2Dx))+b^2x(30A+x(20B+3x(5C+4Dx)))\right)+30a(aC-Ab)\log\left(a+bx^2\right)}{60b^3}-\frac{a^{3/2}(aD-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) + (x*(60*a^
2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*(20*B + 3*x*(5*C + 4
*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2])/(60*b^3)
```

Maple [A] time = 0.006, size = 152, normalized size = 1.2

$$\frac{Dx^5}{5\,b}+\frac{Cx^4}{4\,b}+\frac{Bx^3}{3\,b}-\frac{Dx^3a}{3\,b^2}+\frac{Ax^2}{2\,b}-\frac{Cx^2a}{2\,b^2}-\frac{Bax}{b^2}+\frac{a^2Dx}{b^3}-\frac{a\ln\left(bx^2+a\right)A}{2\,b^2}+\frac{a^2\ln\left(bx^2+a\right)C}{2\,b^3}+\frac{a^2B}{b^2}\arctan\left(bx\frac{1}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/5*D*x^5/b+1/4*C*x^4/b+1/3/b*B*x^3-1/3/b^2*D*x^3*a+1/2/b*A*x^2-1/2/b^2*C*x
^2*a-1/b^2*B*x*a+1/b^3*a^2*D*x-1/2*a/b^2*ln(b*x^2+a)*A+1/2*a^2/b^3*ln(b*x^2
+a)*C+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B-a^3/b^3/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 1.07699, size = 269, normalized size = 2.07

$$\frac{Cx^4}{4b} + \frac{Dx^5}{5b} + \left(\frac{a\left(-Ab + Ca\right)}{2b^3} - \frac{\sqrt{-a^3b^7}\left(-Bb + Da\right)}{2b^7}\right)\log\left(x + \frac{-Aab + Ca^2 - 2b^3\left(\frac{a(-Ab+Ca)}{2b^3} - \frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab + Da^2}\right) + \Bigg(\frac{a\,(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**4/(4*b) + D*x**5/(5*b) + (a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(
-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*
b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b + D*a**2)) + (a*(-
A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a
*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*
a)/(2*b**7)))/(-B*a*b + D*a**2)) - x**3*(-B*b + D*a)/(3*b**2) - x**2*(-A*b
+ C*a)/(2*b**2) + x*(-B*a*b + D*a**2)/b**3
```

Giac [A] time = 1.18655, size = 185, normalized size = 1.42

$$\frac{\left(Ca^2 - Aab\right)\log\left(bx^2 + a\right)}{2\,b^3} - \frac{\left(Da^3 - Ba^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^4x^5 + 15\,Cb^4x^4 - 20\,Dab^3x^3 + 20\,Bb^4x^3 - 30\,Cab^3x^2}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^4 - 20*D*a*b^3*x^3
+ 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2 + 60*D*a^2*b^2*x - 60*B*a*b^
3*x)/b^5
```

3.88 $\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$

Optimal. Leaf size=111

$$\frac{x(Ab-aC)}{b^2}-\frac{\sqrt{a}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^2(bB-aD)}{2b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/(
4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*B
- a*D)*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.113119, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{x(Ab-aC)}{b^2}-\frac{\sqrt{a}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^2(bB-aD)}{2b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/(
4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*B
- a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int \left(\frac{Ab-aC}{b^2}+\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{b}+\frac{Dx^3}{b}-\frac{a(Ab-aC)+a(bB-aD)x}{b^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\int \frac{a(Ab-aC)+a(bB-aD)x}{a+bx^2}\,dx}{b^2}$$

$$= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{(a(Ab-aC))\int \frac{1}{a+bx^2}\,dx}{b^2}-\frac{(a(bB-aD)}{b}$$

$$= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a(bB-aL}{}$$

Mathematica [A] time = 0.0525082, size = 95, normalized size = 0.86

$$\frac{bx\left(-6a(2C+Dx)+12Ab+bx\left(6B+4Cx+3Dx^2\right)\right)+12\sqrt{a}\sqrt{b}(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6a(aD-bB)\log\left(a+bx^2\right)}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 12*Sqrt[a]*
Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-(b*B) + a*D)*Log
[a + b*x^2])/(12*b^3)
```

Maple [A] time = 0.004, size = 128, normalized size = 1.2

$$\frac{Dx^4}{4b}+\frac{Cx^3}{3b}+\frac{Bx^2}{2b}-\frac{Dx^2a}{2b^2}+\frac{Ax}{b}-\frac{aCx}{b^2}-\frac{a\ln\left(bx^2+a\right)B}{2b^2}+\frac{a^2\ln\left(bx^2+a\right)D}{2b^3}-\frac{aA}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2C}{b^2}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/4*D*x^4/b+1/3*C*x^3/b+1/2/b*B*x^2-1/2/b^2*D*x^2*a+1/b*A*x-1/b^2*a*C*x-1/2
*a/b^2*ln(b*x^2+a)*B+1/2*a^2/b^3*ln(b*x^2+a)*D-a/b/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))*A+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 1.03254, size = 243, normalized size = 2.19

$$\frac{Cx^3}{3b} + \frac{Dx^4}{4b} + \left(\frac{a\left(-Bb + Da\right)}{2b^3} - \frac{\sqrt{-ab^7}\left(-Ab + Ca\right)}{2b^6}\right)\log\left(x + \frac{Bab - Da^2 + 2b^3\left(\frac{a(-Bb+Da)}{2b^3} - \frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2 + Cab}\right) + \left(\frac{a\left(-Bb + \right.}{2b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**3/(3*b) + D*x**4/(4*b) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)/(2*b**3
) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b)) + (a*(-B*b + D
*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2
 + 2*b**3*(a*(-B*b + D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/
(-A*b**2 + C*a*b)) - x**2*(-B*b + D*a)/(2*b**2) - x*(-A*b + C*a)/b**2
```

Giac [A] time = 1.18293, size = 151, normalized size = 1.36

$$\frac{\left(Ca^2 - Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{\left(Da^2 - Bab\right)\log\left(bx^2 + a\right)}{2\,b^3} + \frac{3\,Db^3x^4 + 4\,Cb^3x^3 - 6\,Dab^2x^2 + 6\,Bb^3x^2 - 12\,Cab^2x + 12\,A}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^2 - B*a*b)
*log(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 - 6*D*a*b^2*x^2 + 6*B
*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

3.89 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=92

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}+\frac{x(bB-aD)}{b^2}-\frac{\sqrt{a}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0846149, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1802, 635, 205, 260}

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}+\frac{x(bB-aD)}{b^2}-\frac{\sqrt{a}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^2)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx=\int\left(\frac{bB-aD}{b^2}+\frac{Cx}{b}+\frac{Dx^2}{b}-\frac{a(bB-aD)-b(Ab-aC)x}{b^2\left(a+bx^2\right)}\right)dx$$
$$=\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\int\frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{b^2}$$
$$=\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+\frac{(Ab-aC)\int\frac{x}{a+bx^2}\,dx}{b}-\frac{(a(bB-aD))\int\frac{1}{a+bx^2}\,dx}{b^2}$$
$$=\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.0637263, size = 81, normalized size = 0.88

$$\frac{3(Ab-aC)\log\left(a+bx^2\right)+x(-6aD+6bB+bx(3C+2Dx))}{6b^2}+\frac{\sqrt{a}(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x*(6*b*B -
6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^2])/(6*b^2)
```

Maple [A] time = 0.004, size = 106, normalized size = 1.2

$$\frac{Dx^3}{3\,b}+\frac{Cx^2}{2\,b}+\frac{Bx}{b}-\frac{aDx}{b^2}+\frac{\ln\left(bx^2+a\right)A}{2\,b}-\frac{\ln\left(bx^2+a\right)aC}{2\,b^2}-\frac{Ba}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2D}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/3*D*x^3/b+1/2*C*x^2/b+1/b*B*x-1/b^2*a*D*x+1/2/b*ln(b*x^2+a)*A-1/2/b^2*ln(
b*x^2+a)*a*C-1/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B*a+1/b^2/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))*a^2*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 0.968387, size = 211, normalized size = 2.29

$$\frac{Cx^2}{2b} + \frac{Dx^3}{3b} + \left(-\frac{-Ab+Ca}{2b^2} - \frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x + \frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right) + \left(-\frac{-Ab+Ca}{2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**2/(2*b) + D*x**3/(3*b) + (-(-A*b + C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b
 + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) - sq
```

```
rt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) + (-(-A*b + C*a)/(2*b**2)
 + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*
b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) - x
*(-B*b + D*a)/b**2
```

Giac [A] time = 1.18793, size = 119, normalized size = 1.29

$$-\frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,b^2} + \frac{\left(Da^2 - Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Db^2x^3 + 3\,Cb^2x^2 - 6\,Dabx + 6\,Bb^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*a*b*x + 6*B*b^2*x)/
b^3
```

3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx$

Optimal. Leaf size=73

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a
]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0650568, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1810, 635, 205, 260}

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a
]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx &= \int \left(\frac{C}{b}+\frac{Dx}{b}+\frac{Ab-aC+(bB-aD)x}{b\left(a+bx^2\right)}\right)dx\\ &= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{\int \frac{Ab-aC+(bB-aD)x}{a+bx^2}\,dx}{b}\\ &= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\int \frac{1}{a+bx^2}\,dx}{b}+\frac{(bB-aD)\int \frac{x}{a+bx^2}\,dx}{b}\\ &= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0404422, size = 68, normalized size = 0.93

$$\frac{\frac{2\sqrt{b}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log\left(a+bx^2\right)+bx(2C+Dx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.004, size = 83, normalized size = 1.1

$$\frac{Dx^2}{2\,b}+\frac{Cx}{b}+\frac{\ln\left(bx^2+a\right)B}{2\,b}-\frac{\ln\left(bx^2+a\right)aD}{2\,b^2}+A\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{aC}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/2*D*x^2/b+C*x/b+1/2/b*ln(b*x^2+a)*B-1/2/b^2*ln(b*x^2+a)*a*D+1/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*A-1/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 0.959732, size = 219, normalized size = 3.

$$\frac{Cx}{b} + \frac{Dx^2}{2b} + \left(-\frac{-Bb + Da}{2b^2} - \frac{\sqrt{-ab^5}\,(-Ab + Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} - \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2 + Cab}\right) + \left(-\frac{-Bb + D}{2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a)
/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) - s
qrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*a)/
```

```
(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a**2 -
 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))
/(-A*b**2 + C*a*b))
```

Giac [A] time = 1.19558, size = 89, normalized size = 1.22

$$-\frac{(Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{(Da - Bb)\log\left(bx^2 + a\right)}{2\,b^2} + \frac{Dbx^2 + 2\,Cbx}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x^
2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)} dx$

Optimal. Leaf size=72

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A*
Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rubi [A] time = 0.0977419, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A*
Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx &= \int \left(\frac{D}{b}+\frac{A}{ax}+\frac{a(bB-aD)-b(Ab-aC)x}{ab\left(a+bx^2\right)}\right)dx\\ &= \frac{Dx}{b}+\frac{A\log(x)}{a}+\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{ab}\\ &= \frac{Dx}{b}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{a}+\frac{(bB-aD)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{Dx}{b}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}\end{aligned}$$

Mathematica [A] time = 0.0511625, size = 73, normalized size = 1.01

$$\frac{(aC-Ab)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}-\frac{(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
(A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] time = 0.006, size = 80, normalized size = 1.1

$$\frac{Dx}{b}+\frac{A\ln(x)}{a}-\frac{\ln\left(bx^2+a\right)A}{2a}+\frac{\ln\left(bx^2+a\right)C}{2b}+B\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{aD}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x)
```

```
[Out] D*x/b+A*ln(x)/a-1/2/a*ln(b*x^2+a)*A+1/2/b*ln(b*x^2+a)*C+1/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*B-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 25.4488, size = 1268, normalized size = 17.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

```
[Out] A*log(x)/a + D*x/b + ((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/
(2*a**2*b**3))*log(x + (-6*A**3*b**4 + 8*A**2*C*a*b**3 - 6*A**2*a*b**4*((-A
```

```
*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*A*B**2
*a*b**3 - 4*A*B*D*a**2*b**2 - 2*A*C**2*a**2*b**2 - 4*A*C*a**2*b**3*((-A*b +
 C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*A*D**2*a**
3*b + 12*A*a**2*b**4*((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/
(2*a**2*b**3))**2 - 2*B**2*a**2*b**3*((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**
3)*(-B*b + D*a)/(2*a**2*b**3)) + 4*B*D*a**3*b**2*((-A*b + C*a)/(2*a*b) - sq
rt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*C**2*a**3*b**2*((-A*b + C*a)
/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) - 4*C*a**3*b**3*((-
A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3))**2 - 2*D*
*2*a**4*b*((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**
3)))/(-9*A**2*B*b**4 + 9*A**2*D*a*b**3 + 6*A*B*C*a*b**3 - 6*A*C*D*a**2*b**2
 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - B*C**2*a**2*b**2 - 3*B*D**2*a**3*b +
C**2*D*a**3*b + D**3*a**4)) + ((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*
b + D*a)/(2*a**2*b**3))*log(x + (-6*A**3*b**4 + 8*A**2*C*a*b**3 - 6*A**2*a*
b**4*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) +
 2*A*B**2*a*b**3 - 4*A*B*D*a**2*b**2 - 2*A*C**2*a**2*b**2 - 4*A*C*a**2*b**3
*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*A
*D**2*a**3*b + 12*A*a**2*b**4*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*
b + D*a)/(2*a**2*b**3))**2 - 2*B**2*a**2*b**3*((-A*b + C*a)/(2*a*b) + sqrt(
-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 4*B*D*a**3*b**2*((-A*b + C*a)/(2*
a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*C**2*a**3*b**2*((-A
*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) - 4*C*a**3
*b**3*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3))*
*2 - 2*D**2*a**4*b*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2
*a**2*b**3)))/(-9*A**2*B*b**4 + 9*A**2*D*a*b**3 + 6*A*B*C*a*b**3 - 6*A*C*D*
a**2*b**2 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - B*C**2*a**2*b**2 - 3*B*D**2*
a**3*b + C**2*D*a**3*b + D**3*a**4))
```

Giac [A] time = 1.12289, size = 89, normalized size = 1.24

$$\frac{Dx}{b} + \frac{A\log\left(|x|\right)}{a} - \frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] D*x/b + A*log(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) +
 1/2*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=76

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
(B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rubi [A] time = 0.0984595, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
(B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{A}{ax^2}+\frac{B}{ax}+\frac{-Ab+aC-(bB-aD)x}{a\left(a+bx^2\right)}\right)dx\\ &= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{\int \frac{-Ab+aC-(bB-aD)x}{a+bx^2}\,dx}{a}\\ &= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{(-Ab+aC)\int \frac{1}{a+bx^2}\,dx}{a}+\frac{(-bB+aD)\int \frac{x}{a+bx^2}\,dx}{a}\\ &= -\frac{A}{ax}-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}\end{aligned}$$

Mathematica [A] time = 0.0459417, size = 75, normalized size = 0.99

$$\frac{(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}+\frac{(aD-bB)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
 + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] time = 0.007, size = 83, normalized size = 1.1

$$-\frac{A}{ax}+\frac{B\ln(x)}{a}-\frac{\ln\left(bx^2+a\right)B}{2a}+\frac{\ln\left(bx^2+a\right)D}{2b}-\frac{Ab}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+C\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x)
```

```
[Out] -A/a/x+B*ln(x)/a-1/2/a*ln(b*x^2+a)*B+1/2/b*ln(b*x^2+a)*D-1/a/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))*A*b+1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 23.3426, size = 1258, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) + B*log(x)/a + ((-B*b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*
a)/(2*a**3*b**2))*log(x + (-2*A**2*B*a*b**3 + 2*A**2*a**2*b**3*((-B*b + D*a
```

```
)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) + 4*A*B*C*a**2*b**
2 - 4*A*C*a**3*b**2*((-B*b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(
2*a**3*b**2)) + 6*B**3*a**2*b**2 - 8*B**2*D*a**3*b + 6*B**2*a**3*b**2*((-B*
b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 2*B*C**2*
a**3*b + 2*B*D**2*a**4 + 4*B*D*a**4*b*((-B*b + D*a)/(2*a*b) - sqrt(-a**3*b*
*3)*(-A*b + C*a)/(2*a**3*b**2)) - 12*B*a**4*b**2*((-B*b + D*a)/(2*a*b) - sq
rt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))**2 + 2*C**2*a**4*b*((-B*b + D*a)
/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 2*D**2*a**5*((-B*
b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) + 4*D*a**5*
b*((-B*b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))**2)/
(-A**3*b**4 + 3*A**2*C*a*b**3 - 9*A*B**2*a*b**3 + 6*A*B*D*a**2*b**2 - 3*A*C
**2*a**2*b**2 - A*D**2*a**3*b + 9*B**2*C*a**2*b**2 - 6*B*C*D*a**3*b + C**3*
a**3*b + C*D**2*a**4)) + ((-B*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C
*a)/(2*a**3*b**2))*log(x + (-2*A**2*B*a*b**3 + 2*A**2*a**2*b**3*((-B*b + D*
a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) + 4*A*B*C*a**2*b*
*2 - 4*A*C*a**3*b**2*((-B*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/
(2*a**3*b**2)) + 6*B**3*a**2*b**2 - 8*B**2*D*a**3*b + 6*B**2*a**3*b**2*((-B
*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 2*B*C**2
*a**3*b + 2*B*D**2*a**4 + 4*B*D*a**4*b*((-B*b + D*a)/(2*a*b) + sqrt(-a**3*b
**3)*(-A*b + C*a)/(2*a**3*b**2)) - 12*B*a**4*b**2*((-B*b + D*a)/(2*a*b) + s
qrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))**2 + 2*C**2*a**4*b*((-B*b + D*a
)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 2*D**2*a**5*((-B
*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) + 4*D*a**5
*b*((-B*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))**2)
/(-A**3*b**4 + 3*A**2*C*a*b**3 - 9*A*B**2*a*b**3 + 6*A*B*D*a**2*b**2 - 3*A*
C**2*a**2*b**2 - A*D**2*a**3*b + 9*B**2*C*a**2*b**2 - 6*B*C*D*a**3*b + C**3
*a**3*b + C*D**2*a**4))
```

Giac [A] time = 1.15784, size = 92, normalized size = 1.21

$$\frac{B\log\left(|x|\right)}{a} + \frac{(Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{(Da - Bb)\log\left(bx^2 + a\right)}{2\,ab} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*log(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*
a - B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=92

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{2ax^2}-\frac{B}{ax}$$

```
[Out] -A/(2*a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)
*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.108623, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{2ax^2}-\frac{B}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)
*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*a^2)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx &= \int \left(\frac{A}{ax^3}+\frac{B}{ax^2}+\frac{-Ab+aC}{a^2x}+\frac{-a(bB-aD)+b(Ab-aC)x}{a^2\left(a+bx^2\right)}\right)dx\\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{\int \frac{-a(bB-aD)+b(Ab-aC)x}{a+bx^2}\,dx}{a^2}\\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(b(Ab-aC))\int \frac{x}{a+bx^2}\,dx}{a^2}-\frac{(bB-aD)\int \frac{1}{a+bx^2}\,dx}{a}\\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0799407, size = 84, normalized size = 0.91

$$\frac{(Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-Ab)-\frac{aA}{x^2}+\frac{2\sqrt{a}(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)*Log[a + b*x^2])/(2*
a^2)
```

Maple [A] time = 0.009, size = 102, normalized size = 1.1

$$-\frac{A}{2\,ax^2}-\frac{B}{ax}-\frac{A\ln(x)\,b}{a^2}+\frac{\ln(x)\,C}{a}+\frac{b\ln\left(bx^2+a\right)A}{2\,a^2}-\frac{\ln\left(bx^2+a\right)C}{2\,a}-\frac{Bb}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+D\arctan\left(bx\frac{1}{\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-B/a/x-1/a^2*ln(x)*A*b+1/a*ln(x)*C+1/2/a^2*b*ln(b*x^2+a)*A-1/2/
a*ln(b*x^2+a)*C-1/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B*b+1/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 24.6077, size = 1686, normalized size = 18.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

```
[Out] (-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))*log(x + (-
6*A**3*b**4 + 18*A**2*C*a*b**3 + 6*A**2*a**2*b**3*(-(-A*b + C*a)/(2*a**2) -
 sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 2*A*B**2*a*b**3 - 4*A*B*D*a**2*b*
*2 - 18*A*C**2*a**2*b**2 - 12*A*C*a**3*b**2*(-(-A*b + C*a)/(2*a**2) - sqrt(
-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 2*A*D**2*a**3*b + 12*A*a**4*b**2*(-(-A*
b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))**2 - 2*B**2*C*a*
*2*b**2 + 2*B**2*a**3*b**2*(-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b +
D*a)/(2*a**4*b)) + 4*B*C*D*a**3*b - 4*B*D*a**4*b*(-(-A*b + C*a)/(2*a**2) -
sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 6*C**3*a**3*b + 6*C**2*a**4*b*(-(-
A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) - 2*C*D**2*a**
4 - 12*C*a**5*b*(-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**
4*b))**2 + 2*D**2*a**5*(-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)
/(2*a**4*b)))/(-9*A**2*B*b**4 + 9*A**2*D*a*b**3 + 18*A*B*C*a*b**3 - 18*A*C*
D*a**2*b**2 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - 9*B*C**2*a**2*b**2 - 3*B*D
**2*a**3*b + 9*C**2*D*a**3*b + D**3*a**4)) + (-(-A*b + C*a)/(2*a**2) + sqrt
(-a**5*b)*(-B*b + D*a)/(2*a**4*b))*log(x + (-6*A**3*b**4 + 18*A**2*C*a*b**3
 + 6*A**2*a**2*b**3*(-(-A*b + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2
*a**4*b)) + 2*A*B**2*a*b**3 - 4*A*B*D*a**2*b**2 - 18*A*C**2*a**2*b**2 - 12*
A*C*a**3*b**2*(-(-A*b + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*
b)) + 2*A*D**2*a**3*b + 12*A*a**4*b**2*(-(-A*b + C*a)/(2*a**2) + sqrt(-a**5
*b)*(-B*b + D*a)/(2*a**4*b))**2 - 2*B**2*C*a**2*b**2 + 2*B**2*a**3*b**2*(-(
-A*b + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 4*B*C*D*a**
3*b - 4*B*D*a**4*b*(-(-A*b + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*
a**4*b)) + 6*C**3*a**3*b + 6*C**2*a**4*b*(-(-A*b + C*a)/(2*a**2) + sqrt(-a*
*5*b)*(-B*b + D*a)/(2*a**4*b)) - 2*C*D**2*a**4 - 12*C*a**5*b*(-(-A*b + C*a)
/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))**2 + 2*D**2*a**5*(-(-A*b
 + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)))/(-9*A**2*B*b**4
+ 9*A**2*D*a*b**3 + 18*A*B*C*a*b**3 - 18*A*C*D*a**2*b**2 - B**3*a*b**3 + 3*
B**2*D*a**2*b**2 - 9*B*C**2*a**2*b**2 - 3*B*D**2*a**3*b + 9*C**2*D*a**3*b +
 D**3*a**4)) - (A + 2*B*x)/(2*a*x**2) + (-A*b + C*a)*log(x + (-6*A**3*b**4
+ 18*A**2*C*a*b**3 + 6*A**2*b**3*(-A*b + C*a) + 2*A*B**2*a*b**3 - 4*A*B*D*a
**2*b**2 - 18*A*C**2*a**2*b**2 - 12*A*C*a*b**2*(-A*b + C*a) + 2*A*D**2*a**3
*b + 12*A*b**2*(-A*b + C*a)**2 - 2*B**2*C*a**2*b**2 + 2*B**2*a*b**2*(-A*b +
 C*a) + 4*B*C*D*a**3*b - 4*B*D*a**2*b*(-A*b + C*a) + 6*C**3*a**3*b + 6*C**2
*a**2*b*(-A*b + C*a) - 2*C*D**2*a**4 - 12*C*a*b*(-A*b + C*a)**2 + 2*D**2*a*
*3*(-A*b + C*a))/(-9*A**2*B*b**4 + 9*A**2*D*a*b**3 + 18*A*B*C*a*b**3 - 18*A
*C*D*a**2*b**2 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - 9*B*C**2*a**2*b**2 - 3*
B*D**2*a**3*b + 9*C**2*D*a**3*b + D**3*a**4))/a**2
```

Giac [A] time = 1.16396, size = 108, normalized size = 1.17

$$\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,a^2}+\frac{(Ca-Ab)\log\left(|x|\right)}{a^2}-\frac{2\,Bax+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(abs(x))/a^2 - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

3.94 $\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{x^3(3Ab-5aC)}{6ab^2}+\frac{x(3Ab-5aC)}{2b^3}-\frac{\sqrt{a}(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x^2(2bB-3aD)}{2b^3}-$$

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3*A*b - 5*a
*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*D)/b) - (A*b - a*C)*
x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.267927, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x^3(3Ab-5aC)}{6ab^2}+\frac{x(3Ab-5aC)}{2b^3}-\frac{\sqrt{a}(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x^2(2bB-3aD)}{2b^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3*A*b - 5*a
*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*D)/b) - (A*b - a*C)*
x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(3Ab-5aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int\left(-\frac{a(3Ab-5aC)}{b^2}-\frac{2a(2bB-3aD)x}{b^2}+\frac{(3Ab-5aC)x^2}{b}-\frac{2aDx^3}{b}\right.}{2ab}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-a\right.}{2ab\left(a+bx^2\right)}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-a\right.}{2ab\left(a+bx^2\right)}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-a\right.}{2ab\left(a+bx^2\right)}$$

Mathematica [A] time = 0.125929, size = 139, normalized size = 0.79

$$\frac{\frac{6a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{a+bx^2}+12bx(Ab-2aC)+6\sqrt{a}\sqrt{b}(5aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6bx^2(bB-2aD)+6a(3aD-2bB)\log\left(a+\right.}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b^2*D*x^4 +
 (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) + 6*Sqrt[a]*Sqrt[b]*(-
3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-2*b*B + 3*a*D)*Log[a + b
*x^2])/(12*b^4)
```

Maple [A] time = 0.01, size = 201, normalized size = 1.1

$$\frac{Dx^4}{4b^2}+\frac{Cx^3}{3b^2}+\frac{Bx^2}{2b^2}-\frac{Dx^2a}{b^3}+\frac{Ax}{b^2}-2\frac{aCx}{b^3}+\frac{aAx}{2b^2\left(bx^2+a\right)}-\frac{a^2Cx}{2b^3\left(bx^2+a\right)}-\frac{a^2B}{2b^3\left(bx^2+a\right)}+\frac{a^3D}{2b^4\left(bx^2+a\right)}-\frac{\ln\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*D*x^4/b^2+1/3/b^2*C*x^3+1/2*B*x^2/b^2-1/b^3*D*x^2*a+1/b^2*A*x-2/b^3*a*C
*x+1/2*a/b^2/(b*x^2+a)*A*x-1/2*a^2/b^3/(b*x^2+a)*C*x-1/2/b^3*a^2/(b*x^2+a)*
B+1/2*a^3/b^4/(b*x^2+a)*D-1/b^3*ln(b*x^2+a)*B*a+3/2*a^2/b^4*ln(b*x^2+a)*D-3
/2*a/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+5/2*a^2/b^3/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 3.68118, size = 333, normalized size = 1.89

$$\frac{Cx^3}{3b^2}+\frac{Dx^4}{4b^2}+\left(\frac{a\left(-2Bb+3Da\right)}{2b^4}-\frac{\sqrt{-ab^9}\left(-3Ab+5Ca\right)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da^2+4b^4\left(\frac{a(-2Bb+3Da)}{2b^4}-\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**3/(3*b**2) + D*x**4/(4*b**2) + (a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*
b**9)*(-3*A*b + 5*C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-
2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b
**2 + 5*C*a*b)) + (a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*
C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*
b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) -
(B*a**2*b - D*a**3 + x*(-A*a*b**2 + C*a**2*b))/(2*a*b**4 + 2*b**5*x**2) - x
**2*(-B*b + 2*D*a)/(2*b**3) - x*(-A*b + 2*C*a)/b**3
```

Giac [A] time = 1.18027, size = 215, normalized size = 1.22

$$\frac{\left(5\,Ca^2-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}+\frac{\left(3\,Da^2-2\,Bab\right)\log\left(bx^2+a\right)}{2\,b^4}+\frac{Da^3-Ba^2b-\left(Ca^2b-Aab^2\right)x}{2\left(bx^2+a\right)b^4}+\frac{3\,Db^6x^4+4\,Cb^6x^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(3*D*a^
2 - 2*B*a*b)*log(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2
)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4*C*b^6*x^3 - 12*D*a*b^5*x^2 +
 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b^6*x)/b^8
```

3.95 $\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=154

$$-\frac{x^2(Ab-2aC)}{2ab^2}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x(3bB-5aD)}{2b^3}-\frac{\sqrt{a}(3bB-5aD)\tan^{-1}\left(\right.}{2b^{7/2}}$$

```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*x^3)/(3*b^
2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]
*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*
Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.241322, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x^2(Ab-2aC)}{2ab^2}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x(3bB-5aD)}{2b^3}-\frac{\sqrt{a}(3bB-5aD)\tan^{-1}\left(\right.}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```


```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*x^3)/(3*b^
2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]
*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*
Log[a + b*x^2])/(2*b^3)
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)+2(Ab-2aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int\left(-\frac{a(3bB-5aD)}{b^2}+\frac{2(Ab-2aC)x}{b}-\frac{2aDx^2}{b}+\frac{a^2(3bB-5aD)-}{b^2(a+}\right.}{2ab}$$

$$= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{a^2(3bB-5}{}}{}$$

$$= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-2aC}{}$$

$$= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\sqrt{a}(3bB-}{}$$

Mathematica [A] time = 0.0770374, size = 128, normalized size = 0.83

$$\frac{a(-a(C+Dx)+Ab+bBx)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x(bB-2aD)}{b^3}+\frac{\sqrt{a}(5aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A*b + b*B*x
 - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B + 5*a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.008, size = 177, normalized size = 1.2

$$\frac{Dx^3}{3\,b^2}+\frac{Cx^2}{2\,b^2}+\frac{Bx}{b^2}-2\,\frac{aDx}{b^3}+\frac{Bax}{2\,b^2\left(bx^2+a\right)}-\frac{a^2Dx}{2\,b^3\left(bx^2+a\right)}+\frac{aA}{2\,b^2\left(bx^2+a\right)}-\frac{a^2C}{2\,b^3\left(bx^2+a\right)}+\frac{\ln\left(bx^2+a\right)A}{2\,b^2}-\frac{\ln\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*D*x^3/b^2+1/2/b^2*C*x^2+1/b^2*B*x-2/b^3*a*D*x+1/2/b^2/(b*x^2+a)*B*x*a-1
/2/b^3/(b*x^2+a)*a^2*D*x+1/2/b^2*a/(b*x^2+a)*A-1/2/b^3/(b*x^2+a)*a^2*C+1/2/
b^2*ln(b*x^2+a)*A-1/b^3*ln(b*x^2+a)*a*C-3/2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))*B*a+5/2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^2*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 3.46395, size = 287, normalized size = 1.86

$$\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**2/(2*b**2) + D*x**3/(3*b**2) + (-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**
7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*
C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a))
 + (-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log
(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*(-3
*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a)) - (-A*a*b + C*a**2 + x*(-B*a*b +
 D*a**2))/(2*a*b**3 + 2*b**4*x**2) - x*(-B*b + 2*D*a)/b**3
```

Giac [A] time = 1.20425, size = 177, normalized size = 1.15

$$-\frac{(2\,Ca-Ab)\log\left(bx^2+a\right)}{2\,b^3}+\frac{\left(5\,Da^2-3\,Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}-\frac{Ca^2-Aab+\left(Da^2-Bab\right)x}{2\left(bx^2+a\right)b^3}+\frac{2\,Db^4x^3+3\,Cb^4x^2-12\,D}{6\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a - A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 - 12*D*a*b^3*x + 6*B*b^4*x)/b^
6
```

3.96 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=134

$$-\frac{x(Ab-3aC)}{2ab^2}+\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

```
[Out] -((A*b - 3*a*C)*x)/(2*a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*D)/b) - (A
*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.226388, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x(Ab-3aC)}{2ab^2}+\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] -((A*b - 3*a*C)*x)/(2*a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*D)/b) - (A
*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)+(Ab-3aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int\left(A-\frac{3aC}{b}-\frac{2aDx}{b}-\frac{a(Ab-3aC)+2a(bB-2aD)x}{b\left(a+bx^2\right)}\right)dx}{2ab}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{\int \frac{a(Ab-3aC)+2a(bB-2aD)x}{a+bx^2}\,dx}{2ab^2}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-3aC)\int\frac{1}{a+bx^2}\,dx}{2b^2}+\frac{(b\ldots}{\ldots}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\ldots$$

Mathematica [A] time = 0.0749142, size = 100, normalized size = 0.75

$$\frac{\frac{a^2(-D)+ab(B+Cx)-Ab^2x}{a+bx^2}+\frac{\sqrt{b}(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log\left(a+bx^2\right)+2bCx+bDx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*x^2) + (Sq
rt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*B - 2*a*D)*Lo
g[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.008, size = 154, normalized size = 1.2

$$\frac{Dx^2}{2\,b^2}+\frac{Cx}{b^2}-\frac{Ax}{2\,b\left(bx^2+a\right)}+\frac{aCx}{2\,b^2\left(bx^2+a\right)}+\frac{Ba}{2\,b^2\left(bx^2+a\right)}-\frac{a^2D}{2\,b^3\left(bx^2+a\right)}+\frac{\ln\left(bx^2+a\right)B}{2\,b^2}-\frac{\ln\left(bx^2+a\right)aD}{b^3}+\frac{A}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*D*x^2/b^2+1/b^2*C*x-1/2/b/(b*x^2+a)*A*x+1/2/b^2/(b*x^2+a)*a*C*x+1/2/b^2
/(b*x^2+a)*B*a-1/2/b^3/(b*x^2+a)*a^2*D+1/2/b^2*ln(b*x^2+a)*B-1/b^3*ln(b*x^2
+a)*a*D+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A-3/2/b^2/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 3.33235, size = 284, normalized size = 2.12

$$\frac{Cx}{b^2} + \frac{Dx^2}{2b^2} + \left(-\frac{-Bb+2Da}{2b^3} - \frac{\sqrt{-ab^7}\,(-Ab+3Ca)}{4ab^6}\right)\log\left(x + \frac{2Bab - 4Da^2 - 4ab^3\left(-\frac{-Bb+2Da}{2b^3} - \frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right) + \left(-\frac{-B\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D
*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b
)) + (-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6))*l
og(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b
**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a**2 + x
*(-A*b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

Giac [A] time = 1.19379, size = 150, normalized size = 1.12

$$-\frac{(3\,Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} - \frac{(2\,Da - Bb)\log\left(bx^2+a\right)}{2\,b^3} + \frac{Db^2x^2 + 2\,Cb^2x}{2\,b^4} - \frac{Da^2 - Bab - \left(Cab - Ab^2\right)x}{2\left(bx^2+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2*D*a - B*b
)*log(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 - 1/2*(D*a^2 - B*a*b
 - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

3.97 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((b
*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b
*x^2])/(2*b^2)
```

Rubi [A] time = 0.117818, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1804, 1810, 635, 205, 260}

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((b
*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b
*x^2])/(2*b^2)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx &= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2aCx-2aDx^2}{a+bx^2}\,dx}{2ab}\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2aD}{b}-\frac{a(bB-3aD)+2abCx}{b\left(a+bx^2\right)}\right)dx}{2ab}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{\int \frac{a(bB-3aD)+2abCx}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{C\int \frac{x}{a+bx^2}\,dx}{b}+\frac{(bB-3aD)\int \frac{1}{a+bx^2}\,dx}{2b^2}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] time = 0.0485397, size = 92, normalized size = 0.91

$$\frac{aC + aDx - Ab - bBx}{2b^2\left(a + bx^2\right)} - \frac{(3aD - bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}} + \frac{C\log\left(a + bx^2\right)}{2b^2} + \frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) - ((-(b*B) +
 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b*x^2
])/(2*b^2)
```

Maple [A] time = 0.008, size = 127, normalized size = 1.3

$$\frac{Dx}{b^2} - \frac{Bx}{2b\left(bx^2 + a\right)} + \frac{aDx}{2b^2\left(bx^2 + a\right)} - \frac{A}{2b\left(bx^2 + a\right)} + \frac{aC}{2b^2\left(bx^2 + a\right)} + \frac{C\ln\left(bx^2 + a\right)}{2b^2} + \frac{B}{2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3a}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] D*x/b^2-1/2/b/(b*x^2+a)*B*x+1/2/b^2/(b*x^2+a)*a*D*x-1/2/b/(b*x^2+a)*A+1/2/b
^2/(b*x^2+a)*a*C+1/2*C*ln(b*x^2+a)/b^2+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))*B-3/2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 2.65598, size = 212, normalized size = 2.1

$$\frac{Dx}{b^2} + \left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}\,(-Bb+3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right) + \left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}\,(-Bb+3Da)}{4ab^5}\right)\log\Bigg(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(x + (
2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/(-
B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(
x + (2*C*a - 4*a*b**2*(C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)
))/(-B*b + 3*D*a)) + (-A*b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*x**2)
```

Giac [A] time = 1.14644, size = 109, normalized size = 1.08

$$\frac{Dx}{b^2} + \frac{C\log\left(bx^2 + a\right)}{2\,b^2} - \frac{(3\,Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Ca - Ab + (Da - Bb)x}{2\left(bx^2 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] D*x/b^2 + 1/2*C*log(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*x^2 + a)*b^2)
```

3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=93

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0654198, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1814, 635, 205, 260}

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a + b*x^2])/(2*b^2)
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-\frac{Ab+aC}{b}-\frac{2aDx}{b}}{a+bx^2}\,dx}{2a}\\ &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aC)\int \frac{1}{a+bx^2}\,dx}{2ab}+\frac{D\int \frac{x}{a+bx^2}\,dx}{b}\\ &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0812819, size = 83, normalized size = 0.89

$$\frac{\frac{a^2D-ab(B+Cx)+Ab^2x}{a\left(a+bx^2\right)}+\frac{\sqrt{b}(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] ((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.006, size = 97, normalized size = 1.

$$\frac{1}{bx^2+a}\left(\frac{(Ab-aC)\,x}{2\,ab}-\frac{Bb-aD}{2\,b^2}\right)+\frac{D\ln\left(bx^2+a\right)}{2\,b^2}+\frac{A}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{C}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] (1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2*D*ln(b*x^2+a)/b^2+1/2
/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 2.15369, size = 233, normalized size = 2.51

$$\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)\log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right) + \left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)\log\left(x + \frac{-2Da}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] (D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a**
2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4)))/
(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4
))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C
*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) - (B*a*b - D*a**2 + x*(-A*b**2 + C*a*
b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

Giac [A] time = 1.17426, size = 119, normalized size = 1.28

$$\frac{D\log\left(bx^2+a\right)}{2\,b^2}+\frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}-\frac{(Ca-Ab)x-\frac{Da^2-Bab}{b}}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/
(2*a^2)
```

Rubi [A] time = 0.121648, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 801, 635, 205, 260}

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/
(2*a^2)
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx &= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-2A-\frac{(bB+aD)x}{b}}{x\left(a+bx^2\right)}\,dx}{2a}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax}+\frac{-abB-a^2D+2Ab^2x}{ab\left(a+bx^2\right)}\right)dx}{2a}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{\int \frac{-abB-a^2D+2Ab^2x}{a+bx^2}\,dx}{2a^2b}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^2}+\frac{(bB+aD)\int \frac{1}{a+bx^2}\,dx}{2ab}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{(bB+aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0706707, size = 85, normalized size = 0.89

$$\frac{\frac{a(-a(C+Dx)+Ab+bBx)}{b(a+bx^2)} - A\log\left(a+bx^2\right) + \frac{\sqrt{a}(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + 2A\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B + a*D)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.01, size = 125, normalized size = 1.3

$$\frac{A\ln(x)}{a^2} + \frac{Bx}{2a(bx^2+a)} - \frac{xD}{(2bx^2+2a)b} + \frac{A}{2a(bx^2+a)} - \frac{C}{(2bx^2+2a)b} - \frac{A\ln(bx^2+a)}{2a^2} + \frac{B}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x)
```

```
[Out] A*ln(x)/a^2+1/2/a*x/(b*x^2+a)*B-1/2/(b*x^2+a)/b*x*D+1/2/a/(b*x^2+a)*A-1/2/(
b*x^2+a)/b*C-1/2*A*ln(b*x^2+a)/a^2+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
)*B+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 8.1076, size = 797, normalized size = 8.39

$$\frac{A\log(x)}{a^2}+\left(-\frac{A}{2a^2}-\frac{\sqrt{-a^5b^3}\,(Bb+Da)}{4a^4b^3}\right)\log\left(x+\frac{48A^3b^4+48A^2a^2b^4\left(-\frac{A}{2a^2}-\frac{\sqrt{-a^5b^3}(Bb+Da)}{4a^4b^3}\right)-4AB^2ab^3-8ABDa^2b^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

```
[Out] A*log(x)/a**2 + (-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3))*
log(x + (48*A**3*b**4 + 48*A**2*a**2*b**4*(-A/(2*a**2) - sqrt(-a**5*b**3)*(
B*b + D*a)/(4*a**4*b**3)) - 4*A*B**2*a*b**3 - 8*A*B*D*a**2*b**2 - 4*A*D**2*
a**3*b - 96*A*a**4*b**4*(-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4
*b**3))**2 + 4*B**2*a**3*b**3*(-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)/(
4*a**4*b**3)) + 8*B*D*a**4*b**2*(-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)
/(4*a**4*b**3)) + 4*D**2*a**5*b*(-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)
/(4*a**4*b**3)))/(36*A**2*B*b**4 + 36*A**2*D*a*b**3 + B**3*a*b**3 + 3*B**2*
D*a**2*b**2 + 3*B*D**2*a**3*b + D**3*a**4)) + (-A/(2*a**2) + sqrt(-a**5*b**
3)*(B*b + D*a)/(4*a**4*b**3))*log(x + (48*A**3*b**4 + 48*A**2*a**2*b**4*(-A
/(2*a**2) + sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)) - 4*A*B**2*a*b**3 -
 8*A*B*D*a**2*b**2 - 4*A*D**2*a**3*b - 96*A*a**4*b**4*(-A/(2*a**2) + sqrt(-
a**5*b**3)*(B*b + D*a)/(4*a**4*b**3))**2 + 4*B**2*a**3*b**3*(-A/(2*a**2) +
sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)) + 8*B*D*a**4*b**2*(-A/(2*a**2)
+ sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)) + 4*D**2*a**5*b*(-A/(2*a**2)
+ sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)))/(36*A**2*B*b**4 + 36*A**2*D*
a*b**3 + B**3*a*b**3 + 3*B**2*D*a**2*b**2 + 3*B*D**2*a**3*b + D**3*a**4)) -
 (-A*b + C*a + x*(-B*b + D*a))/(2*a**2*b + 2*a*b**2*x**2)
```

Giac [A] time = 1.20465, size = 126, normalized size = 1.33

$$-\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log\left(|x|\right)}{a^2}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}-\frac{Ca^2-Aab+\left(Da^2-Bab\right)x}{2\left(bx^2+a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^2 + A*log(abs(x))/a^2 + 1/2*(D*a + B*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*a^2*b)
```

3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) - ((3*A*
b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x])/a^2
- (B*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.141581, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```



```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) - ((3*A*
b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x])/a^2
- (B*Log[a + b*x^2])/(2*a^2)
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-2A-2Bx+\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{2a}\\
&= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax^2}-\frac{2B}{ax}+\frac{3Ab-aC+2bBx}{a\left(a+bx^2\right)}\right)dx}{2a}\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}+\frac{B\log(x)}{a^2}-\frac{\int \frac{3Ab-aC+2bBx}{a+bx^2}\,dx}{2a^2}\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}+\frac{B\log(x)}{a^2}-\frac{(bB)\int \frac{x}{a+bx^2}\,dx}{a^2}-\frac{(3Ab-aC)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{B\log(x)}{a^2}-\frac{B\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0704589, size = 110, normalized size = 1.

$$\frac{a^2(-D)+abB+abCx-Ab^2x}{2a^2b\left(a+bx^2\right)}+\frac{(aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a + b*x^2)) +
((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x
])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.012, size = 136, normalized size = 1.2

$$-\frac{A}{a^2x}+\frac{B\ln(x)}{a^2}-\frac{Abx}{2\,a^2\left(bx^2+a\right)}+\frac{Cx}{2\,a\left(bx^2+a\right)}+\frac{B}{2\,a\left(bx^2+a\right)}-\frac{D}{\left(2\,bx^2+2\,a\right)b}-\frac{B\ln\left(bx^2+a\right)}{2\,a^2}-\frac{3\,Ab}{2\,a^2}\arctan\left(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -A/a^2/x+B*ln(x)/a^2-1/2/a^2*x/(b*x^2+a)*A*b+1/2/a/(b*x^2+a)*C*x+1/2/a/(b*x
^2+a)*B-1/2/(b*x^2+a)/b*D-1/2*B*ln(b*x^2+a)/a^2-3/2/a^2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2))*A*b+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 8.45161, size = 782, normalized size = 7.11

$$\frac{B\log(x)}{a^2} + \left(-\frac{B}{2a^2} - \frac{\sqrt{-a^5b}\,(-3Ab+Ca)}{4a^5b}\right)\log\left(x + \frac{-36A^2Bab^2 + 36A^2a^3b^2\left(-\frac{B}{2a^2} - \frac{\sqrt{-a^5b}(-3Ab+Ca)}{4a^5b}\right) + 24ABCa^2b - 24A\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] B*log(x)/a**2 + (-B/(2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b))*log
(x + (-36*A**2*B*a*b**2 + 36*A**2*a**3*b**2*(-B/(2*a**2) - sqrt(-a**5*b)*(-
3*A*b + C*a)/(4*a**5*b)) + 24*A*B*C*a**2*b - 24*A*C*a**4*b*(-B/(2*a**2) - s
qrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)) + 48*B**3*a**2*b + 48*B**2*a**4*b*(
-B/(2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)) - 4*B*C**2*a**3 - 96
*B*a**6*b*(-B/(2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b))**2 + 4*C*
*2*a**5*(-B/(2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)))/(-27*A**3*
b**3 + 27*A**2*C*a*b**2 - 108*A*B**2*a*b**2 - 9*A*C**2*a**2*b + 36*B**2*C*a
**2*b + C**3*a**3)) + (-B/(2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b
))*log(x + (-36*A**2*B*a*b**2 + 36*A**2*a**3*b**2*(-B/(2*a**2) + sqrt(-a**5
*b)*(-3*A*b + C*a)/(4*a**5*b)) + 24*A*B*C*a**2*b - 24*A*C*a**4*b*(-B/(2*a**
2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)) + 48*B**3*a**2*b + 48*B**2*a*
*4*b*(-B/(2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)) - 4*B*C**2*a**
3 - 96*B*a**6*b*(-B/(2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b))**2
+ 4*C**2*a**5*(-B/(2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)))/(-27
*A**3*b**3 + 27*A**2*C*a*b**2 - 108*A*B**2*a*b**2 - 9*A*C**2*a**2*b + 36*B*
*2*C*a**2*b + C**3*a**3)) + (-2*A*a*b + x**2*(-3*A*b**2 + C*a*b) + x*(B*a*b
 - D*a**2))/(2*a**3*b*x + 2*a**2*b**2*x**3)
```

Giac [A] time = 1.18563, size = 139, normalized size = 1.26

$$-\frac{B\log\left(bx^2+a\right)}{2\,a^2}+\frac{B\log\left(|x|\right)}{a^2}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}+\frac{Cabx^2-3\,Ab^2x^2-Da^2x+Babx-2\,Aab}{2\left(bx^3+ax\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x^2 - D*a^2*x + B*a
*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

3.101 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=135

$$\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

```
[Out] -A/(2*a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a*(a + b*x^
2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) - ((2
*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.204027, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```


```
[Out] -A/(2*a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a*(a + b*x^
2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) - ((2
*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2A-2Bx+2\left(\frac{Ab}{a}-C\right)x^2+\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax^3}-\frac{2B}{ax^2}-\frac{2(-2Ab+aC)}{a^2x}+\frac{a(3bB-aD)-2b(2Ab-aC)x}{a^2\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}-\frac{\int \frac{a(3bB-aD)-2b(2Ab-aC)x}{a+bx^2}\,dx}{2a^3}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(b(2Ab-aC))\int \frac{x}{a+bx^2}\,dx}{a^3}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{(2Ab-aC)\log(x)}{a^3}+$$

Mathematica [A] time = 0.101593, size = 112, normalized size = 0.83

$$\frac{\frac{a(a(C+Dx)-Ab-bBx)}{a+bx^2}+(2Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-2Ab)-\frac{aA}{x^2}+\frac{\sqrt{a}(aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)
+ (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-2*A*b
+ a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.015, size = 169, normalized size = 1.3

$$-\frac{A}{2\,a^2x^2}-\frac{B}{a^2x}-2\,\frac{A\ln(x)\,b}{a^3}+\frac{\ln(x)\,C}{a^2}-\frac{bBx}{2\,a^2\left(bx^2+a\right)}+\frac{Dx}{2\,a\left(bx^2+a\right)}-\frac{Ab}{2\,a^2\left(bx^2+a\right)}+\frac{C}{2\,a\left(bx^2+a\right)}+\frac{b\ln\left(bx^2+a\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*A/a^2/x^2-B/a^2/x-2/a^3*ln(x)*A*b+1/a^2*ln(x)*C-1/2/a^2/(b*x^2+a)*B*x*
b+1/2/a/(b*x^2+a)*D*x-1/2/a^2/(b*x^2+a)*A*b+1/2/a/(b*x^2+a)*C+1/a^3*b*ln(b*
x^2+a)*A-1/2/a^2*ln(b*x^2+a)*C-3/2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
B*b+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 39.5762, size = 1807, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b))*log(x
+ (-384*A**3*b**4 + 576*A**2*C*a*b**3 + 192*A**2*a**3*b**3*(-(-2*A*b + C*a)
/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) + 72*A*B**2*a*b**3 - 4
8*A*B*D*a**2*b**2 - 288*A*C**2*a**2*b**2 - 192*A*C*a**4*b**2*(-(-2*A*b + C*
a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) + 8*A*D**2*a**3*b +
192*A*a**6*b**2*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4
*a**6*b))**2 - 36*B**2*C*a**2*b**2 + 36*B**2*a**4*b**2*(-(-2*A*b + C*a)/(2*
a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) + 24*B*C*D*a**3*b - 24*B*D
*a**5*b*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)
) + 48*C**3*a**3*b + 48*C**2*a**5*b*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*
b)*(-3*B*b + D*a)/(4*a**6*b)) - 4*C*D**2*a**4 - 96*C*a**7*b*(-(-2*A*b + C*a
)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b))**2 + 4*D**2*a**6*(-(-
2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)))/(-432*A**
2*B*b**4 + 144*A**2*D*a*b**3 + 432*A*B*C*a*b**3 - 144*A*C*D*a**2*b**2 - 27*
B**3*a*b**3 + 27*B**2*D*a**2*b**2 - 108*B*C**2*a**2*b**2 - 9*B*D**2*a**3*b
+ 36*C**2*D*a**3*b + D**3*a**4)) + (-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b
)*(-3*B*b + D*a)/(4*a**6*b))*log(x + (-384*A**3*b**4 + 576*A**2*C*a*b**3 +
192*A**2*a**3*b**3*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a)
/(4*a**6*b)) + 72*A*B**2*a*b**3 - 48*A*B*D*a**2*b**2 - 288*A*C**2*a**2*b**2
 - 192*A*C*a**4*b**2*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*
a)/(4*a**6*b)) + 8*A*D**2*a**3*b + 192*A*a**6*b**2*(-(-2*A*b + C*a)/(2*a**3
) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b))**2 - 36*B**2*C*a**2*b**2 + 36*
```

```
B**2*a**4*b**2*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*
a**6*b)) + 24*B*C*D*a**3*b - 24*B*D*a**5*b*(-(-2*A*b + C*a)/(2*a**3) + sqrt
(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) + 48*C**3*a**3*b + 48*C**2*a**5*b*(-(-
2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) - 4*C*D**2
*a**4 - 96*C*a**7*b*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a
)/(4*a**6*b))**2 + 4*D**2*a**6*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-
3*B*b + D*a)/(4*a**6*b)))/(-432*A**2*B*b**4 + 144*A**2*D*a*b**3 + 432*A*B*C
*a*b**3 - 144*A*C*D*a**2*b**2 - 27*B**3*a*b**3 + 27*B**2*D*a**2*b**2 - 108*
B*C**2*a**2*b**2 - 9*B*D**2*a**3*b + 36*C**2*D*a**3*b + D**3*a**4)) + (-A*a
 - 2*B*a*x + x**3*(-3*B*b + D*a) + x**2*(-2*A*b + C*a))/(2*a**3*x**2 + 2*a*
*2*b*x**4) + (-2*A*b + C*a)*log(x + (-384*A**3*b**4 + 576*A**2*C*a*b**3 + 1
92*A**2*b**3*(-2*A*b + C*a) + 72*A*B**2*a*b**3 - 48*A*B*D*a**2*b**2 - 288*A
*C**2*a**2*b**2 - 192*A*C*a*b**2*(-2*A*b + C*a) + 8*A*D**2*a**3*b + 192*A*b
**2*(-2*A*b + C*a)**2 - 36*B**2*C*a**2*b**2 + 36*B**2*a*b**2*(-2*A*b + C*a)
 + 24*B*C*D*a**3*b - 24*B*D*a**2*b*(-2*A*b + C*a) + 48*C**3*a**3*b + 48*C**
2*a**2*b*(-2*A*b + C*a) - 4*C*D**2*a**4 - 96*C*a*b*(-2*A*b + C*a)**2 + 4*D*
*2*a**3*(-2*A*b + C*a))/(-432*A**2*B*b**4 + 144*A**2*D*a*b**3 + 432*A*B*C*a
*b**3 - 144*A*C*D*a**2*b**2 - 27*B**3*a*b**3 + 27*B**2*D*a**2*b**2 - 108*B*
C**2*a**2*b**2 - 9*B*D**2*a**3*b + 36*C**2*D*a**3*b + D**3*a**4))/a**3
```

Giac [A] time = 1.16835, size = 170, normalized size = 1.26

$$\frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}-\frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3}+\frac{(Ca-2\,Ab)\log\left(|x|\right)}{a^3}-\frac{2\,Ba^2x-\left(Da^2-3\,Bab\right)x^3+Aa^2-\left(Ca^2\right.}{2\left(bx^2+a\right)a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a - 2*A*b)
*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(abs(x))/a^3 - 1/2*(2*B*a^2*x - (D*a
^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2)/((b*x^2 + a)*a^3*x^2)
```

3.102 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=185

$$\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2(a+bx^2)}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}-\frac{x^2(bB-}{2a}$$

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) - (x^4*(a*(B
 - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (x^3*(A*b - 5*a*C + 4
*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3*(A*b - 5*a*C)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.338183, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 801, 635, 205, 260}

$$\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2(a+bx^2)}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}-\frac{x^2(bB-}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) - (x^4*(a*(B
 - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (x^3*(A*b - 5*a*C + 4
*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3*(A*b - 5*a*C)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(Ab-5aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} + \frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{\int \frac{x^2(-3a(Ab-5aC)-}{a+bx}}{8a^2}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} + \frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{\int\left(-\frac{3a(Ab-5aC)}{b}\right.}{}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3} - \frac{(bB-3aD)x^2}{2ab^3} - \frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} + \frac{x^3(Ab-5aC+}{8ab^2(a}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3} - \frac{(bB-3aD)x^2}{2ab^3} - \frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} + \frac{x^3(Ab-5aC+}{8ab^2(a}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3} - \frac{(bB-3aD)x^2}{2ab^3} - \frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} + \frac{x^3(Ab-5aC+}{8ab^2(a}$$

Mathematica [A] time = 0.116181, size = 139, normalized size = 0.75

$$\frac{\frac{2a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{\left(a+bx^2\right)^2} + \frac{-12a^2D+8abB+9abCx-5Ab^2x}{a+bx^2} + \frac{3\sqrt{b}(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + 4(bB-3aD)\log\left(a+bx^2\right) + 8bCx + 4bDx^2}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*x)/(a + b*
x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2)^2 + (3*Sqrt[b]*(
A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 4*(b*B - 3*a*D)*Log[a +
 b*x^2])/(8*b^4)
```

Maple [A] time = 0.01, size = 235, normalized size = 1.3

$$\frac{Dx^2}{2b^3}+\frac{Cx}{b^3}-\frac{5Ax^3}{8b(bx^2+a)^2}+\frac{9Cx^3a}{8b^2(bx^2+a)^2}+\frac{Bax^2}{b^2(bx^2+a)^2}-\frac{3Dx^2a^2}{2b^3(bx^2+a)^2}-\frac{3aAx}{8b^2(bx^2+a)^2}+\frac{7a^2Cx}{8b^3(bx^2+a)^2}+\frac{}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/2/b^3*D*x^2+1/b^3*C*x-5/8/b/(b*x^2+a)^2*A*x^3+9/8/b^2/(b*x^2+a)^2*C*x^3*a
+1/b^2/(b*x^2+a)^2*B*x^2*a-3/2/b^3/(b*x^2+a)^2*D*x^2*a^2-3/8/b^2/(b*x^2+a)^
2*A*a*x+7/8/b^3/(b*x^2+a)^2*a^2*C*x+3/4/b^3/(b*x^2+a)^2*a^2*B-5/4/b^4/(b*x^
2+a)^2*a^3*D+1/2/b^3*ln(b*x^2+a)*B-3/2/b^4*ln(b*x^2+a)*a*D+3/8/b^2/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*A-15/8/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 23.3071, size = 357, normalized size = 1.93

$$\frac{Cx}{b^3} + \frac{Dx^2}{2b^3} + \left(-\frac{-Bb + 3Da}{2b^4} - \frac{3\sqrt{-ab^9}\,(-Ab + 5Ca)}{16ab^8}\right)\log\left(x + \frac{8Bab - 24Da^2 - 16ab^4\left(-\frac{-Bb+3Da}{2b^4} - \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2 + 15Cab}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-
A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b
+ 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2
 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/
(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)/(2*b
**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C*a*b))
 + (6*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8*B*a*b*
*2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b**4 + 16*a*b**5*
x**2 + 8*b**6*x**4)
```

Giac [A] time = 1.19736, size = 212, normalized size = 1.15

$$-\frac{3\,(5\,Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} - \frac{(3\,Da - Bb)\log\left(bx^2 + a\right)}{2\,b^4} + \frac{Db^3x^2 + 2\,Cb^3x}{2\,b^6} - \frac{10\,Da^3 - 6\,Ba^2b - \left(9\,Cab^2 - 5\,Ab^3\right)x^3 -}{8\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3*D*a - B*b
)*log(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 - 1/8*(10*D*a^3 - 6*
B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B*a*b^2)*x^2 - (7*C*
a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2*b^4)
```

3.103 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-x(3bB-7aD))}{8ab^2\left(a+bx^2\right)}-\frac{3x(bB-5aD)}{8ab^3}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{C\log\left(a+bx\right.}{2b^3}$$

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4
*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(8*a*b^2*(a + b*x^2
)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C
*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.232444, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 774, 635, 205, 260}

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-x(3bB-7aD))}{8ab^2\left(a+bx^2\right)}-\frac{3x(bB-5aD)}{8ab^3}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{C\log\left(a+bx\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4
*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(8*a*b^2*(a + b*x^2
)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C
*Log[a + b*x^2])/(2*b^3)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx=-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int\frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)-4aCx-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$=-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int\frac{x\left(8a^2C-3a(bB-5aD)x\right)}{a+bx^2}\,d}{8a^2b^2}$$

$$=-\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int\frac{3a^2}{}}{}$$

$$=-\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{C\int}{}$$

$$=-\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{3(bB}{}$$

Mathematica [A] time = 0.0758234, size = 126, normalized size = 0.81

$$\frac{a(-a(C+Dx)+Ab+bBx)}{4b^3\left(a+bx^2\right)^2}+\frac{8aC+9aDx-4Ab-5bBx}{8b^3\left(a+bx^2\right)}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{C\log\left(a+bx^2\right)}{2b^3}+\frac{Dx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^2)) + (a*(
A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*(b*B - 5*a*D)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.01, size = 206, normalized size = 1.3

$$\frac{Dx}{b^3}-\frac{5\,Bx^3}{8\,b\left(bx^2+a\right)^2}+\frac{9\,Dx^3a}{8\,b^2\left(bx^2+a\right)^2}-\frac{Ax^2}{2\,b\left(bx^2+a\right)^2}+\frac{Cx^2a}{b^2\left(bx^2+a\right)^2}-\frac{3\,Bax}{8\,b^2\left(bx^2+a\right)^2}+\frac{7\,a^2Dx}{8\,b^3\left(bx^2+a\right)^2}-\frac{Aa}{4\,b^2\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] D/b^3*x-5/8/b/(b*x^2+a)^2*B*x^3+9/8/b^2/(b*x^2+a)^2*D*x^3*a-1/2/b/(b*x^2+a)
^2*A*x^2+1/b^2/(b*x^2+a)^2*C*x^2*a-3/8/b^2/(b*x^2+a)^2*B*x*a+7/8/b^3/(b*x^2
+a)^2*a^2*D*x-1/4/b^2/(b*x^2+a)^2*A*a+3/4/b^3/(b*x^2+a)^2*a^2*C+1/2*C*ln(b*
x^2+a)/b^3+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B-15/8/b^3/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))*a*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 21.8294, size = 282, normalized size = 1.82

$$\frac{Dx}{b^3}+\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)\log\left(x+\frac{8Ca-16ab^3\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)+\left(\frac{C}{2b^3}+\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x
+ (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**
7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*
a*b**7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5
*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(-5*B*
b**2 + 9*D*a*b) + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2))/(8*
a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

Giac [A] time = 1.13859, size = 165, normalized size = 1.06

$$\frac{Dx}{b^3}+\frac{C\log\left(bx^2+a\right)}{2\,b^3}-\frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}+\frac{\left(9\,Dab-5\,Bb^2\right)x^3+6\,Ca^2-2\,Aab+4\left(2\,Cab-Ab^2\right)x^2+\left(7\,D\right.}{8\left(bx^2+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2
*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/((b*x^2 + a)^2*b^3)
```

3.104 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=136

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{D\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] -(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (x*(A*b +
3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A*b + 3*a*C)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.158462, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1804, 635, 205, 260}

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{D\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (x*(A*b +
3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A*b + 3*a*C)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D*Log[a + b*x^2])/(2*b^3)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)-(Ab+3aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{a(Ab+3aC)+8a^2Dx}{a+bx^2}}{8a^2b^2}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3aC)\int \frac{1}{a+bx}}{8ab^2}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3aC)\tan^{-1}}{8a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.0932766, size = 122, normalized size = 0.9

$$\frac{\frac{-2a^2D+2ab(B+Cx)-2Ab^2x}{\left(a+bx^2\right)^2}+\frac{8a^2D-ab(4B+5Cx)+Ab^2x}{a\left(a+bx^2\right)}+\frac{\sqrt{b}(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+4D\log\left(a+bx^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

[Out] ((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*D + A*b^2*x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2])/(8*b^3)

Maple [A] time = 0.009, size = 133, normalized size = 1.

$$\frac{1}{\left(bx^2+a\right)^2}\left(\frac{(Ab-5\,aC)\,x^3}{8\,ab}-\frac{(Bb-2\,aD)\,x^2}{2\,b^2}-\frac{(Ab+3\,aC)\,x}{8\,b^2}-\frac{a\,(Bb-3\,aD)}{4\,b^3}\right)+\frac{D\ln\left(bx^2+a\right)}{2\,b^3}+\frac{A}{8\,ab}\,\mathrm{arctan}\left(bx\frac{1}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)

[Out] (1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/b^2*x-1/4*a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/2*D*ln(b*x^2+a)/b^3+1/8/a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*C

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [B] time = 18.1106, size = 303, normalized size = 2.23

$$\left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + \frac{-8Da^2 + 16a^2b^3\left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right) + \left(\frac{D}{2b^3} + \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] (D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D*
a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b
**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a)/(
16*a**3*b**6))*log(x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) + sqrt(-a**3*b
**7)*(A*b + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) - (2*B*a**2*b - 6*D
*a**3 + x**3*(-A*b**3 + 5*C*a*b**2) + x**2*(4*B*a*b**2 - 8*D*a**2*b) + x*(A
*a*b**2 + 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5*x**4)
```

Giac [A] time = 1.19087, size = 173, normalized size = 1.27

$$\frac{D\log\left(bx^2+a\right)}{2\,b^3} + \frac{(3\,Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{\left(5\,Cab - Ab^2\right)x^3 - 4\left(2\,Da^2 - Bab\right)x^2 + \left(3\,Ca^2 + Aab\right)x - \frac{2\left(3\,Da^3 - Ba^2b\right)}{b}}{8\left(bx^2+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*a*b)*x^2 + (3*C*a^2
 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2 + a)^2*a*b^2)
```

3.105 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=119

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] -(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (2*(A*b + a*
C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.114108, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1804, 1814, 12, 205}

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (2*(A*b + a*
C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx &= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int\frac{-a\left(B-\frac{aD}{b}\right)-2(Ab+aC)x-4aDx^2}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)}+\frac{\int\frac{a\left(B+\frac{3aD}{b}\right)}{a+bx^2}\,dx}{8a^2b}\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)}+\frac{(bB+3aD)\int\frac{1}{a+bx^2}\,dx}{8ab^2}\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)}+\frac{(bB+3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.111549, size = 99, normalized size = 0.83

$$\frac{\frac{\sqrt{b}\left(-a^2(2C+3Dx)-ab\left(2A+x\left(B+4Cx+5Dx^2\right)\right)+b^2Bx^3\right)}{a\left(a+bx^2\right)^2}+\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*x + 5*D*x^
2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/
2))/(8*b^(5/2))
```

Maple [A] time = 0.008, size = 110, normalized size = 0.9

$$\frac{1}{\left(bx^2+a\right)^2}\left(\frac{(Bb-5\,aD)\,x^3}{8\,ab}-\frac{Cx^2}{2\,b}-\frac{(Bb+3\,aD)\,x}{8\,b^2}-\frac{Ab+aC}{4\,b^2}\right)+\frac{B}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,D}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(B*b-5*D*a)/a/b*x^3-1/2*C*x^2/b-1/8*(B*b+3*D*a)/b^2*x-1/4*(A*b+C*a)/b^
2)/(b*x^2+a)^2+1/8/a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B+3/8/b^2/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 13.4165, size = 177, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}-\frac{2Aab+2Ca^2+4Cabx^2+x}{8a^3b^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 - (2*A*a*b + 2*C*a**2 + 4*C*a*b*x**2 + x**3*(-B*b**2 + 5*D*a*b) +
 x*(B*a*b + 3*D*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

Giac [A] time = 1.19612, size = 131, normalized size = 1.1

$$\frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}-\frac{5\,Dabx^3-Bb^2x^3+4\,Cabx^2+3\,Da^2x+Babx+2\,Ca^2+2\,Aab}{8\left(bx^2+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*D*a*b*x^
3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C*a^2 + 2*A*a*b)/((b*
x^2 + a)^2*a*b^2)
```

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=116

$$-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2\left(a+bx^2\right)}+\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(4*a*b*(a + b*x^2)^2) - (4*a^2*D - b*(3*
A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0682323, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1814, 639, 205}

$$-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2\left(a+bx^2\right)}+\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(4*a*b*(a + b*x^2)^2) - (4*a^2*D - b*(3*
A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
```

```
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{-3A-\frac{aC}{b}-\frac{4aDx}{b}}{\left(a+bx^2\right)^2}\,dx}{4a}\\ &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\int \frac{1}{a+bx^2}\,dx}{8a^2b}\\ &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0990765, size = 104, normalized size = 0.9

$$\frac{\frac{\sqrt{a}\left(-a^2b(2B+x(C+4Dx))-2a^3D+ab^2x\left(5A+Cx^2\right)+3Ab^3x^3\right)}{\left(a+bx^2\right)^2}+\sqrt{b}(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x*
(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*b^2)
```

Maple [A] time = 0.007, size = 111, normalized size = 1.

$$\frac{1}{\left(bx^2+a\right)^2}\left(\frac{(3\,Ab+aC)\,x^3}{8\,a^2}-\frac{Dx^2}{2\,b}+\frac{(5\,Ab-aC)\,x}{8\,ab}-\frac{Bb+aD}{4\,b^2}\right)+\frac{3\,A}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{C}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b^
2)/(b*x^2+a)^2+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+1/8/a/b/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 7.9541, size = 184, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{-2Ba^2b-2Da^3-4Da^2bx^2}{8a^4b^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b**2
) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b*
*4*x**4)
```

Giac [A] time = 1.21808, size = 143, normalized size = 1.23

$$\frac{(Ca + 3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Cab^2x^3 + 3\,Ab^3x^3 - 4\,Da^2bx^2 - Ca^2bx + 5\,Aab^2x - 2\,Da^3 - 2\,Ba^2b}{8\left(bx^2 + a\right)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x^
3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B*a
^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=130

$$\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b*B + a*D)*
x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.134627, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1805, 823, 801, 635, 205, 260}

$$\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b*B + a*D)*
x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-\frac{(3bB+aD)x}{b}}{x\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{\int \frac{8aAb+a(3bB+aD)x}{x\left(a+bx^2\right)}\,dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{\int \left(\frac{8Ab}{x}+\frac{3abB+a^2D-8Ab^2x}{a+bx^2}\right)dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{A\log(x)}{a^3} + \frac{\int \frac{3abB+a^2D-8Ab^2x}{a+bx^2}\,dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{A\log(x)}{a^3} - \frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^3} + \frac{(3bB+a}{8}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{(3bB+aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}} + \frac{A\log(x)}{a^3} -$$

Mathematica [A] time = 0.102998, size = 117, normalized size = 0.9

$$\frac{\frac{2a^2(-a(C+Dx)+Ab+bBx)}{b\left(a+bx^2\right)^2} + \frac{a(aDx+4Ab+3bBx)}{b\left(a+bx^2\right)} - 4A\log\left(a+bx^2\right) + \frac{\sqrt{a}(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + 8A\log(x)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*B*x - a*(C
 + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x^2])/(8*a^3)
```

Maple [A] time = 0.013, size = 184, normalized size = 1.4

$$\frac{A\ln(x)}{a^3} + \frac{3\,bBx^3}{8\,a^2\left(bx^2+a\right)^2} + \frac{Dx^3}{8\,a\left(bx^2+a\right)^2} + \frac{Ax^2b}{2\,a^2\left(bx^2+a\right)^2} + \frac{5\,Bx}{8\,a\left(bx^2+a\right)^2} - \frac{xD}{8\left(bx^2+a\right)^2b} + \frac{3\,A}{4\,a\left(bx^2+a\right)^2} - \frac{}{4\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x)
```

```
[Out] A*ln(x)/a^3+3/8/a^2/(b*x^2+a)^2*B*x^3*b+1/8/a/(b*x^2+a)^2*D*x^3+1/2/a^2/(b*
x^2+a)^2*A*x^2*b+5/8/a/(b*x^2+a)^2*B*x-1/8/(b*x^2+a)^2/b*x*D+3/4/a/(b*x^2+a
)^2*A-1/4/(b*x^2+a)^2/b*C-1/2*A*ln(b*x^2+a)/a^3+3/8/a^2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2))*B+1/8/a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 16.3593, size = 872, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

```
[Out] A*log(x)/a**3 + (-A/(2*a**3) - sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3
))*log(x + (3072*A**3*b**4 + 3072*A**2*a**3*b**4*(-A/(2*a**3) - sqrt(-a**7*
b**3)*(3*B*b + D*a)/(16*a**6*b**3)) - 144*A*B**2*a*b**3 - 96*A*B*D*a**2*b**
2 - 16*A*D**2*a**3*b - 6144*A*a**6*b**4*(-A/(2*a**3) - sqrt(-a**7*b**3)*(3*
B*b + D*a)/(16*a**6*b**3))**2 + 144*B**2*a**4*b**3*(-A/(2*a**3) - sqrt(-a**
7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)) + 96*B*D*a**5*b**2*(-A/(2*a**3) - sqr
t(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)) + 16*D**2*a**6*b*(-A/(2*a**3) -
 sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)))/(1728*A**2*B*b**4 + 576*A*
*2*D*a*b**3 + 27*B**3*a*b**3 + 27*B**2*D*a**2*b**2 + 9*B*D**2*a**3*b + D**3
*a**4)) + (-A/(2*a**3) + sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3))*log
(x + (3072*A**3*b**4 + 3072*A**2*a**3*b**4*(-A/(2*a**3) + sqrt(-a**7*b**3)*
(3*B*b + D*a)/(16*a**6*b**3)) - 144*A*B**2*a*b**3 - 96*A*B*D*a**2*b**2 - 16
*A*D**2*a**3*b - 6144*A*a**6*b**4*(-A/(2*a**3) + sqrt(-a**7*b**3)*(3*B*b +
D*a)/(16*a**6*b**3))**2 + 144*B**2*a**4*b**3*(-A/(2*a**3) + sqrt(-a**7*b**3
)*(3*B*b + D*a)/(16*a**6*b**3)) + 96*B*D*a**5*b**2*(-A/(2*a**3) + sqrt(-a**
7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)) + 16*D**2*a**6*b*(-A/(2*a**3) + sqrt(
-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)))/(1728*A**2*B*b**4 + 576*A**2*D*a
*b**3 + 27*B**3*a*b**3 + 27*B**2*D*a**2*b**2 + 9*B*D**2*a**3*b + D**3*a**4)
) + (6*A*a*b + 4*A*b**2*x**2 - 2*C*a**2 + x**3*(3*B*b**2 + D*a*b) + x*(5*B*
a*b - D*a**2))/(8*a**4*b + 16*a**3*b**2*x**2 + 8*a**2*b**3*x**4)
```

Giac [A] time = 1.67072, size = 173, normalized size = 1.33

$$-\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log\left(|x|\right)}{a^3}+\frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}+\frac{4\,Aab^2x^2-2\,Ca^3+6\,Aa^2b+\left(Da^2b+3\,Bab^2\right)x^3-(Da^3}{8\left(bx^2+a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^3 + A*log(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b +
 (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b)
```

3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=144

$$\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + (4*B
 - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b - a*C)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b*x^2])/
(2*a^3)
```

Rubi [A] time = 0.228149, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```


```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + (4*B
 - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b - a*C)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b*x^2])/
(2*a^3)
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-4Bx+3\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \frac{8A+8Bx-\left(\frac{7Ab}{a}-3C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \left(\frac{8A}{ax^2}+\frac{8B}{ax}+\frac{-15Ab+3aC-8bBx}{a\left(a+bx^2\right)}\right)dx}{8a^2}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{B\log(x)}{a^3} + \frac{\int \frac{-15Ab+3aC-8bBx}{a+bx^2}\,dx}{8a^3}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{B\log(x)}{a^3} - \frac{(bB)\int \frac{x}{a+bx^2}\,dx}{a^3} - \frac{(3(5\cdots}{\cdots}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} - \frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} + \frac{B\log\cdots}{a^3\cdots}$$

Mathematica [A] time = 0.0961463, size = 141, normalized size = 0.98

$$\frac{a^2(-D)+abB+abCx-Ab^2x}{4a^2b\left(a+bx^2\right)^2} + \frac{4aB+3aCx-7Abx}{8a^3\left(a+bx^2\right)} + \frac{3(aC-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} - \frac{A}{a^3x} - \frac{B\log\left(a+bx^2\right)}{2a^3} + \frac{B\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a + b*x^2)^2)
+ (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b
*x^2])/(2*a^3)
```

Maple [A] time = 0.013, size = 195, normalized size = 1.4

$$-\frac{A}{a^{3}x}+\frac{B\ln(x)}{a^{3}}-\frac{7\,Ax^{3}b^{2}}{8\,a^{3}\left(bx^{2}+a\right)^{2}}+\frac{3\,bCx^{3}}{8\,a^{2}\left(bx^{2}+a\right)^{2}}+\frac{Bx^{2}b}{2\,a^{2}\left(bx^{2}+a\right)^{2}}-\frac{9\,Abx}{8\,a^{2}\left(bx^{2}+a\right)^{2}}+\frac{5\,Cx}{8\,a\left(bx^{2}+a\right)^{2}}+\frac{3\,B}{4\,a\left(bx^{2}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -A/a^3/x+B*ln(x)/a^3-7/8/a^3/(b*x^2+a)^2*A*x^3*b^2+3/8/a^2/(b*x^2+a)^2*C*x^
3*b+1/2/a^2/(b*x^2+a)^2*B*x^2*b-9/8/a^2/(b*x^2+a)^2*A*b*x+5/8/a/(b*x^2+a)^2
*C*x+3/4/a/(b*x^2+a)^2*B-1/4/(b*x^2+a)^2/b*D-1/2*B*ln(b*x^2+a)/a^3-15/8/a^3
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A*b+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 20.804, size = 860, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] B*log(x)/a**3 + (-B/(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b))*
log(x + (-1200*A**2*B*a*b**2 + 1200*A**2*a**4*b**2*(-B/(2*a**3) - 3*sqrt(-a
**7*b)*(-5*A*b + C*a)/(16*a**7*b)) + 480*A*B*C*a**2*b - 480*A*C*a**5*b*(-B/
(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)) + 1024*B**3*a**2*b +
 1024*B**2*a**5*b*(-B/(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)
) - 48*B*C**2*a**3 - 2048*B*a**8*b*(-B/(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b +
 C*a)/(16*a**7*b))**2 + 48*C**2*a**6*(-B/(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b
 + C*a)/(16*a**7*b)))/(-1125*A**3*b**3 + 675*A**2*C*a*b**2 - 2880*A*B**2*a*
b**2 - 135*A*C**2*a**2*b + 576*B**2*C*a**2*b + 9*C**3*a**3)) + (-B/(2*a**3)
 + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b))*log(x + (-1200*A**2*B*a*b**2
 + 1200*A**2*a**4*b**2*(-B/(2*a**3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a*
*7*b)) + 480*A*B*C*a**2*b - 480*A*C*a**5*b*(-B/(2*a**3) + 3*sqrt(-a**7*b)*(
-5*A*b + C*a)/(16*a**7*b)) + 1024*B**3*a**2*b + 1024*B**2*a**5*b*(-B/(2*a**
3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)) - 48*B*C**2*a**3 - 2048*B*
a**8*b*(-B/(2*a**3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b))**2 + 48*C
**2*a**6*(-B/(2*a**3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)))/(-1125
*A**3*b**3 + 675*A**2*C*a*b**2 - 2880*A*B**2*a*b**2 - 135*A*C**2*a**2*b + 5
76*B**2*C*a**2*b + 9*C**3*a**3)) + (-8*A*a**2*b + 4*B*a*b**2*x**3 + x**4*(-
15*A*b**3 + 3*C*a*b**2) + x**2*(-25*A*a*b**2 + 5*C*a**2*b) + x*(6*B*a**2*b
- 2*D*a**3))/(8*a**5*b*x + 16*a**4*b**2*x**3 + 8*a**3*b**3*x**5)
```

Giac [A] time = 1.70337, size = 190, normalized size = 1.32

$$-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(|x|\right)}{a^3}+\frac{3\left(Ca-5\,Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}+\frac{4\,Bab^2x^3+3\left(Cab^2-5\,Ab^3\right)x^4-8\,Aa^2b+5\left(Ca^2b-5\,A\right.}{8\left(bx^2+a\right)^2a^3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^3 + B*log(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x
```

```
^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/((b
*x^2 + a)^2*a^3*b*x)
```

3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=174

$$-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{A}{2a^3x^2}-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-$$

```
[Out] -A/(2*a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a*(a + b*x^
2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a + b*x^2)) - (3*(5*b
*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) - ((3*A*b - a*C)
*Log[x])/a^4 + ((3*A*b - a*C)*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.311011, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{A}{2a^3x^2}-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -A/(2*a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a*(a + b*x^
2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a + b*x^2)) - (3*(5*b
*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) - ((3*A*b - a*C)
*Log[x])/a^4 + ((3*A*b - a*C)*Log[a + b*x^2])/(2*a^4)
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx = -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-4Bx+4\left(\frac{Ab}{a}-C\right)x^2+3\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)^2}dx}{4a}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} + \frac{\int \frac{8A+8Bx-8\left(\frac{2Ab}{a}-C\right)x^2-\left(\frac{7bB}{a}-3D\right)x^3}{x^3\left(a+bx^2\right)} d}{8a^2}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} + \frac{\int\left(\frac{8A}{ax^3}+\frac{8B}{ax^2}+\frac{8(-3Ab+aC)}{a^2x}+\frac{-3a(5}{}\right.}{8a^2}$$

$$= -\frac{A}{2a^3x^2} - \frac{B}{a^3x} - \frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} - \frac{(3Ab-aC)\log(x}{a^4}$$

$$= -\frac{A}{2a^3x^2} - \frac{B}{a^3x} - \frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} - \frac{(3Ab-aC)\log(x}{a^4}$$

$$= -\frac{A}{2a^3x^2} - \frac{B}{a^3x} - \frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} - \frac{3(5bB-aD)\tan}{8a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.150128, size = 147, normalized size = 0.84

$$\frac{\frac{2a^2(a(C+Dx)-Ab-bBx)}{\left(a+bx^2\right)^2} + \frac{a(4aC+3aDx-8Ab-7bBx)}{a+bx^2} + 4(3Ab-aC)\log\left(a+bx^2\right) + 8\log(x)(aC-3Ab) - \frac{4aA}{x^2} + \frac{3\sqrt{a}(aD-5bB)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{\sqrt{b}}}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*x))/(a + b
*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)^2 + (3*Sqrt[a]*(
-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 8*(-3*A*b + a*C)*Log[x
] + 4*(3*A*b - a*C)*Log[a + b*x^2])/(8*a^4)
```

Maple [A] time = 0.017, size = 250, normalized size = 1.4

$$-\frac{A}{2\,a^3x^2}-\frac{B}{a^3x}-3\,\frac{A\ln(x)\,b}{a^4}+\frac{\ln(x)\,C}{a^3}-\frac{7\,Bx^3b^2}{8\,a^3\left(bx^2+a\right)^2}+\frac{3\,bDx^3}{8\,a^2\left(bx^2+a\right)^2}-\frac{Ax^2b^2}{a^3\left(bx^2+a\right)^2}+\frac{bCx^2}{2\,a^2\left(bx^2+a\right)^2}-\frac{9\,b}{8\,a^2\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2*A/a^3/x^2-B/a^3/x-3/a^4*ln(x)*A*b+1/a^3*ln(x)*C-7/8/a^3/(b*x^2+a)^2*B*
x^3*b^2+3/8/a^2/(b*x^2+a)^2*D*x^3*b-1/a^3/(b*x^2+a)^2*A*x^2*b^2+1/2/a^2/(b*
x^2+a)^2*C*x^2*b-9/8/a^2/(b*x^2+a)^2*B*x*b+5/8/a/(b*x^2+a)^2*D*x-5/4/a^2/(b
*x^2+a)^2*A*b+3/4/a/(b*x^2+a)^2*C+3/2/a^4*b*ln(b*x^2+a)*A-1/2/a^3*ln(b*x^2+
a)*C-15/8/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B*b+3/8/a^2/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))*D
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [B] time = 60.1891, size = 1904, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b))*log
(x + (-27648*A**3*b**4 + 27648*A**2*C*a*b**3 + 9216*A**2*a**4*b**3*(-(-3*A*
b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) + 3600*A*B*
*2*a*b**3 - 1440*A*B*D*a**2*b**2 - 9216*A*C**2*a**2*b**2 - 6144*A*C*a**5*b*
*2*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b))
+ 144*A*D**2*a**3*b + 6144*A*a**8*b**2*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-
a**9*b)*(-5*B*b + D*a)/(16*a**8*b))**2 - 1200*B**2*C*a**2*b**2 + 1200*B**2*
a**5*b**2*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a*
*8*b)) + 480*B*C*D*a**3*b - 480*B*D*a**6*b*(-(-3*A*b + C*a)/(2*a**4) - 3*sq
rt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) + 1024*C**3*a**3*b + 1024*C**2*a**6
*b*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b))
- 48*C*D**2*a**4 - 2048*C*a**9*b*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b
)*(-5*B*b + D*a)/(16*a**8*b))**2 + 48*D**2*a**7*(-(-3*A*b + C*a)/(2*a**4) -
 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)))/(-25920*A**2*B*b**4 + 5184*A*
*2*D*a*b**3 + 17280*A*B*C*a*b**3 - 3456*A*C*D*a**2*b**2 - 1125*B**3*a*b**3
+ 675*B**2*D*a**2*b**2 - 2880*B*C**2*a**2*b**2 - 135*B*D**2*a**3*b + 576*C*
*2*D*a**3*b + 9*D**3*a**4)) + (-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(
-5*B*b + D*a)/(16*a**8*b))*log(x + (-27648*A**3*b**4 + 27648*A**2*C*a*b**3
+ 9216*A**2*a**4*b**3*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b +
 D*a)/(16*a**8*b)) + 3600*A*B**2*a*b**3 - 1440*A*B*D*a**2*b**2 - 9216*A*C**
2*a**2*b**2 - 6144*A*C*a**5*b**2*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b
)*(-5*B*b + D*a)/(16*a**8*b)) + 144*A*D**2*a**3*b + 6144*A*a**8*b**2*(-(-3*
A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b))**2 - 1200
*B**2*C*a**2*b**2 + 1200*B**2*a**5*b**2*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(
-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) + 480*B*C*D*a**3*b - 480*B*D*a**6*b*(-
(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) + 102
4*C**3*a**3*b + 1024*C**2*a**6*b*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b
)*(-5*B*b + D*a)/(16*a**8*b)) - 48*C*D**2*a**4 - 2048*C*a**9*b*(-(-3*A*b +
C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b))**2 + 48*D**2*a*
*7*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)))
/(-25920*A**2*B*b**4 + 5184*A**2*D*a*b**3 + 17280*A*B*C*a*b**3 - 3456*A*C*D
*a**2*b**2 - 1125*B**3*a*b**3 + 675*B**2*D*a**2*b**2 - 2880*B*C**2*a**2*b**
2 - 135*B*D**2*a**3*b + 576*C**2*D*a**3*b + 9*D**3*a**4)) + (-4*A*a**2 - 8*
B*a**2*x + x**5*(-15*B*b**2 + 3*D*a*b) + x**4*(-12*A*b**2 + 4*C*a*b) + x**3
*(-25*B*a*b + 5*D*a**2) + x**2*(-18*A*a*b + 6*C*a**2))/(8*a**5*x**2 + 16*a*
*4*b*x**4 + 8*a**3*b**2*x**6) + (-3*A*b + C*a)*log(x + (-27648*A**3*b**4 +
```

```
27648*A**2*C*a*b**3 + 9216*A**2*b**3*(-3*A*b + C*a) + 3600*A*B**2*a*b**3 -
1440*A*B*D*a**2*b**2 - 9216*A*C**2*a**2*b**2 - 6144*A*C*a*b**2*(-3*A*b + C*
a) + 144*A*D**2*a**3*b + 6144*A*b**2*(-3*A*b + C*a)**2 - 1200*B**2*C*a**2*b
**2 + 1200*B**2*a*b**2*(-3*A*b + C*a) + 480*B*C*D*a**3*b - 480*B*D*a**2*b*(
-3*A*b + C*a) + 1024*C**3*a**3*b + 1024*C**2*a**2*b*(-3*A*b + C*a) - 48*C*D
**2*a**4 - 2048*C*a*b*(-3*A*b + C*a)**2 + 48*D**2*a**3*(-3*A*b + C*a))/(-25
920*A**2*B*b**4 + 5184*A**2*D*a*b**3 + 17280*A*B*C*a*b**3 - 3456*A*C*D*a**2
*b**2 - 1125*B**3*a*b**3 + 675*B**2*D*a**2*b**2 - 2880*B*C**2*a**2*b**2 - 1
35*B*D**2*a**3*b + 576*C**2*D*a**3*b + 9*D**3*a**4))/a**4
```

Giac [A] time = 1.19892, size = 219, normalized size = 1.26

$$\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(|x|\right)}{a^4}+\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-12\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*a - 3*A*b)
*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(abs(x))/a^4 + 1/8*(3*D*a*b*x^5 - 15
*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a^2*x^3 - 25*B*a*b*x^3 + 6*C*
a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x - 4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

3.110 $\int \frac{-x+4x^3}{(5+x^2)^2}\,dx$

Optimal. Leaf size=20

$$\frac{21}{2(x^2+5)}+2\log(x^2+5)$$

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rubi [A] time = 0.0230683, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 444, 43}

$$\frac{21}{2(x^2+5)}+2\log(x^2+5)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{-x+4x^3}{\left(5+x^2\right)^2}\,dx &= \int \frac{x\left(-1+4x^2\right)}{\left(5+x^2\right)^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+4x}{(5+x)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{21}{(5+x)^2}+\frac{4}{5+x}\right)dx, x, x^2\right)\\
&= \frac{21}{2\left(5+x^2\right)}+2\log\left(5+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0071059, size = 20, normalized size = 1.

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Maple [A] time = 0.007, size = 19, normalized size = 1.

$$\frac{21}{2\,x^2+10}+2\,\ln\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-x)/(x^2+5)^2,x)
```

```
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

Maxima [A] time = 0.987198, size = 24, normalized size = 1.2

$$\frac{21}{2\left(x^2+5\right)}+2\,\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="maxima")
```

```
[Out] 21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

Fricas [A] time = 1.39457, size = 63, normalized size = 3.15

$$\frac{4\left(x^2+5\right)\log\left(x^2+5\right)+21}{2\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

Sympy [A] time = 0.092506, size = 15, normalized size = 0.75

$$2\log\left(x^2+5\right)+\frac{21}{2x^2+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-x)/(x**2+5)**2,x)
```

```
[Out] 2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

Giac [A] time = 1.15171, size = 34, normalized size = 1.7

$$-\frac{4\,x^2-1}{2\left(x^2+5\right)}+2\,\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*x^2 - 1)/(x^2 + 5) + 2*log(x^2 + 5)
```

3.111 $\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0211766, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 444, 43}

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx &= \int \frac{x\left(-1+x^2\right)}{\sqrt{-2+x^2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{\sqrt{-2+x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{-2+x}}+\sqrt{-2+x}\right)dx,x,x^2\right)\\ &= \sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0081222, size = 18, normalized size = 0.78

$$\frac{1}{3}\sqrt{x^2-2}\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] (Sqrt[-2 + x^2]*(1 + x^2))/3
```

Maple [A] time = 0.005, size = 15, normalized size = 0.7

$$\frac{x^2+1}{3}\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(x^2-2)^(1/2),x)
```

```
[Out] 1/3*(x^2+1)*(x^2-2)^(1/2)
```

Maxima [A] time = 0.970398, size = 30, normalized size = 1.3

$$\frac{1}{3}\sqrt{x^2-2}x^2+\frac{1}{3}\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

Fricas [A] time = 1.42835, size = 39, normalized size = 1.7

$$\frac{1}{3}\left(x^2+1\right)\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + 1)*sqrt(x^2 - 2)
```

Sympy [A] time = 0.343196, size = 22, normalized size = 0.96

$$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(x**2-2)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

Giac [A] time = 1.14376, size = 23, normalized size = 1.

$$\frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}}+\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

3.112 $\int \frac{-x^2+2x^4}{1+2x^2} dx$

Optimal. Leaf size=25

$$\frac{x^3}{3} - x + \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0241169, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1593, 459, 321, 203}

$$\frac{x^3}{3} - x + \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{-x^2+2x^4}{1+2x^2}\,dx &= \int \frac{x^2\left(-1+2x^2\right)}{1+2x^2}\,dx \\
&= \frac{x^3}{3}-2\int \frac{x^2}{1+2x^2}\,dx \\
&= -x+\frac{x^3}{3}+\int \frac{1}{1+2x^2}\,dx \\
&= -x+\frac{x^3}{3}+\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0101867, size = 25, normalized size = 1.

$$\frac{x^3}{3}-x+\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$-x+\frac{x^3}{3}+\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-x^2)/(2*x^2+1),x)
```

```
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.52928, size = 27, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Fricas [A] time = 1.40986, size = 61, normalized size = 2.44

$$\frac{1}{3}x^3 + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Sympy [A] time = 0.087976, size = 20, normalized size = 0.8

$$\frac{x^3}{3} - x + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-x**2)/(2*x**2+1),x)
```

```
[Out] x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

<u>Giac</u> [A] time = 1.17975, size = 27, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

3.113 $\int \frac{x^3+x^4}{1+x^2} \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0252424, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 801, 635, 203, 260}

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3+x^4}{1+x^2}\,dx &= \int \frac{x^3(1+x)}{1+x^2}\,dx\\
&= \int \left(-1+x+x^2+\frac{1-x}{1+x^2}\right)dx\\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1-x}{1+x^2}\,dx\\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0053304, size = 30, normalized size = 1.

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.002, size = 25, normalized size = 0.8

$$-x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3)/(x^2+1),x)
```

```
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.52162, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.45196, size = 73, normalized size = 2.43

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.089859, size = 22, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} - x - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3)/(x**2+1),x)
```

```
[Out] x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

Giac [A] time = 1.13593, size = 32, normalized size = 1.07

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

3.114 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=210

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4}-\frac{ax^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^6}-\frac{a^{5/2}\tan^{-1}\left(\right.}{}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (
f*x^11)/(11*b) - (a^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(13/2)
```

Rubi [A] time = 0.161395, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4}-\frac{ax^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^6}-\frac{a^{5/2}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (
f*x^11)/(11*b) - (a^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(13/2)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx = \int \left(\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{b^5} + \frac{\left(b^3c-ab^2d+a^2be\right.}{b^4}\right.$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^5} + \frac{\left(b^3c-ab^2d+a^2be-\right.}{5b^4}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^5} + \frac{\left(b^3c-ab^2d+a^2be-\right.}{5b^4}$$

Mathematica [A] time = 0.14573, size = 210, normalized size = 1.

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4} + \frac{ax^3\left(-a^2be+a^3f+ab^2d-b^3c\right)}{3b^5} - \frac{a^2x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{b^6} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9
*b^2) + (f*x^11)/(11*b) + (a^(5/2)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

Maple [A] time = 0.006, size = 278, normalized size = 1.3

$$\frac{fx^{11}}{11\,b} - \frac{x^9af}{9\,b^2} + \frac{x^9e}{9\,b} + \frac{x^7a^2f}{7\,b^3} - \frac{x^7ae}{7\,b^2} + \frac{x^7d}{7\,b} - \frac{x^5a^3f}{5\,b^4} + \frac{x^5a^2e}{5\,b^3} - \frac{x^5ad}{5\,b^2} + \frac{x^5c}{5\,b} + \frac{x^3a^4f}{3\,b^5} - \frac{x^3a^3e}{3\,b^4} + \frac{x^3a^2d}{3\,b^3} - \frac{ax^3c}{3\,b^2} - \frac{a^5fx}{b^6} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/11*f*x^11/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/7/b^3*x^7*a^2*f-1/7/b^2*x^7*a*e
+1/7/b*x^7*d-1/5/b^4*x^5*a^3*f+1/5/b^3*x^5*a^2*e-1/5/b^2*x^5*a*d+1/5/b*x^5*
c+1/3/b^5*x^3*a^4*f-1/3/b^4*x^3*a^3*e+1/3/b^3*x^3*a^2*d-1/3/b^2*x^3*a*c-1/b
^6*a^5*f*x+1/b^5*a^4*e*x-1/b^4*a^3*d*x+1/b^3*a^2*c*x+a^6/b^6/(a*b)^(1/2)*ar
```

```
ctan(b*x/(a*b)^(1/2))*f-a^5/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+a^4/b
^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51908, size = 953, normalized size = 4.54

$$\left[\frac{630\,b^5 f x^{11} + 770\left(b^5 e - ab^4 f\right)x^9 + 990\left(b^5 d - ab^4 e + a^2b^3 f\right)x^7 + 1386\left(b^5 c - ab^4 d + a^2b^3 e - a^3b^2 f\right)x^5 - 2310\left(ab^4 c - \right.}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6930*(630*b^5*f*x^11 + 770*(b^5*e - a*b^4*f)*x^9 + 990*(b^5*d - a*b^4*e
+ a^2*b^3*f)*x^7 + 1386*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 231
0*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b
^2*d + a^4*b*e - a^5*f)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) + 6930*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6, 1/3465*(31
5*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a*b^4*e + a^2*b^3*f
)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 1155*(a*b^4*c -
 a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b^2*d + a^4*b
*e - a^5*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3465*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*x)/b^6]
```

Sympy [A] time = 0.900026, size = 366, normalized size = 1.74

$$-\frac{\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}+\frac{\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\left(\ldots\right.}{a^5f-a^4\ldots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**6*sqrt(-
a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*f - a**4*b*e + a*
*3*b**2*d - a**2*b**3*c) + x)/2 + sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*
b**2*d - b**3*c)*log(b**6*sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + f*x**11/(
11*b) - x**9*(a*f - b*e)/(9*b**2) + x**7*(a**2*f - a*b*e + b**2*d)/(7*b**3)
 - x**5*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(5*b**4) + x**3*(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c)/(3*b**5) - x*(a**5*f - a**4*b*e + a**3*b*
*2*d - a**2*b**3*c)/b**6
```

Giac [A] time = 1.19992, size = 338, normalized size = 1.61

$$-\frac{\left(a^3b^3c-a^4b^2d-a^6f+a^5be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6}+\frac{315\,b^{10}fx^{11}-385\,ab^9fx^9+385\,b^{10}x^9e+495\,b^{10}dx^7+495\,a^2b^8fx^7-495\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^10*f*x^11 - 385*a*b^9*f*x^9 + 385*b^10*x^9*e + 495*b^
10*d*x^7 + 495*a^2*b^8*f*x^7 - 495*a*b^9*x^7*e + 693*b^10*c*x^5 - 693*a*b^9
*d*x^5 - 693*a^3*b^7*f*x^5 + 693*a^2*b^8*x^5*e - 1155*a*b^9*c*x^3 + 1155*a^
2*b^8*d*x^3 + 1155*a^4*b^6*f*x^3 - 1155*a^3*b^7*x^3*e + 3465*a^2*b^8*c*x -
3465*a^3*b^7*d*x - 3465*a^5*b^5*f*x + 3465*a^4*b^6*x*e)/b^11
```

3.115 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=172

$$\frac{x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}-\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{11/2}}+$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e -
 a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rubi [A] time = 0.122713, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}-\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{11/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```


```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e -
 a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```


Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx = \int\left(-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x}{b^3}\right.$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}$$

Mathematica [A] time = 0.113696, size = 162, normalized size = 0.94

$$\frac{x\left(21a^2b^2\left(15d+5ex^2+3fx^4\right)-105a^3b\left(3e+fx^2\right)+315a^4f-3ab^3\left(105c+35dx^2+21ex^4+15fx^6\right)+b^4x^2\left(105c+63d\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(315*a^4*f - 105*a^3*b*(3*e + f*x^2) + 21*a^2*b^2*(15*d + 5*e*x^2 + 3*f*
x^4) - 3*a*b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6) + b^4*x^2*(105*c +
63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5) - (a^(3/2)*(-(b^3*c) + a*b^2*d
- a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Maple [A] time = 0.004, size = 230, normalized size = 1.3

$$\frac{fx^9}{9\,b}-\frac{x^7af}{7\,b^2}+\frac{x^7e}{7\,b}+\frac{x^5a^2f}{5\,b^3}-\frac{x^5ae}{5\,b^2}+\frac{x^5d}{5\,b}-\frac{x^3a^3f}{3\,b^4}+\frac{x^3a^2e}{3\,b^3}-\frac{ax^3d}{3\,b^2}+\frac{x^3c}{3\,b}+\frac{a^4fx}{b^5}-\frac{a^3ex}{b^4}+\frac{a^2dx}{b^3}-\frac{acx}{b^2}-\frac{a^5f}{b^5}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/9*f*x^9/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/5/b^3*x^5*a^2*f-1/5/b^2*x^5*a*e+1
/5/b*x^5*d-1/3/b^4*x^3*a^3*f+1/3/b^3*x^3*a^2*e-1/3/b^2*x^3*a*d+1/3/b*x^3*c+
1/b^5*a^4*f*x-1/b^4*a^3*e*x+1/b^3*a^2*d*x-1/b^2*a*c*x-a^5/b^5/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))*f+a^4/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-a^3/
b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50748, size = 770, normalized size = 4.48

$$\left[\frac{70\,b^4 f x^9 + 90\left(b^4 e - ab^3 f\right)x^7 + 126\left(b^4 d - ab^3 e + a^2 b^2 f\right)x^5 + 210\left(b^4 c - ab^3 d + a^2 b^2 e - a^3 b f\right)x^3 - 315\left(ab^3 c - a^2 b^2 d + \cdots\right.}{630\,b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*f*x^9 + 90*(b^4*e - a*b^3*f)*x^7 + 126*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^5 + 210*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b)
- a)/(b*x^2 + a)) - 630*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5, 1/3
15*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 + 315*(a*b^3*c -
a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5]
```

Sympy [A] time = 0.867167, size = 325, normalized size = 1.89

$$\frac{\sqrt{-\frac{a^3}{b^{11}}}\left(a^3 f - a^2 be + ab^2 d - b^3 c\right)\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\left(a^3 f - a^2 be + ab^2 d - b^3 c\right)}{a^4 f - a^3 be + a^2 b^2 d - ab^3 c} + x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^{11}}}\left(a^3 f - a^2 be + ab^2 d - b^3 c\right)\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\left(\cdots\right.}{a^4 f - \cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**5*sqrt(-a
**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a**3*b*e + a**
2*b**2*d - a*b**3*c) + x)/2 - sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2
*d - b**3*c)*log(b**5*sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)/(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c) + x)/2 + f*x**9/(9*b) - x
**7*(a*f - b*e)/(7*b**2) + x**5*(a**2*f - a*b*e + b**2*d)/(5*b**3) - x**3*(
a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**4) + x*(a**4*f - a**3*b*e + a*
*2*b**2*d - a*b**3*c)/b**5
```

Giac [A] time = 1.17473, size = 270, normalized size = 1.57

$$\frac{\left(a^2b^3c - a^3b^2d - a^5f + a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^8fx^9 - 45\,ab^7fx^7 + 45\,b^8x^7e + 63\,b^8dx^5 + 63\,a^2b^6fx^5 - 63\,ab^7x^5e + 10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^8*f*x^9 - 45*a*b^7*f*x^7 + 45*b^8*x^7*e + 63*b^8*d*x^5 +
 63*a^2*b^6*f*x^5 - 63*a*b^7*x^5*e + 105*b^8*c*x^3 - 105*a*b^7*d*x^3 - 105*
a^3*b^5*f*x^3 + 105*a^2*b^6*x^3*e - 315*a*b^7*c*x + 315*a^2*b^6*d*x + 315*a
^4*b^4*f*x - 315*a^3*b^5*x*e)/b^9
```

3.116 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=136

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{x^5(be-af)}{5b^2}+$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^3)
/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) - (Sqrt[a]*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi [A] time = 0.105374, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{x^5(be-af)}{5b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^3)
/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) - (Sqrt[a]*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx = \int\left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^2}{b^3}+\frac{(be-af)x^4}{b^2}+\frac{fx^6}{b}+\frac{-ab^3c+a^2\cdots}{b^4\cdots}\right.$$

$$=\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{\left(a\left(b^3c-ab\cdots\right.\right.}{\cdots}$$

$$=\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{\sqrt{a}\left(b^3c-a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.0885713, size = 128, normalized size = 0.94

$$\frac{x\left(35a^2b\left(3e+fx^2\right)-105a^3f-7ab^2\left(15d+5ex^2+3fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2\cdots\right.}{b\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(-105*a^3*f + 35*a^2*b*(3*e + f*x^2) - 7*a*b^2*(15*d + 5*e*x^2 + 3*f*x^4
) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4) + (Sqrt[a]*(-(
b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A] time = 0.004, size = 182, normalized size = 1.3

$$\frac{fx^7}{7\,b}-\frac{x^5af}{5\,b^2}+\frac{x^5e}{5\,b}+\frac{x^3a^2f}{3\,b^3}-\frac{ax^3e}{3\,b^2}+\frac{x^3d}{3\,b}-\frac{a^3fx}{b^4}+\frac{a^2ex}{b^3}-\frac{adx}{b^2}+\frac{cx}{b}+\frac{a^4f}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{a^3e}{b^3}\arctan\left(bx\frac{1}{\sqrt{a\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/7*f*x^7/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/3/b^3*x^3*a^2*f-1/3/b^2*x^3*a*e+1
/3/b*x^3*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+1/b*c*x+a^4/b^4/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*f-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e
+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-a/b/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56202, size = 606, normalized size = 4.46

$$\left[\frac{30\,b^3fx^7+42\left(b^3e-ab^2f\right)x^5+70\left(b^3d-ab^2e+a^2bf\right)x^3-105\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*f*x^7 + 42*(b^3*e - a*b^2*f)*x^5 + 70*(b^3*d - a*b^2*e + a^2
*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a/b)*log((b*x^2 +
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x)/b^4, 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b
^2*e + a^2*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a/b)*arc
tan(b*x*sqrt(a/b)/a) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4]
```

Sympy [A] time = 0.813254, size = 180, normalized size = 1.32

$$-\frac{\sqrt{-\frac{a}{b^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2}+\frac{fx^7}{7b}-\frac{x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -sqrt(-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**4*sqrt(-a/b*
*9) + x)/2 + sqrt(-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**4
*sqrt(-a/b**9) + x)/2 + f*x**7/(7*b) - x**5*(a*f - b*e)/(5*b**2) + x**3*(a*
*2*f - a*b*e + b**2*d)/(3*b**3) - x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
/b**4
```

Giac [A] time = 1.16904, size = 205, normalized size = 1.51

$$-\frac{\left(ab^3c - a^2b^2d - a^4f + a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6fx^7 - 21\,ab^5fx^5 + 21\,b^6x^5e + 35\,b^6dx^3 + 35\,a^2b^4fx^3 - 35\,ab^5x^3e + 10}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^6*f*x^7 - 21*a*b^5*f*x^5 + 21*b^6*x^5*e + 35*b^6*d*x^3 +
35*a^2*b^4*f*x^3 - 35*a*b^5*x^3*e + 105*b^6*c*x - 105*a*b^5*d*x - 105*a^3*b
^3*f*x + 105*a^2*b^4*x*e)/b^7
```

3.117 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} dx$

Optimal. Leaf size=100

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{a}b^{7/2}}+\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*x^5)/(5*b)
 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```

Rubi [A] time = 0.0615987, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1810, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{a}b^{7/2}}+\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```


```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*x^5)/(5*b)
 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```


Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \int \left(\frac{b^2d-abe+a^2f}{b^3} + \frac{(be-af)x^2}{b^2} + \frac{fx^4}{b} + \frac{b^3c-ab^2d+a^2be-a^3f}{b^3\left(a+bx^2\right)}\right) dx$$
$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3} + \frac{(be-af)x^3}{3b^2} + \frac{fx^5}{5b} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^2}\,dx}{b^3}$$
$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3} + \frac{(be-af)x^3}{3b^2} + \frac{fx^5}{5b} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

Mathematica [A] time = 0.0799274, size = 98, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{a}b^{7/2}} + \frac{x\left(15a^2f-5ab\left(3e+fx^2\right)+b^2\left(15d+5ex^2+3fx^4\right)\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] (x*(15*a^2*f - 5*a*b*(3*e + f*x^2) + b^2*(15*d + 5*e*x^2 + 3*f*x^4)))/(15*b
^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sq
rt[a]*b^(7/2))
```

Maple [A] time = 0.002, size = 135, normalized size = 1.4

$$\frac{fx^5}{5\,b} - \frac{ax^3f}{3\,b^2} + \frac{x^3e}{3\,b} + \frac{a^2fx}{b^3} - \frac{aex}{b^2} + \frac{dx}{b} - \frac{a^3f}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{a^2e}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{ad}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/5*f*x^5/b-1/3/b^2*x^3*a*f+1/3/b*x^3*e+1/b^3*a^2*f*x-1/b^2*a*e*x+1/b*d*x-1
/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^3*f+1/b^2/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))*a^2*e-1/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*d+1/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50391, size = 505, normalized size = 5.05

$$\left[\frac{6\,ab^3fx^5+10\left(ab^3e-a^2b^2f\right)x^3+15\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+30\left(ab^3d-a^2b^2e+a^3bf\right)x}{30\,ab^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*f*x^5 + 10*(a*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a))
+ 30*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4), 1/15*(3*a*b^3*f*x^5 + 5*(a
*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a) + 15*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4)]
```

Sympy [A] time = 0.926852, size = 158, normalized size = 1.58

$$\frac{\sqrt{-\frac{1}{ab^7}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^7}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}+\frac{fx^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*b**3*sqrt(
-1/(a*b**7)) + x)/2 - sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)*log(a*b**3*sqrt(-1/(a*b**7)) + x)/2 + f*x**5/(5*b) - x**3*(a*f - b*e)/
```

```
(3*b**2) + x*(a**2*f - a*b*e + b**2*d)/b**3
```

Giac [A] time = 1.14538, size = 143, normalized size = 1.43

$$\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}+\frac{3\,b^4fx^5-5\,ab^3fx^3+5\,b^4x^3e+15\,b^4dx+15\,a^2b^2fx-15\,ab^3xe}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^4*f*x^5 - 5*a*b^3*f*x^3 + 5*b^4*x^3*e + 15*b^4*d*x + 15*a^2*b^2*
f*x - 15*a*b^3*x*e)/b^5
```

3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.0940384, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx = \int \left(\frac{be-af}{b^2}+\frac{c}{ax^2}+\frac{fx^2}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^2\right)}\right)dx$$
$$= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{ab^2}$$
$$= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.0633793, size = 83, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) + ((-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Maple [A] time = 0.005, size = 114, normalized size = 1.4

$$\frac{fx^3}{3\,b}-\frac{afx}{b^2}+\frac{ex}{b}-\frac{c}{ax}+\frac{a^2f}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{ae}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+d\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{bc}{a}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x)
```

```
[Out] 1/3*f*x^3/b-1/b^2*a*f*x+1/b*x*e-c/a/x+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2))*f-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+1/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))*d-1/a*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5066, size = 444, normalized size = 5.29

$$\left[\frac{2\,a^2b^2fx^4 - 6\,ab^3c + 3\left(b^3c - ab^2d + a^2be - a^3f\right)\sqrt{-ab}x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right) + 6\left(a^2b^2e - a^3bf\right)x^2}{6\,a^2b^3x}, \frac{a^2b^2fx^4 - 3\,ab^3c - 3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*f*x^4 - 6*a*b^3*c + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*s
qrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a^2*b^2*e -
a^3*b*f)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*f*x^4 - 3*a*b^3*c - 3*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(a^2*b^2*e - a
^3*b*f)*x^2)/(a^2*b^3*x)]
```

Sympy [B] time = 1.30436, size = 150, normalized size = 1.79

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**2*b**
2*sqrt(-1/(a**3*b**5)) + x)/2 + sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a
*b**2*d - b**3*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/2 + f*x**3/(3*b)
- x*(a*f - b*e)/b**2 - c/(a*x)
```

Giac [A] time = 1.1902, size = 116, normalized size = 1.38

$$-\frac{c}{ax}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^{2}}+\frac{b^{2}fx^{3}-3\,abfx+3\,b^{2}xe}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/(a*x) - (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a*b^2) + 1/3*(b^2*f*x^3 - 3*a*b*f*x + 3*b^2*x*e)/b^3
```

3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}+\frac{fx}{b}$$

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.087426, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)}\,dx = \int \left(\frac{f}{b}+\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b\left(a+bx^2\right)}\right)dx$$
$$= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^2}\,dx}{a^2b}$$
$$= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

Mathematica [A] time = 0.0798082, size = 83, normalized size = 1.01

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{5/2}b^{3/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b - ((-(b^3*c) + a*b^2*d - a^2*b
*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Maple [A] time = 0.007, size = 115, normalized size = 1.4

$$\frac{fx}{b}-\frac{c}{3\,ax^3}-\frac{d}{ax}+\frac{bc}{a^2x}-\frac{af}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+e\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{bd}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^2c}{a^2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x)
```

```
[Out] f*x/b-1/3*c/a/x^3-1/a/x*d+1/a^2/x*b*c-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*f+1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-1/a*b/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))*d+1/a^2*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56508, size = 455, normalized size = 5.55

$$\left[\frac{6\,a^3bfx^4+3\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}x^3\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,a^2b^2c+6\left(ab^3c-a^2b^2d\right)x^2}{6\,a^3b^2x^3},\frac{3\,a^3bfx^4+3\left(b^3c-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^3*
log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*a^2*b^2*c + 6*(a*b^3*c -
a^2*b^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) - a^2*b^2*c + 3*(a*b^3*c
 - a^2*b^2*d)*x^2)/(a^3*b^2*x^3)]
```

Sympy [B] time = 2.42794, size = 151, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{a^5b^3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^5b^3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2}+\frac{f}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**3*b*sq
rt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**
2*d - b**3*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 + f*x/b - (a*c + x**2*
(3*a*d - 3*b*c))/(3*a**2*x**3)
```

Giac [A] time = 1.16674, size = 109, normalized size = 1.33

$$\frac{fx}{b} + \frac{\left(b^{3}c - ab^{2}d - a^{3}f + a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^{2}b} + \frac{3\,bcx^{2} - 3\,adx^{2} - ac}{3\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

3.120 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)} dx$

Optimal. Leaf size=104

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{bc-ad}{3a^2x^3}-\frac{c}{5ax^5}$$

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/(a^3*x) -
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*
Sqrt[b])
```

Rubi [A] time = 0.102393, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{bc-ad}{3a^2x^3}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/(a^3*x) -
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*
Sqrt[b])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^2\right)}\right)dx$$
$$= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int\frac{1}{a+bx^2}\,dx}{a^3}$$
$$= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.0826154, size = 103, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{7/2}\sqrt{b}}+\frac{a^2(-e)+abd-b^2c}{a^3x}+\frac{bc-ad}{3a^2x^3}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) + (-(b^2*c) + a*b*d - a^2*e)/(a^3*x)
 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^
(7/2)*Sqrt[b])
```

Maple [A] time = 0.007, size = 142, normalized size = 1.4

$$-\frac{c}{5\,ax^5}-\frac{d}{3\,ax^3}+\frac{bc}{3\,x^3a^2}-\frac{e}{ax}+\frac{bd}{a^2x}-\frac{b^2c}{a^3x}+f\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{be}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^2d}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x)
```

```
[Out] -1/5*c/a/x^5-1/3/a/x^3*d+1/3/a^2/x^3*b*c-1/a/x*e+1/a^2/x*b*d-1/a^3/x*b^2*c+
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f-1/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2))*b*e+1/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*b^2*d-1/a^3/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))*b^3*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50064, size = 522, normalized size = 5.02

$$\left[\frac{15\left(b^3c - ab^2d + a^2be - a^3f\right)\sqrt{-ab}x^5\log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right) - 6a^3bc - 30\left(ab^3c - a^2b^2d + a^3be\right)x^4 + 10\left(a^2b^2c - a^3bd\right)x^2}{30a^4bx^5}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^5*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*a^3*b*c - 30*(a*b^3*c - a^2*b^2*d + a^3
*b*e)*x^4 + 10*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4*b*x^5), -1/15*(15*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x^5*arctan(sqrt(a*b)*x/a) + 3*a^3*b*c +
 15*(a*b^3*c - a^2*b^2*d + a^3*b*e)*x^4 - 5*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4
*b*x^5)]
```

Sympy [A] time = 5.3041, size = 167, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^7b}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(-a^4\sqrt{-\frac{1}{a^7b}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^7b}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(a^4\sqrt{-\frac{1}{a^7b}} + x\right)}{2} - \frac{3a^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**4*sqrt(-
1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3
```

```
*c)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 - (3*a**2*c + x**4*(15*a**2*e - 15*a*
b*d + 15*b**2*c) + x**2*(5*a**2*d - 5*a*b*c))/(15*a**3*x**5)
```

Giac [A] time = 1.15126, size = 142, normalized size = 1.37

$$-\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\,b^2cx^4 - 15\,abdx^4 + 15\,a^2x^4e - 5\,abcx^2 + 5\,a^2dx^2 + 3\,a^2c}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*b^2*c*x^4 - 15*a*b*d*x^4 + 15*a^2*x^4*e - 5*a*b*c*x^2 + 5*a^2*d*
x^2 + 3*a^2*c)/(a^3*x^5)
```

3.121 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx$

Optimal. Leaf size=137

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{9/2}}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{bc-ad}{5a^2x^5}-\frac{c}{7ax^7}$$

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt[b]*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rubi [A] time = 0.130352, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{9/2}}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{bc-ad}{5a^2x^5}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt[b]*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^8}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^4}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}-\frac{b\left(-b^3c+ab^2d-a^2be\right.}{a^4\left(a+bx^2\right)}\right.$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{\left(b\left(b^3c-ab^2d+a^2be-a^3\right.\right.}{a^4}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{\sqrt{b}\left(b^3c-ab^2d+a^2be-a\right.}{a^{9/2}}$$

Mathematica [A] time = 0.115209, size = 139, normalized size = 1.01

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{9/2}}+\frac{a^2(-e)+abd-b^2c}{3a^3x^3}+\frac{bc-ad}{5a^2x^5}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^3*
x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (Sqrt[b]*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Maple [A] time = 0.009, size = 190, normalized size = 1.4

$$-\frac{c}{7\,ax^7}-\frac{d}{5\,ax^5}+\frac{bc}{5\,x^5a^2}-\frac{e}{3\,ax^3}+\frac{bd}{3\,x^3a^2}-\frac{b^2c}{3\,a^3x^3}-\frac{f}{ax}+\frac{be}{a^2x}-\frac{b^2d}{a^3x}+\frac{b^3c}{a^4x}-\frac{bf}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^2e}{a^2}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x)
```

```
[Out] -1/7*c/a/x^7-1/5/a/x^5*d+1/5/a^2/x^5*b*c-1/3/a/x^3*e+1/3/a^2/x^3*b*d-1/3/a^
3/x^3*b^2*c-1/a/x*f+1/a^2/x*b*e-1/a^3/x*b^2*d+1/a^4/x*b^3*c-b/a/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*f+b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-b^
3/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+b^4/a^4/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51032, size = 624, normalized size = 4.55

$$\left[-\frac{105\left(b^3c-ab^2d+a^2be-a^3f\right)x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-210\left(b^3c-ab^2d+a^2be-a^3f\right)x^6+70\left(ab^2c-a^2bd+a^3e\right)\cdots}{210\,a^4x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(-b/a)*log((b*x^2
- 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 210*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x^6 + 70*(a*b^2*c - a^2*b*d + a^3*e)*x^4 + 30*a^3*c - 42*(a^2*b*c - a^3
*d)*x^2)/(a^4*x^7), 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt
(b/a)*arctan(x*sqrt(b/a)) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 3
5*(a*b^2*c - a^2*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a
^4*x^7)]
```

Sympy [B] time = 13.2633, size = 301, normalized size = 2.2

$$\frac{\sqrt{-\frac{b}{a^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}\left(a^3f\cdots\right.}{a^3bf-a^2b\cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**5*sqrt(-b/a**
9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f - a**2*b**2*e + a*b**3
*d - b**4*c) + x)/2 - sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a**5*sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f -
 a**2*b**2*e + a*b**3*d - b**4*c) + x)/2 - (15*a**3*c + x**6*(105*a**3*f -
105*a**2*b*e + 105*a*b**2*d - 105*b**3*c) + x**4*(35*a**3*e - 35*a**2*b*d +
 35*a*b**2*c) + x**2*(21*a**3*d - 21*a**2*b*c))/(105*a**4*x**7)
```

Giac [A] time = 1.18216, size = 204, normalized size = 1.49

$$\frac{\left(b^4c - ab^3d - a^3bf + a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105\,b^3cx^6 - 105\,ab^2dx^6 - 105\,a^3fx^6 + 105\,a^2bx^6e - 35\,ab^2cx^4 + 35\,a^2bdx^4 -}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*b^3*c*x^6 - 105*a*b^2*d*x^6 - 105*a^3*f*x^6 + 105*a^2*b*x^6
*e - 35*a*b^2*c*x^4 + 35*a^2*b*d*x^4 - 35*a^3*x^4*e + 21*a^2*b*c*x^2 - 21*a
^3*d*x^2 - 15*a^3*c)/(a^4*x^7)
```

3.122 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=175

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5x}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{11/2}}-\frac{a^2e-}{5}$$

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x^5
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (b*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Rubi [A] time = 0.145924, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5x}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{11/2}}-\frac{a^2e-}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x^5
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (b*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx=\int\left(\frac{c}{ax^{10}}+\frac{-bc+ad}{a^2x^8}+\frac{b^2c-abd+a^2e}{a^3x^6}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^4}-\frac{b\left(-b^3c+ab^2d-a^2b\right.}{a^5x^2}\right.$$

$$=-\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}-\frac{b\left(b^3c-ab^2d+a^2be-a^3f\right.}{a^5x}$$

$$=-\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}-\frac{b\left(b^3c-ab^2d+a^2be-a^3f\right.}{a^5x}$$

Mathematica [A] time = 0.132944, size = 174, normalized size = 0.99

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}+\frac{b\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^5x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{11/2}}+\frac{a^2(-e)+a}{5a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) + (-(b^2*c) + a*b*d - a^2*e)/(5*a^3*
x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(3/2)*(-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Maple [A] time = 0.008, size = 238, normalized size = 1.4

$$-\frac{c}{9\,ax^9}-\frac{d}{7\,ax^7}+\frac{bc}{7\,a^2x^7}-\frac{e}{5\,ax^5}+\frac{bd}{5\,x^5a^2}-\frac{b^2c}{5\,a^3x^5}-\frac{f}{3\,ax^3}+\frac{be}{3\,x^3a^2}-\frac{b^2d}{3\,a^3x^3}+\frac{b^3c}{3\,a^4x^3}+\frac{bf}{a^2x}-\frac{b^2e}{a^3x}+\frac{b^3d}{a^4x}-\frac{b^4c}{a^5x}+\frac{b^2f}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x)
```

```
[Out] -1/9*c/a/x^9-1/7/a/x^7*d+1/7/a^2/x^7*b*c-1/5/a/x^5*e+1/5/a^2/x^5*b*d-1/5/a^
3/x^5*b^2*c-1/3/a/x^3*f+1/3/a^2/x^3*b*e-1/3/a^3/x^3*b^2*d+1/3/a^4/x^3*b^3*c
+1/a^2*b/x*f-1/a^3*b^2/x*e+1/a^4*b^3/x*d-1/a^5*b^4/x*c+b^2/a^2/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))*f-b^3/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+b^4
/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-b^5/a^5/(a*b)^(1/2)*arctan(b*x/(
```

```
a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50432, size = 788, normalized size = 4.5

$$\left[-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+630\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-210\left(ab^3c-a^2b^2d\right.}{630\,a^5x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/630*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 630*(b^4*c - a*b^3*d + a^2*b^2*e
 - a^3*b*f)*x^8 - 210*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 70*a^4*
c + 126*(a^2*b^2*c - a^3*b*d + a^4*e)*x^4 - 90*(a^3*b*c - a^4*d)*x^2)/(a^5*
x^9), -1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(b/a)*arc
tan(x*sqrt(b/a)) + 315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*
b*d + a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)]
```

Sympy [B] time = 27.4595, size = 354, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{a^3b^2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**6*sqrt(-
b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**2*f - a**2*b**
3*e + a*b**4*d - b**5*c) + x)/2 + sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*
b**2*d - b**3*c)*log(a**6*sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(a**3*b**2*f - a**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + (-35*a**4
*c + x**8*(315*a**3*b*f - 315*a**2*b**2*e + 315*a*b**3*d - 315*b**4*c) + x*
*6*(-105*a**4*f + 105*a**3*b*e - 105*a**2*b**2*d + 105*a*b**3*c) + x**4*(-6
3*a**4*e + 63*a**3*b*d - 63*a**2*b**2*c) + x**2*(-45*a**4*d + 45*a**3*b*c))
/(315*a**5*x**9)
```

Giac [A] time = 1.18385, size = 271, normalized size = 1.55

$$-\frac{\left(b^5c - ab^4d - a^3b^2f + a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{315\,b^4cx^8 - 315\,ab^3dx^8 - 315\,a^3bfx^8 + 315\,a^2b^2x^8e - 105\,ab^3cx^6 + 105\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*b^4*c*x^8 - 315*a*b^3*d*x^8 - 315*a^3*b*f*x^8 + 315*a^2*
b^2*x^8*e - 105*a*b^3*c*x^6 + 105*a^2*b^2*d*x^6 + 105*a^4*f*x^6 - 105*a^3*b
*x^6*e + 63*a^2*b^2*c*x^4 - 63*a^3*b*d*x^4 + 63*a^4*x^4*e - 45*a^3*b*c*x^2
+ 45*a^4*d*x^2 + 35*a^4*c)/(a^5*x^9)
```

3.123 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx$

Optimal. Leaf size=211

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5}+\frac{b^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^6x}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\right.}{}$$

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3*x
^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/a^(13/2)
```

Rubi [A] time = 0.175188, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5}+\frac{b^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^6x}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3*x
^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/a^(13/2)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}dx = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^{10}} + \frac{b^2c-abd+a^2e}{a^3x^8} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} - \frac{b\left(-b^3c+ab^2d-a^2b\right.}{a^5x^4}\right.$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{3a^5x^3}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{3a^5x^3}$$

Mathematica [A] time = 0.160742, size = 211, normalized size = 1.

$$\frac{b\left(-a^2be+a^3f+ab^2d-b^3c\right)}{3a^5x^3} + \frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5} + \frac{b^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^6x} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be\right.}{a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3*x
^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/a^(13/2)
```

Maple [A] time = 0.011, size = 286, normalized size = 1.4

$$-\frac{c}{11\,ax^{11}} - \frac{d}{9\,ax^9} + \frac{bc}{9\,a^2x^9} - \frac{e}{7\,ax^7} + \frac{bd}{7\,a^2x^7} - \frac{b^2c}{7\,a^3x^7} - \frac{f}{5\,ax^5} + \frac{be}{5\,x^5a^2} - \frac{b^2d}{5\,a^3x^5} + \frac{b^3c}{5\,a^4x^5} - \frac{b^2f}{a^3x} + \frac{b^3e}{a^4x} - \frac{b^4d}{a^5x} + \frac{b^5c}{a^6x} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x)
```

```
[Out] -1/11*c/a/x^11-1/9/a/x^9*d+1/9/a^2/x^9*b*c-1/7/a/x^7*e+1/7/a^2/x^7*b*d-1/7/
a^3/x^7*b^2*c-1/5/a/x^5*f+1/5/a^2/x^5*b*e-1/5/a^3/x^5*b^2*d+1/5/a^4/x^5*b^3
*c-1/a^3*b^2/x*f+1/a^4*b^3/x*e-1/a^5*b^4/x*d+1/a^6*b^5/x*c+1/3/a^2*b/x^3*f-
1/3/a^3*b^2/x^3*e+1/3/a^4*b^3/x^3*d-1/3/a^5*b^4/x^3*c-b^3/a^3/(a*b)^(1/2)*a
```

```
rctan(b*x/(a*b)^(1/2))*f+b^4/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-b^5/
a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+b^6/a^6/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53972, size = 976, normalized size = 4.63

$$\left[-\frac{3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{11}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6930\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{10}+2310\left(ab^4c-\cdots\right.}{6\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6930*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(-b/a)*lo
g((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6930*(b^5*c - a*b^4*d + a^2
*b^3*e - a^3*b^2*f)*x^10 + 2310*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)
*x^8 - 1386*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^6 + 630*a^5*c + 990
*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4 - 770*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11),
 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(b/a)*arct
an(x*sqrt(b/a)) + 3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 115
5*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^
2*d + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*
x^4 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)]
```

Sympy [A] time = 51.001, size = 398, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\left(a^3\right.}{a^3b^3f-a^2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**7*sqrt(-b
**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**3*f - a**2*b**4
*e + a*b**5*d - b**6*c) + x)/2 - sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b
**2*d - b**3*c)*log(a**7*sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)/(a**3*b**3*f - a**2*b**4*e + a*b**5*d - b**6*c) + x)/2 - (315*a**5*
c + x**10*(3465*a**3*b**2*f - 3465*a**2*b**3*e + 3465*a*b**4*d - 3465*b**5*
c) + x**8*(-1155*a**4*b*f + 1155*a**3*b**2*e - 1155*a**2*b**3*d + 1155*a*b*
*4*c) + x**6*(693*a**5*f - 693*a**4*b*e + 693*a**3*b**2*d - 693*a**2*b**3*c
) + x**4*(495*a**5*e - 495*a**4*b*d + 495*a**3*b**2*c) + x**2*(385*a**5*d -
 385*a**4*b*c))/(3465*a**6*x**11)
```

Giac [A] time = 1.17518, size = 336, normalized size = 1.59

$$\frac{\left(b^6c-ab^5d-a^3b^3f+a^2b^4e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6}+\frac{3465\,b^5cx^{10}-3465\,ab^4dx^{10}-3465\,a^3b^2fx^{10}+3465\,a^2b^3x^{10}e-1155\,ab^4cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*b^5*c*x^10 - 3465*a*b^4*d*x^10 - 3465*a^3*b^2*f*x^10 +
3465*a^2*b^3*x^10*e - 1155*a*b^4*c*x^8 + 1155*a^2*b^3*d*x^8 + 1155*a^4*b*f*
x^8 - 1155*a^3*b^2*x^8*e + 693*a^2*b^3*c*x^6 - 693*a^3*b^2*d*x^6 - 693*a^5*
f*x^6 + 693*a^4*b*x^6*e - 495*a^3*b^2*c*x^4 + 495*a^4*b*d*x^4 - 495*a^5*x^4
*e + 385*a^4*b*c*x^2 - 385*a^5*d*x^2 - 315*a^5*c)/(a^6*x^11)
```

3.124 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2} dx$

Optimal. Leaf size=240

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x^5\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{10ab^4}+\frac{x^3\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{6b^5}-\frac{ax\left(9a^2be-11a^3f-\right.}{2b^6}$$

```
[Out] -(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/(2*b^6) + ((5*b^3*c - 7
*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3*c - 7*a*b^2*d + 9*a
^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/
(9*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (
a^(3/2)*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*b^(13/2))
```

Rubi [A] time = 0.294288, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x^5\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{10ab^4}+\frac{x^3\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{6b^5}-\frac{ax\left(9a^2be-11a^3f-\right.}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```



```
[Out] -(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/(2*b^6) + ((5*b^3*c - 7
*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3*c - 7*a*b^2*d + 9*a
^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/
(9*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (
a^(3/2)*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*b^(13/2))
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6 \left(c + dx^2 + ex^4 + fx^6\right)}{\left(a + bx^2\right)^2} dx = \frac{\left(c - \frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right) x^7}{2a\left(a + bx^2\right)} - \frac{\int \frac{x^5\left(\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2} dx}{2ab}$$

$$= \frac{\left(c - \frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right) x^7}{2a\left(a + bx^2\right)} - \frac{\int \frac{x^6\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2} dx}{2ab}$$

$$= \frac{\left(c - \frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right) x^7}{2a\left(a + bx^2\right)} - \frac{\int \left(\frac{a^2\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)}{b^5} - \frac{a\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x^2}{b^4} + \frac{\left(5b^3c\right.}{}\right.}{}$$

$$= -\frac{a\left(5b^3c - 7ab^2d + 9a^2be - 11a^3f\right)x}{2b^6} + \frac{\left(5b^3c - 7ab^2d + 9a^2be - 11a^3f\right)x^3}{6b^5} - \frac{\left(5b^3c - \right.}{}$$

$$= -\frac{a\left(5b^3c - 7ab^2d + 9a^2be - 11a^3f\right)x}{2b^6} + \frac{\left(5b^3c - 7ab^2d + 9a^2be - 11a^3f\right)x^3}{6b^5} - \frac{\left(5b^3c - \right.}{}$$

Mathematica [A] time = 0.123684, size = 227, normalized size = 0.95

$$\frac{x^3\left(3a^2be - 4a^3f - 2ab^2d + b^3c\right)}{3b^5} - \frac{x\left(a^2b^3c - a^3b^2d + a^4be + a^5(-f)\right)}{2b^6\left(a + bx^2\right)} + \frac{ax\left(-4a^2be + 5a^3f + 3ab^2d - 2b^3c\right)}{b^6} - \frac{a^{3/2}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)/(
5*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) - ((a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*x)/(2*b^6*(a + b*x^2)) - (a^(3/2)*(-5*b^3*c + 7*a*
b^2*d - 9*a^2*b*e + 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Maple [A] time = 0.011, size = 309, normalized size = 1.3

$$\frac{fx^9}{9b^2} - \frac{2x^7af}{7b^3} + \frac{x^7e}{7b^2} + \frac{3x^5a^2f}{5b^4} - \frac{2x^5ae}{5b^3} + \frac{x^5d}{5b^2} - \frac{4x^3a^3f}{3b^5} + \frac{x^3a^2e}{b^4} - \frac{2ax^3d}{3b^3} + \frac{x^3c}{3b^2} + 5\frac{a^4fx}{b^6} - 4\frac{a^3ex}{b^5} + 3\frac{a^2dx}{b^4} - 2\frac{ac}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/9*f*x^9/b^2-2/7/b^3*x^7*a*f+1/7/b^2*x^7*e+3/5/b^4*x^5*a^2*f-2/5/b^3*x^5*a
*e+1/5/b^2*x^5*d-4/3/b^5*x^3*a^3*f+1/b^4*x^3*a^2*e-2/3/b^3*x^3*a*d+1/3/b^2*
x^3*c+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d*x-2/b^3*a*c*x+1/2*a^5/b^6*x/(
b*x^2+a)*f-1/2*a^4/b^5*x/(b*x^2+a)*e+1/2*a^3/b^4*x/(b*x^2+a)*d-1/2*a^2/b^3*
x/(b*x^2+a)*c-11/2*a^5/b^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f+9/2*a^4/b^
5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-7/2*a^3/b^4/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*d+5/2*a^2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58722, size = 1260, normalized size = 5.25

$$\left[\frac{140\,b^5 f x^{11} + 20\left(9\,b^5 e - 11\,ab^4 f\right)x^9 + 36\left(7\,b^5 d - 9\,ab^4 e + 11\,a^2b^3 f\right)x^7 + 84\left(5\,b^5 c - 7\,ab^4 d + 9\,a^2b^3 e - 11\,a^3b^2 f\right)x^5 - 42}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*f*x^11 + 20*(9*b^5*e - 11*a*b^4*f)*x^9 + 36*(7*b^5*d - 9*a
*b^4*e + 11*a^2*b^3*f)*x^7 + 84*(5*b^5*c - 7*a*b^4*d + 9*a^2*b^3*e - 11*a^3
*b^2*f)*x^5 - 420*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^3
- 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f + (5*a*b^4*c - 7*a^
2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt
```

```
(-a/b) - a)/(b*x^2 + a)) - 630*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*
a^5*f)*x)/(b^7*x^2 + a*b^6), 1/630*(70*b^5*f*x^11 + 10*(9*b^5*e - 11*a*b^4*
f)*x^9 + 18*(7*b^5*d - 9*a*b^4*e + 11*a^2*b^3*f)*x^7 + 42*(5*b^5*c - 7*a*b^
4*d + 9*a^2*b^3*e - 11*a^3*b^2*f)*x^5 - 210*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^
3*b^2*e - 11*a^4*b*f)*x^3 + 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f + (5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f)*x)/(b^7*x^2 + a*b^6)]
```

Sympy [A] time = 2.44927, size = 430, normalized size = 1.79

$$\frac{x\left(a^5f-a^4be+a^3b^2d-a^2b^3c\right)}{2ab^6+2b^7x^2}+\frac{\sqrt{-\frac{a^3}{b^{13}}}\left(11a^3f-9a^2be+7ab^2d-5b^3c\right)\log\left(-\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}\left(11a^3f-9a^2be+7ab^2d-5b^3c\right)}{11a^4f-9a^3be+7a^2b^2d-5ab^3c}+x\right)}{4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(2*a*b**6 + 2*b**7*x**2)
+ sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)*log(-b
**6*sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*
a**4*f - 9*a**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 - sqrt(-a**3/b**13
)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)*log(b**6*sqrt(-a**3/b**1
3)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*a**4*f - 9*a**3*b*e
 + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 + f*x**9/(9*b**2) - x**7*(2*a*f - b*e
)/(7*b**3) + x**5*(3*a**2*f - 2*a*b*e + b**2*d)/(5*b**4) - x**3*(4*a**3*f -
 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(3*b**5) + x*(5*a**4*f - 4*a**3*b*e + 3*
a**2*b**2*d - 2*a*b**3*c)/b**6
```

Giac [A] time = 1.15514, size = 340, normalized size = 1.42

$$\frac{\left(5\,a^2b^3c-7\,a^3b^2d-11\,a^5f+9\,a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6}-\frac{a^2b^3cx-a^3b^2dx-a^5fx+a^4bxe}{2\left(bx^2+a\right)b^6}+\frac{35\,b^{16}fx^9-90\,ab^{15}fx^7+45\,b\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + 9*a^4*b*e)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^6) - 1/2*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/(
(b*x^2 + a)*b^6) + 1/315*(35*b^16*f*x^9 - 90*a*b^15*f*x^7 + 45*b^16*x^7*e +
 63*b^16*d*x^5 + 189*a^2*b^14*f*x^5 - 126*a*b^15*x^5*e + 105*b^16*c*x^3 - 2
10*a*b^15*d*x^3 - 420*a^3*b^13*f*x^3 + 315*a^2*b^14*x^3*e - 630*a*b^15*c*x
+ 945*a^2*b^14*d*x + 1575*a^4*b^12*f*x - 1260*a^3*b^13*x*e)/b^18
```

3.125 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=202

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x^3\left(7a^2be-9a^3f-5ab^2d+3b^3c\right)}{6ab^4}+\frac{x\left(7a^2be-9a^3f-5ab^2d+3b^3c\right)}{2b^5}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(7a^2be\right.}{2b}$$

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3*c - 5*a*b
^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) +
(f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(2*a*(a + b*x
^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi [A] time = 0.234874, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x^3\left(7a^2be-9a^3f-5ab^2d+3b^3c\right)}{6ab^4}+\frac{x\left(7a^2be-9a^3f-5ab^2d+3b^3c\right)}{2b^5}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(7a^2be\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3*c - 5*a*b
^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) +
(f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(2*a*(a + b*x
^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^3\left(\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^4\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{a\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)}{b^4}+\frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^2}{b^3}-\frac{2a(be-2af}{b^2}\right.}{2ab}$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be-2af)x^5}{5b^3} +$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be-2af)x^5}{5b^3} +$$

Mathematica [A] time = 0.101106, size = 187, normalized size = 0.93

$$\frac{x\left(-a^2b^2d+a^3be+a^4(-f)+ab^3c\right)}{2b^5\left(a+bx^2\right)}+\frac{x\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{b^5}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-7a^2be+9a^3f+5ab^2d-3b^3c\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((a*b^
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/(2*b^5*(a + b*x^2)) + (Sqrt[a]*(-3*b^
3*c + 5*a*b^2*d - 7*a^2*b*e + 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(1
1/2))
```

Maple [A] time = 0.01, size = 258, normalized size = 1.3

$$\frac{fx^7}{7\,b^2}-\frac{2\,x^5af}{5\,b^3}+\frac{x^5e}{5\,b^2}+\frac{x^3a^2f}{b^4}-\frac{2\,ax^3e}{3\,b^3}+\frac{x^3d}{3\,b^2}-4\,\frac{a^3fx}{b^5}+3\,\frac{a^2ex}{b^4}-2\,\frac{adx}{b^3}+\frac{cx}{b^2}-\frac{a^4xf}{2\,b^5\left(bx^2+a\right)}+\frac{a^3xe}{2\,b^4\left(bx^2+a\right)}-\frac{}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/7*f*x^7/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+1/b^4*x^3*a^2*f-2/3/b^3*x^3*a*e
+1/3/b^2*x^3*d-4/b^5*a^3*f*x+3/b^4*a^2*e*x-2/b^3*a*d*x+1/b^2*c*x-1/2*a^4/b^
5*x/(b*x^2+a)*f+1/2*a^3/b^4*x/(b*x^2+a)*e-1/2*a^2/b^3*x/(b*x^2+a)*d+1/2*a/b
^2*x/(b*x^2+a)*c+9/2*a^4/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f-7/2*a^3/
b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+5/2*a^2/b^3/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2))*d-3/2*a/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49243, size = 1041, normalized size = 5.15

$$\left[\frac{60\,b^4 f x^9 + 12\left(7\,b^4 e - 9\,a b^3 f\right)x^7 + 28\left(5\,b^4 d - 7\,a b^3 e + 9\,a^2 b^2 f\right)x^5 + 140\left(3\,b^4 c - 5\,a b^3 d + 7\,a^2 b^2 e - 9\,a^3 b f\right)x^3 - 105\left(3\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*b^4*f*x^9 + 12*(7*b^4*e - 9*a*b^3*f)*x^7 + 28*(5*b^4*d - 7*a*b^3
*e + 9*a^2*b^2*f)*x^5 + 140*(3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)
*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f + (3*b^4*c - 5*a*
b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a
/b) - a)/(b*x^2 + a)) + 210*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)
*x)/(b^6*x^2 + a*b^5), 1/210*(30*b^4*f*x^9 + 6*(7*b^4*e - 9*a*b^3*f)*x^7 +
14*(5*b^4*d - 7*a*b^3*e + 9*a^2*b^2*f)*x^5 + 70*(3*b^4*c - 5*a*b^3*d + 7*a^
2*b^2*e - 9*a^3*b*f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4
*f + (3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*x)/(
b^6*x^2 + a*b^5)]
```

Sympy [A] time = 2.5218, size = 250, normalized size = 1.24

$$-\frac{x\left(a^4 f - a^3 be + a^2 b^2 d - ab^3 c\right)}{2ab^5 + 2b^6 x^2} - \frac{\sqrt{-\frac{a}{b^{11}}}\left(9a^3 f - 7a^2 be + 5ab^2 d - 3b^3 c\right)\log\left(-b^5\sqrt{-\frac{a}{b^{11}}} + x\right)}{4} + \frac{\sqrt{-\frac{a}{b^{11}}}\left(9a^3 f - 7a^2 be + 5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(2*a*b**5 + 2*b**6*x**2) -
sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*b**3*c)*log(-b**5*sq
rt(-a/b**11) + x)/4 + sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d -
3*b**3*c)*log(b**5*sqrt(-a/b**11) + x)/4 + f*x**7/(7*b**2) - x**5*(2*a*f -
```

```
b*e)/(5*b**3) + x**3*(3*a**2*f - 2*a*b*e + b**2*d)/(3*b**4) - x*(4*a**3*f -
 3*a**2*b*e + 2*a*b**2*d - b**3*c)/b**5
```

Giac [A] time = 1.1531, size = 271, normalized size = 1.34

$$-\frac{\left(3\,ab^{3}c-5\,a^{2}b^{2}d-9\,a^{4}f+7\,a^{3}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^{5}}+\frac{ab^{3}cx-a^{2}b^{2}dx-a^{4}fx+a^{3}bxe}{2\,\left(bx^{2}+a\right)b^{5}}+\frac{15\,b^{12}fx^{7}-42\,ab^{11}fx^{5}+21\,b^{12}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^3*c - 5*a^2*b^2*d - 9*a^4*f + 7*a^3*b*e)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/2*(a*b^3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x
^2 + a)*b^5) + 1/105*(15*b^12*f*x^7 - 42*a*b^11*f*x^5 + 21*b^12*x^5*e + 35*
b^12*d*x^3 + 105*a^2*b^10*f*x^3 - 70*a*b^11*x^3*e + 105*b^12*c*x - 210*a*b^
11*d*x - 420*a^3*b^9*f*x + 315*a^2*b^10*x*e)/b^14
```

3.126 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=163

$$\frac{x^3\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2ab^4}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2\sqrt{a}b^{9/2}}+\frac{x^3(be-2af)}{3b^3}+$$

```
[Out] -((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(2*a*b^4) + ((b*e - 2*a*f)*x
^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)
/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rubi [A] time = 0.228794, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^3\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2ab^4}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2\sqrt{a}b^{9/2}}+\frac{x^3(be-2af)}{3b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(2*a*b^4) + ((b*e - 2*a*f)*x
^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)
/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \frac{x\left(\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^2\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \left(c-\frac{a\left(3b^2d-5abe+7a^2f\right)}{b^3}-\frac{2a(be-2af)x^2}{b^2}-\frac{2afx^4}{b}+\frac{-ab^3c+3a^2b^2d-5a^3\ldots}{b^3\left(a+bx^2\right)}\ldots}{2ab}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)}+\frac{(b\ldots}{\ldots}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)}+\frac{(b\ldots}{\ldots}$$

Mathematica [A] time = 0.0817565, size = 148, normalized size = 0.91

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2b^4\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-5a^2be+7a^3f+3ab^2d-b^3c\right)}{2\sqrt{a}b^{9/2}}+\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4}+\frac{x^3(be-2af)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^5)
/(5*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*b^4*(a + b*x^2)) - ((
-(b^3*c) + 3*a*b^2*d - 5*a^2*b*e + 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*Sqrt[a]*b^(9/2))
```

Maple [A] time = 0.009, size = 212, normalized size = 1.3

$$\frac{fx^5}{5\,b^2}-\frac{2\,ax^3f}{3\,b^3}+\frac{x^3e}{3\,b^2}+3\,\frac{a^2fx}{b^4}-2\,\frac{aex}{b^3}+\frac{dx}{b^2}+\frac{xa^3f}{2\,b^4\left(bx^2+a\right)}-\frac{a^2xe}{2\,b^3\left(bx^2+a\right)}+\frac{axd}{2\,b^2\left(bx^2+a\right)}-\frac{cx}{2\,b\left(bx^2+a\right)}-\frac{7\,a^3f}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/5*f*x^5/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+3/b^4*a^2*f*x-2/b^3*a*e*x+1/b^2
*d*x+1/2/b^4*x/(b*x^2+a)*a^3*f-1/2/b^3*x/(b*x^2+a)*a^2*e+1/2/b^2*x/(b*x^2+a
)*a*d-1/2/b*x/(b*x^2+a)*c-7/2/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^3*f
+5/2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^2*e-3/2/b^2/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*a*d+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49616, size = 902, normalized size = 5.53

$$\left[\frac{12\,ab^4fx^7+4\left(5\,ab^4e-7\,a^2b^3f\right)x^5+20\left(3\,ab^4d-5\,a^2b^3e+7\,a^3b^2f\right)x^3+15\left(ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+\left(b^4c-3\right.\right.}{60\left(ab^6x^2+a^2b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*f*x^7 + 4*(5*a*b^4*e - 7*a^2*b^3*f)*x^5 + 20*(3*a*b^4*d - 5
*a^2*b^3*e + 7*a^3*b^2*f)*x^3 + 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a
^4*f + (b^4*c - 3*a*b^3*d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(-a*b)*log((b
*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(a*b^4*c - 3*a^2*b^3*d + 5*a^3
*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*f*x^7 + 2*(5*a*
b^4*e - 7*a^2*b^3*f)*x^5 + 10*(3*a*b^4*d - 5*a^2*b^3*e + 7*a^3*b^2*f)*x^3 +
 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + (b^4*c - 3*a*b^3*d + 5*a
^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 15*(a*b^4*c -
3*a^2*b^3*d + 5*a^3*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [A] time = 2.43083, size = 216, normalized size = 1.33

$$\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{1}{ab^9}}\left(7a^3f-5a^2be+3ab^2d-b^3c\right)\log\left(-ab^4\sqrt{-\frac{1}{ab^9}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^9}}\left(7a^3f-5a^2be+3a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-
1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*c)*log(-a*b**4*sqrt(
-1/(a*b**9)) + x)/4 - sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d
 - b**3*c)*log(a*b**4*sqrt(-1/(a*b**9)) + x)/4 + f*x**5/(5*b**2) - x**3*(2*
a*f - b*e)/(3*b**3) + x*(3*a**2*f - 2*a*b*e + b**2*d)/b**4
```

Giac [A] time = 1.16362, size = 205, normalized size = 1.26

$$\frac{\left(b^{3}c-3\,ab^{2}d-7\,a^{3}f+5\,a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^{4}}-\frac{b^{3}cx-ab^{2}dx-a^{3}fx+a^{2}bxe}{2\left(bx^{2}+a\right)b^{4}}+\frac{3\,b^{8}fx^{5}-10\,ab^{7}fx^{3}+5\,b^{8}x^{3}e+15\,b^{8}dx}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 3*a*b^2*d - 7*a^3*f + 5*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*f*x^5 - 10*a*b^7*f*x^3 + 5*b^8*x^3*e + 15*b^8*d*x + 45*a^2*
b^6*f*x - 30*a*b^7*x*e)/b^10
```

3.127 $\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=118

$$\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^2be+5a^3f+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi [A] time = 0.119856, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 1153, 205}

$$\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^2be+5a^3f+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^2}\,dx &= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-\frac{b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{2a(be-af)x^2}{b^2}-\frac{2afx^4}{b}}{a+bx^2}\,dx}{2a}\\
&= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2a(be-2af)}{b^3}-\frac{2afx^2}{b^2}+\frac{-b^3c-ab^2d+3a^2be-5a^3f}{b^3\left(a+bx^2\right)}\right)dx}{2a}\\
&= \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\int \frac{1}{a+bx^2}\,dx}{2ab^3}\\
&= \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0878644, size = 122, normalized size = 1.03

$$-\frac{x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2ab^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^2be+5a^3f+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Maple [A] time = 0.009, size = 177, normalized size = 1.5

$$\frac{fx^3}{3\,b^2}-2\,\frac{afx}{b^3}+\frac{ex}{b^2}-\frac{a^2xf}{2\,b^3\left(bx^2+a\right)}+\frac{axe}{2\,b^2\left(bx^2+a\right)}-\frac{dx}{2\,b\left(bx^2+a\right)}+\frac{cx}{2\,a\left(bx^2+a\right)}+\frac{5\,a^2f}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*f*x^3/b^2-2/b^3*a*f*x+1/b^2*x*e-1/2/b^3*a^2*x/(b*x^2+a)*f+1/2/b^2*a*x/(
b*x^2+a)*e-1/2/b*x/(b*x^2+a)*d+1/2/a*x/(b*x^2+a)*c+5/2/b^3*a^2/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))*f-3/2/b^2*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+1
/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70085, size = 761, normalized size = 6.45

$$\left[\frac{4\,a^2b^3fx^5+4\left(3\,a^2b^3e-5\,a^3b^2f\right)x^3-3\left(ab^3c+a^2b^2d-3\,a^3be+5\,a^4f+\left(b^4c+ab^3d-3\,a^2b^2e+5\,a^3bf\right)x^2\right)\sqrt{-ab}\log\left(\right.}{12\left(a^2b^5x^2+a^3b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*f*x^5 + 4*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 - 3*(a*b^3*c + a
^2*b^2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f
```

```
)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*
c - a^2*b^3*d + 3*a^3*b^2*e - 5*a^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2
*a^2*b^3*f*x^5 + 2*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 + 3*(a*b^3*c + a^2*b^2*d
 - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f)*x^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c - a^2*b^3*d + 3*a^3*b^2*e - 5*a
^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [A] time = 1.94261, size = 199, normalized size = 1.69

$$-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^2b^3+2ab^4x^2}-\frac{\sqrt{-\frac{1}{a^3b^7}}\left(5a^3f-3a^2be+ab^2d+b^3c\right)\log\left(-a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^7}}\left(5a^3f-3a^2be+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a**2*b**3 + 2*a*b**4*x**2) -
sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d + b**3*c)*log(-a**2*
b**3*sqrt(-1/(a**3*b**7)) + x)/4 + sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*
b*e + a*b**2*d + b**3*c)*log(a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4 + f*x**3
/(3*b**2) - x*(2*a*f - b*e)/b**3
```

Giac [A] time = 1.16407, size = 170, normalized size = 1.44

$$\frac{\left(b^3c+ab^2d+5\,a^3f-3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)ab^3}+\frac{b^4fx^3-6\,ab^3fx+3\,b^4xe}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c + a*b^2*d + 5*a^3*f - 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b^3) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a*b
^3) + 1/3*(b^4*f*x^3 - 6*a*b^3*f*x + 3*b^4*x*e)/b^6
```

3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2} \, dx$

Optimal. Leaf size=112

$$-\frac{x\left(-\frac{a^2 f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+3a^3f-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}-\frac{c}{a^2x}+\frac{fx}{b^2}$$

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(2*a*(
a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rubi [A] time = 0.131882, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1261, 205}

$$-\frac{x\left(-\frac{a^2 f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+3a^3f-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}-\frac{c}{a^2x}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(2*a*(
a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx &= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2c+\left(\frac{bc}{a}-d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{2afx^4}{b}}{x^2\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2af}{b^2}-\frac{2c}{ax^2}+\frac{3b^3c-ab^2d-a^2be+3a^3f}{ab^2\left(a+bx^2\right)}\right)dx}{2a}\\
&= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\int \frac{1}{a+bx^2}\,dx}{2a^2b^2}\\
&= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0617208, size = 115, normalized size = 1.03

$$\frac{x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2a^2b^2\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+3a^3f-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}-\frac{c}{a^2x}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^
2*b^2*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Maple [A] time = 0.013, size = 165, normalized size = 1.5

$$\frac{fx}{b^2}-\frac{c}{a^2x}+\frac{axf}{2\,b^2\left(bx^2+a\right)}-\frac{ex}{2\,b\left(bx^2+a\right)}+\frac{dx}{2\,a\left(bx^2+a\right)}-\frac{bcx}{2\,a^2\left(bx^2+a\right)}-\frac{3\,af}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{e}{2\,b}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] f*x/b^2-c/a^2/x+1/2*a/b^2*x/(b*x^2+a)*f-1/2/b*x/(b*x^2+a)*e+1/2/a*x/(b*x^2+
a)*d-1/2/a^2*b*x/(b*x^2+a)*c-3/2*a/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
f+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+1/2/a/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*d-3/2/a^2*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83527, size = 714, normalized size = 6.38

$$\left[\frac{4\,a^3b^2fx^4-4\,a^2b^3c-2\left(3\,ab^4c-a^2b^3d+a^3b^2e-3\,a^4bf\right)x^2-\left(\left(3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf\right)x^3+\left(3\,ab^3c-a^2b^2d-a\right.\right.}{4\left(a^3b^4x^3+a^4b^3x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*b^2*f*x^4 - 4*a^2*b^3*c - 2*(3*a*b^4*c - a^2*b^3*d + a^3*b^2*e
- 3*a^4*b*f)*x^2 - ((3*b^4*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*
b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a
```

```
*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b^3*x), 1/2*(2*a^3*b^2*f*x^4 -
2*a^2*b^3*c - (3*a*b^4*c - a^2*b^3*d + a^3*b^2*e - 3*a^4*b*f)*x^2 - ((3*b^4
*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*b^3*c - a^2*b^2*d - a^3*b*
e + 3*a^4*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]
```

Sympy [A] time = 4.6105, size = 197, normalized size = 1.76

$$\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f - a^2be - ab^2d + 3b^3c\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f - a^2be - ab^2d + 3b^3c\right)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}} + x\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*log(-a**3*
b**2*sqrt(-1/(a**5*b**5)) + x)/4 - sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*
e - a*b**2*d + 3*b**3*c)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/4 + (-2*a*
b**2*c + x**2*(a**3*f - a**2*b*e + a*b**2*d - 3*b**3*c))/(2*a**3*b**2*x + 2
*a**2*b**3*x**3) + f*x/b**2
```

Giac [A] time = 1.1664, size = 165, normalized size = 1.47

$$\frac{fx}{b^2} - \frac{\left(3\,b^3c - ab^2d + 3\,a^3f - a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b^2} - \frac{3\,b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e + 2\,ab^2c}{2\left(bx^3 + ax\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d + 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x
^2*e + 2*a*b^2*c)/((b*x^3 + a*x)*a^2*b^2)
```

3.129 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=121

$$\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3f-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.157549, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1261, 205}

$$\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3f-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{b^2c}{a^2}+\frac{bd}{a}-e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)}\,dx}{2a}$$

$$=\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2c}{ax^4}-\frac{2(-2bc+ad)}{a^2x^2}+\frac{-5b^3c+3ab^2d-a^2be-a^3f}{a^2b\left(a+bx^2\right)}\right)dx}{2a}$$

$$=-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{2a^3b}$$

$$=-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

Mathematica [A] time = 0.0723972, size = 125, normalized size = 1.03

$$-\frac{x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2a^3b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3f-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) - ((-(b^3*c) + a*b^2*d - a^2*b*e + a
^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Maple [A] time = 0.011, size = 182, normalized size = 1.5

$$-\frac{c}{3\,x^3a^2}-\frac{d}{a^2x}+2\,\frac{bc}{a^3x}-\frac{fx}{2\,b\left(bx^2+a\right)}+\frac{ex}{2\,a\left(bx^2+a\right)}-\frac{bdx}{2\,a^2\left(bx^2+a\right)}+\frac{b^2xc}{2\,a^3\left(bx^2+a\right)}+\frac{f}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c/x^3/a^2-1/a^2/x*d+2/a^3/x*b*c-1/2/b*x/(b*x^2+a)*f+1/2/a*x/(b*x^2+a)*
e-1/2/a^2*b*x/(b*x^2+a)*d+1/2/a^3*b^2*x/(b*x^2+a)*c+1/2/b/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*f+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-3/2/a^2*b/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46541, size = 786, normalized size = 6.5

$$\left[-\frac{4\,a^3b^2c-6\left(5\,ab^4c-3\,a^2b^3d+a^3b^2e-a^4bf\right)x^4-4\left(5\,a^2b^3c-3\,a^3b^2d\right)x^2+3\left(\left(5\,b^4c-3\,ab^3d+a^2b^2e+a^3bf\right)x^5+\left(5\right.\right.}{12\left(a^4b^3x^5+a^5b^2x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b^2*c - 6*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4
 - 4*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 + 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
```

```
+ a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3
), -1/6*(2*a^3*b^2*c - 3*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
4 - 2*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 - 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
 + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(a*b
)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^3)]
```

Sympy [A] time = 12.2098, size = 212, normalized size = 1.75

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\left(a^3f+a^2be-3ab^2d+5b^3c\right)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\left(a^3f+a^2be-3ab^2d+5b^3c\right)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)*log(-a**4
*b*sqrt(-1/(a**7*b**3)) + x)/4 + sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e -
3*a*b**2*d + 5*b**3*c)*log(a**4*b*sqrt(-1/(a**7*b**3)) + x)/4 - (2*a**2*b*c
 + x**4*(3*a**3*f - 3*a**2*b*e + 9*a*b**2*d - 15*b**3*c) + x**2*(6*a**2*b*d
 - 10*a*b**2*c))/(6*a**4*b*x**3 + 6*a**3*b**2*x**5)
```

Giac [A] time = 1.25384, size = 166, normalized size = 1.37

$$\frac{\left(5\,b^3c-3\,ab^2d+a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)a^3b}+\frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c - 3*a*b^2*d + a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^3*b) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^3
*b) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3)
```

3.130 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^2}\,dx$

Optimal. Leaf size=152

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be+a^3(-f)-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{2bc-ad}{3a^3x^3}-\frac{5a}{}$$

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d + a^2*e)/(a
^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*(a + b*x^2)) - ((7*b
^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/
2)*Sqrt[b])
```

Rubi [A] time = 0.213107, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1802, 205}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be+a^3(-f)-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{2bc-ad}{3a^3x^3}-\frac{5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```



```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d + a^2*e)/(a
^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*(a + b*x^2)) - ((7*b
^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/
2)*Sqrt[b])
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6\left(a+bx^2\right)}dx}{2a}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^6}-\frac{2(-2bc+ad)}{a^2x^4}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^2}+\frac{7b^3c-5ab^2d+3a^2be-a^3f}{a^3\left(a+bx^2\right)}\right)}{2a}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\left(7b^3c-5ab^2d+3\right.}{}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\left(7b^3c-5ab^2d+3\right.}{2}$$

Mathematica [A] time = 0.0824897, size = 151, normalized size = 0.99

$$\frac{x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2a^4\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^2be+a^3f+5ab^2d-7b^3c\right)}{2a^{9/2}\sqrt{b}}+\frac{a^2(-e)+2abd-3b^2c}{a^4x}+\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) + (-3*b^2*c + 2*a*b*d - a^2*e)/(
a^4*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^4*(a + b*x^2)) + (
(-7*b^3*c + 5*a*b^2*d - 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
```

a^(9/2)*Sqrt[b])

Maple [A] time = 0.014, size = 219, normalized size = 1.4

$$-\frac{c}{5\,x^5a^2}-\frac{d}{3\,x^3a^2}+\frac{2\,bc}{3\,a^3x^3}-\frac{e}{a^2x}+2\,\frac{bd}{a^3x}-3\,\frac{b^2c}{a^4x}+\frac{fx}{2\,a\left(bx^2+a\right)}-\frac{bxe}{2\,a^2\left(bx^2+a\right)}+\frac{xb^2d}{2\,a^3\left(bx^2+a\right)}-\frac{xb^3c}{2\,a^4\left(bx^2+a\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/5*c/x^5/a^2-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c-1/a^2/x*e+2/a^3/x*b*d-3/a^4/x*
b^2*c+1/2/a*x/(b*x^2+a)*f-1/2/a^2*x/(b*x^2+a)*b*e+1/2/a^3*x/(b*x^2+a)*b^2*d
-1/2/a^4*x/(b*x^2+a)*b^3*c+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f-3/2/
a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*b*e+5/2/a^3/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*b^2*d-7/2/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*b^3*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39414, size = 926, normalized size = 6.09

$$\left[-\frac{30\left(7\,ab^4c-5\,a^2b^3d+3\,a^3b^2e-a^4bf\right)x^6+12\,a^4bc+20\left(7\,a^2b^3c-5\,a^3b^2d+3\,a^4be\right)x^4-4\left(7\,a^3b^2c-5\,a^4bd\right)x^2-15\,(}{60\left(a^5b^2x^7+a^6bx^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(7*a*b^4*c - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 12*a^4*b
*c + 20*(7*a^2*b^3*c - 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 4*(7*a^3*b^2*c - 5*a^
4*b*d)*x^2 - 15*((7*b^4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b
^3*c - 5*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^2*x^7 + a^6*b*x^5), -1/30*(15*(7*a*b^4*c
 - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 6*a^4*b*c + 10*(7*a^2*b^3*c -
 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 2*(7*a^3*b^2*c - 5*a^4*b*d)*x^2 + 15*((7*b^
4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b^2*d + 3
*a^3*b*e - a^4*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^2*x^7 + a^6*
b*x^5)]
```

Sympy [A] time = 30.5347, size = 226, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{a^9b}}\left(a^3f-3a^2be+5ab^2d-7b^3c\right)\log\left(-a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^9b}}\left(a^3f-3a^2be+5ab^2d-7b^3c\right)\log\left(a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*log(-a**5*
sqrt(-1/(a**9*b)) + x)/4 + sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**
2*d - 7*b**3*c)*log(a**5*sqrt(-1/(a**9*b)) + x)/4 + (-6*a**3*c + x**6*(15*a
**3*f - 45*a**2*b*e + 75*a*b**2*d - 105*b**3*c) + x**4*(-30*a**3*e + 50*a**
2*b*d - 70*a*b**2*c) + x**2*(-10*a**3*d + 14*a**2*b*c))/(30*a**5*x**5 + 30*
a**4*b*x**7)
```

Giac [A] time = 1.1733, size = 204, normalized size = 1.34

$$-\frac{\left(7\,b^3c-5\,ab^2d-a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)a^4}-\frac{45\,b^2cx^4-30\,abdx^4+15\,a^2x^4e-10\,abc}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^3*c - 5*a*b^2*d - a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^
```

```
4) - 1/15*(45*b^2*c*x^4 - 30*a*b*d*x^4 + 15*a^2*x^4*e - 10*a*b*c*x^2 + 5*a^
2*d*x^2 + 3*a^2*c)/(a^4*x^5)
```

3.131 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=189

$$\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{a^5x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^2be-3a^3f-7ab^2d+9b^3c\right)}{2a^{11/2}}-$$

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(3
*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*x^2)) + (Sqrt[b]*(9*b^3*c - 7*
a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rubi [A] time = 0.293351, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1802, 205}

$$\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{a^5x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^2be-3a^3f-7ab^2d+9b^3c\right)}{2a^{11/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(3
*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*x^2)) + (Sqrt[b]*(9*b^3*c - 7*
a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{b\left(b^3c-ab^2d+a^2b\right.}{a^4}}{x^8\left(a+bx^2\right)}}{2a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^8}-\frac{2(-2bc+ad)}{a^2x^6}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^4}-\frac{2\left(-4b^3c+3ab^2d-2a^2be+\right.}{a^4x^2}\right.}{2a}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-ab^2d+\right.}{2a^5\left(a+\right.}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-ab^2d+\right.}{2a^5\left(a+\right.}$$

Mathematica [A] time = 0.101657, size = 190, normalized size = 1.01

$$-\frac{bx\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{a^5x}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-5a^2be+3a^3f+7ab^2d-9b^3c\right)}{2a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) + (-3*b^2*c + 2*a*b*d - a^2*e)/(
3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) - (b*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^5*(a + b*x^2)) - (Sqrt[b]*(-9*b^3*c
 + 7*a*b^2*d - 5*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2
```

```
))
```

Maple [A] time = 0.016, size = 268, normalized size = 1.4

$$-\frac{c}{7\,a^{2}x^{7}}-\frac{d}{5\,x^{5}a^{2}}+\frac{2\,bc}{5\,a^{3}x^{5}}-\frac{e}{3\,x^{3}a^{2}}+\frac{2\,bd}{3\,a^{3}x^{3}}-\frac{b^{2}c}{a^{4}x^{3}}-\frac{f}{a^{2}x}+2\,\frac{be}{a^{3}x}-3\,\frac{b^{2}d}{a^{4}x}+4\,\frac{b^{3}c}{a^{5}x}-\frac{bxf}{2\,a^{2}\left(bx^{2}+a\right)}+\frac{b^{2}xe}{2\,a^{3}\left(bx^{2}+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x)
```

```
[Out] -1/7*c/a^2/x^7-1/5/a^2/x^5*d+2/5/a^3/x^5*b*c-1/3/a^2/x^3*e+2/3/a^3/x^3*b*d-
1/a^4/x^3*b^2*c-1/a^2/x*f+2/a^3/x*b*e-3/a^4/x*b^2*d+4/a^5/x*b^3*c-1/2/a^2*b
*x/(b*x^2+a)*f+1/2/a^3*b^2*x/(b*x^2+a)*e-1/2/a^4*b^3*x/(b*x^2+a)*d+1/2/a^5*
b^4*x/(b*x^2+a)*c-3/2/a^2*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f+5/2/a^3*b
^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-7/2/a^4*b^3/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))*d+9/2/a^5*b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37974, size = 1057, normalized size = 5.59

$$\left[\frac{210\left(9\,b^{4}c-7\,ab^{3}d+5\,a^{2}b^{2}e-3\,a^{3}bf\right)x^{8}+140\left(9\,ab^{3}c-7\,a^{2}b^{2}d+5\,a^{3}be-3\,a^{4}f\right)x^{6}-60\,a^{4}c-28\left(9\,a^{2}b^{2}c-7\,a^{3}bd+5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 140*(9*a*
b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 60*a^4*c - 28*(9*a^2*b^2*c
 - 7*a^3*b*d + 5*a^4*e)*x^4 + 12*(9*a^3*b*c - 7*a^4*d)*x^2 - 105*((9*b^4*c
- 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d + 5*a
^3*b*e - 3*a^4*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)))/(a^5*b*x^9 + a^6*x^7), 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*
e - 3*a^3*b*f)*x^8 + 70*(9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6
 - 30*a^4*c - 14*(9*a^2*b^2*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7
*a^4*d)*x^2 + 105*((9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9
*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(
b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

Sympy [B] time = 76.7361, size = 394, normalized size = 2.08

$$\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)\log\left(-\frac{a^6\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{3a^3bf-5a^2b^2e+7ab^3d-9b^4c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)*log(-a**6*sq
rt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)/(3*a**3*b*f -
5*a**2*b**2*e + 7*a*b**3*d - 9*b**4*c) + x)/4 - sqrt(-b/a**11)*(3*a**3*f -
5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)*log(a**6*sqrt(-b/a**11)*(3*a**3*f - 5*a
**2*b*e + 7*a*b**2*d - 9*b**3*c)/(3*a**3*b*f - 5*a**2*b**2*e + 7*a*b**3*d -
 9*b**4*c) + x)/4 - (30*a**4*c + x**8*(315*a**3*b*f - 525*a**2*b**2*e + 735
*a*b**3*d - 945*b**4*c) + x**6*(210*a**4*f - 350*a**3*b*e + 490*a**2*b**2*d
 - 630*a*b**3*c) + x**4*(70*a**4*e - 98*a**3*b*d + 126*a**2*b**2*c) + x**2*
(42*a**4*d - 54*a**3*b*c))/(210*a**6*x**7 + 210*a**5*b*x**9)
```

Giac [A] time = 1.19663, size = 271, normalized size = 1.43

$$\frac{\left(9\,b^4c-7\,ab^3d-3\,a^3bf+5\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5}+\frac{b^4cx-ab^3dx-a^3bfx+a^2b^2xe}{2\left(bx^2+a\right)a^5}+\frac{420\,b^3cx^6-315\,ab^2dx^6-105\,a^3f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f + 5*a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) + 1/2*(b^4*c*x - a*b^3*d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^
2 + a)*a^5) + 1/105*(420*b^3*c*x^6 - 315*a*b^2*d*x^6 - 105*a^3*f*x^6 + 210*
a^2*b*x^6*e - 105*a*b^2*c*x^4 + 70*a^2*b*d*x^4 - 35*a^3*x^4*e + 42*a^2*b*c*
x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^5*x^7)
```

3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}-\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{a^6x}-\frac{b^{3/2}\tan^{-1}\left(\right.}{}$$

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/(5
*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) - (b*(5*b
^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d +
7*a^2*b*e - 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rubi [A] time = 0.376065, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1802, 205}

$$-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}-\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{a^6x}-\frac{b^{3/2}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/(5
*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) - (b*(5*b
^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d +
7*a^2*b*e - 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^6\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{2b\left(b^3c-ab^2d+a\right.}{a^4}}{x^{10}\left(a+bx^2\right)}}{2a}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^6\left(a+bx^2\right)} - \frac{\int\left(-\frac{2c}{ax^{10}}-\frac{2(-2bc+ad)}{a^2x^8}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^6}-\frac{2\left(-4b^3c+3ab^2d-2a^2b\right.}{a^4x^4}\right.}{}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b\left(5b^3c-4ab^2d+\right.}{a^6x}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b\left(5b^3c-4ab^2d+\right.}{a^6x}$$

Mathematica [A] time = 0.113852, size = 230, normalized size = 1.

$$\frac{b^2x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2a^6\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}+\frac{b\left(-3a^2be+2a^3f+4ab^2d-5b^3c\right)}{a^6x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) + (-3*b^2*c + 2*a*b*d - a^2*e)/(
5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(-5
```

```
*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) + a*b^2
*d - a^2*b*e + a^3*f)*x)/(2*a^6*(a + b*x^2)) + (b^(3/2)*(-11*b^3*c + 9*a*b^
2*d - 7*a^2*b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Maple [A] time = 0.016, size = 318, normalized size = 1.4

$$-\frac{c}{9a^2x^9}-\frac{d}{7a^2x^7}+\frac{2bc}{7a^3x^7}-\frac{e}{5x^5a^2}+\frac{2bd}{5a^3x^5}-\frac{3b^2c}{5a^4x^5}-\frac{f}{3x^3a^2}+\frac{2be}{3a^3x^3}-\frac{b^2d}{a^4x^3}+\frac{4b^3c}{3a^5x^3}+2\frac{fb}{a^3x}-3\frac{eb^2}{a^4x}+4\frac{db^3}{a^5x}-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x)
```

```
[Out] -1/9*c/a^2/x^9-1/7/a^2/x^7*d+2/7/a^3/x^7*b*c-1/5/a^2/x^5*e+2/5/a^3/x^5*b*d-
3/5/a^4/x^5*b^2*c-1/3/a^2/x^3*f+2/3/a^3/x^3*b*e-1/a^4/x^3*b^2*d+4/3/a^5/x^3
*b^3*c+2*b/a^3/x*f-3*b^2/a^4/x*e+4*b^3/a^5/x*d-5*b^4/a^6/x*c+1/2*b^2/a^3*x/
(b*x^2+a)*f-1/2*b^3/a^4*x/(b*x^2+a)*e+1/2*b^4/a^5*x/(b*x^2+a)*d-1/2*b^5/a^6
*x/(b*x^2+a)*c+5/2*b^2/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f-7/2*b^3/a^
4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+9/2*b^4/a^5/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*d-11/2*b^5/a^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30327, size = 1284, normalized size = 5.58

$$\left[-\frac{630\left(11b^5c-9ab^4d+7a^2b^3e-5a^3b^2f\right)x^{10}+420\left(11ab^4c-9a^2b^3d+7a^3b^2e-5a^4bf\right)x^8-84\left(11a^2b^3c-9a^3b^2d+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/1260*(630*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 420
*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 84*(11*a^2*b^3*
c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 140*a^5*c + 36*(11*a^3*b^2*c -
 9*a^4*b*d + 7*a^5*e)*x^4 - 20*(11*a^4*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c
- 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d +
 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(a^6*b*x^11 + a^7*x^9), -1/630*(315*(11*b^5*c - 9*a*b^4*d +
 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^
2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f
)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4
*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*
f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(b/
a)*arctan(x*sqrt(b/a)))/(a^6*b*x^11 + a^7*x^9)]
```

Sympy [A] time = 144.033, size = 449, normalized size = 1.95

$$-\frac{\sqrt{-\frac{b^3}{a^{13}}}\left(5a^3f-7a^2be+9ab^2d-11b^3c\right)\log\left(-\frac{a^7\sqrt{-\frac{b^3}{a^{13}}}\left(5a^3f-7a^2be+9ab^2d-11b^3c\right)}{5a^3b^2f-7a^2b^3e+9ab^4d-11b^5c}+x\right)}{4}+\frac{\sqrt{-\frac{b^3}{a^{13}}}\left(5a^3f-7a^2be+9ab^2d-11b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-b**3/a**13)*(5*a**3*f - 7*a**2*b*e + 9*a*b**2*d - 11*b**3*c)*log(-a*
*7*sqrt(-b**3/a**13)*(5*a**3*f - 7*a**2*b*e + 9*a*b**2*d - 11*b**3*c)/(5*a*
*3*b**2*f - 7*a**2*b**3*e + 9*a*b**4*d - 11*b**5*c) + x)/4 + sqrt(-b**3/a**
13)*(5*a**3*f - 7*a**2*b*e + 9*a*b**2*d - 11*b**3*c)*log(a**7*sqrt(-b**3/a*
*13)*(5*a**3*f - 7*a**2*b*e + 9*a*b**2*d - 11*b**3*c)/(5*a**3*b**2*f - 7*a*
*2*b**3*e + 9*a*b**4*d - 11*b**5*c) + x)/4 + (-70*a**5*c + x**10*(1575*a**3
*b**2*f - 2205*a**2*b**3*e + 2835*a*b**4*d - 3465*b**5*c) + x**8*(1050*a**4
*b*f - 1470*a**3*b**2*e + 1890*a**2*b**3*d - 2310*a*b**4*c) + x**6*(-210*a*
*5*f + 294*a**4*b*e - 378*a**3*b**2*d + 462*a**2*b**3*c) + x**4*(-126*a**5*
e + 162*a**4*b*d - 198*a**3*b**2*c) + x**2*(-90*a**5*d + 110*a**4*b*c))/(63
0*a**7*x**9 + 630*a**6*b*x**11)
```

Giac [A] time = 1.24262, size = 340, normalized size = 1.48

$$-\frac{\left(11\,b^5c-9\,ab^4d-5\,a^3b^2f+7\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6}-\frac{b^5cx-ab^4dx-a^3b^2fx+a^2b^3xe}{2\left(bx^2+a\right)a^6}-\frac{1575\,b^4cx^8-1260\,ab^3dx^8-6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f + 7*a^2*b^3*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) - 1/2*(b^5*c*x - a*b^4*d*x - a^3*b^2*f*x + a^2*b^3*x*e)/
((b*x^2 + a)*a^6) - 1/315*(1575*b^4*c*x^8 - 1260*a*b^3*d*x^8 - 630*a^3*b*f*
x^8 + 945*a^2*b^2*x^8*e - 420*a*b^3*c*x^6 + 315*a^2*b^2*d*x^6 + 105*a^4*f*x
^6 - 210*a^3*b*x^6*e + 189*a^2*b^2*c*x^4 - 126*a^3*b*d*x^4 + 63*a^4*x^4*e -
 90*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^6*x^9)
```

3.133 $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=287

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x^5\left(17a^2be-29a^3f-9ab^2d+5b^3c\right)}{20ab^5}+\frac{x^3\left(15a^2be-23a^3f-9ab^2d+5b^3c\right)}{6b^6}-\frac{a^2x\left(13a^2be-17a\right.}{8b^7\left(a\right.}$$

```
[Out] -(a*(15*b^3*c - 27*a*b^2*d + 43*a^2*b*e - 63*a^3*f)*x)/(4*b^7) + ((5*b^3*c
- 9*a*b^2*d + 15*a^2*b*e - 23*a^3*f)*x^3)/(6*b^6) - ((5*b^3*c - 9*a*b^2*d +
 17*a^2*b*e - 29*a^3*f)*x^5)/(20*a*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^9)/(4*a*(a + b*x^2)
^2) - (a^2*(5*b^3*c - 9*a*b^2*d + 13*a^2*b*e - 17*a^3*f)*x)/(8*b^7*(a + b*x
^2)) + (a^(3/2)*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*b^(15/2))
```

Rubi [A] time = 0.492374, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x^5\left(17a^2be-29a^3f-9ab^2d+5b^3c\right)}{20ab^5}+\frac{x^3\left(15a^2be-23a^3f-9ab^2d+5b^3c\right)}{6b^6}-\frac{a^2x\left(13a^2be-17a\right.}{8b^7\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```


```
[Out] -(a*(15*b^3*c - 27*a*b^2*d + 43*a^2*b*e - 63*a^3*f)*x)/(4*b^7) + ((5*b^3*c
- 9*a*b^2*d + 15*a^2*b*e - 23*a^3*f)*x^3)/(6*b^6) - ((5*b^3*c - 9*a*b^2*d +
 17*a^2*b*e - 29*a^3*f)*x^5)/(20*a*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^9)/(4*a*(a + b*x^2)
^2) - (a^2*(5*b^3*c - 9*a*b^2*d + 13*a^2*b*e - 17*a^3*f)*x)/(8*b^7*(a + b*x
^2)) + (a^(3/2)*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*b^(15/2))
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^7\left(\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^8\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x}{8b^7\left(a+bx^2\right)} + \frac{\int \frac{a^3\left(5b^3c-9ab^2d+13a^2be-1\right.}{}}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x}{8b^7\left(a+bx^2\right)} + \frac{\int\left(-2a^2\left(15b^3c-27a\right.\right.}{}$$

$$= -\frac{a\left(15b^3c-27ab^2d+43a^2be-63a^3f\right)x}{4b^7} + \frac{\left(5b^3c-9ab^2d+15a^2be-23a^3f\right)x^3}{6b^6} - \frac{\left(5b^3\right.}{}$$

$$= -\frac{a\left(15b^3c-27ab^2d+43a^2be-63a^3f\right)x}{4b^7} + \frac{\left(5b^3c-9ab^2d+15a^2be-23a^3f\right)x^3}{6b^6} - \frac{\left(5b^3\right.}{}$$

Mathematica [A] time = 0.154838, size = 272, normalized size = 0.95

$$\frac{x^3\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{3b^6} + \frac{a^2x\left(-21a^2be+25a^3f+17ab^2d-13b^3c\right)}{8b^7\left(a+bx^2\right)} + \frac{a^3x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^7\left(a+bx^2\right)^2} + \frac{ax\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5
)/(5*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^9)/(9*b^3) + (a^3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*x)/(4*b^7*(a + b*x^2)^2) + (a^2*(-13*b^3*c + 17*a
*b^2*d - 21*a^2*b*e + 25*a^3*f)*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-35*b^3*
c + 63*a*b^2*d - 99*a^2*b*e + 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
```

```
(15/2))
```

Maple [A] time = 0.014, size = 394, normalized size = 1.4

$$\frac{x^7 e}{7 b^3}+\frac{x^5 d}{5 b^3}+\frac{x^3 c}{3 b^3}+\frac{15 a^4 d x}{8 b^5 \left(b x^2+a\right)^2}+\frac{25 a^5 x^3 f}{8 b^6 \left(b x^2+a\right)^2}+\frac{f x^9}{9 b^3}-\frac{11 a^3 c x}{8 b^4 \left(b x^2+a\right)^2}-\frac{143 a^5 f}{8 b^7} \arctan\left(b x \frac{1}{\sqrt{a b}}\right) \frac{1}{\sqrt{a b}}+\frac{99 a}{8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/7/b^3*x^7*e+1/5/b^3*x^5*d+1/3/b^3*x^3*c+15/8*a^4/b^5/(b*x^2+a)^2*d*x+25/8
*a^5/b^6/(b*x^2+a)^2*x^3*f+1/9*f*x^9/b^3-11/8*a^3/b^4/(b*x^2+a)^2*c*x-143/8
*a^5/b^7/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f+99/8*a^4/b^6/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*e-63/8*a^3/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+3
5/8*a^2/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c-21/8*a^4/b^5/(b*x^2+a)^2*
x^3*e+17/8*a^3/b^4/(b*x^2+a)^2*x^3*d-13/8*a^2/b^3/(b*x^2+a)^2*x^3*c+23/8*a^
6/b^7/(b*x^2+a)^2*f*x-19/8*a^5/b^6/(b*x^2+a)^2*e*x-10/3/b^6*x^3*a^3*f+2/b^5
*x^3*a^2*e-1/b^4*x^3*a*d+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*
a*c*x-3/7/b^4*x^7*a*f+6/5/b^5*x^5*a^2*f-3/5/b^4*x^5*a*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25273, size = 1748, normalized size = 6.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/5040*(560*b^6*f*x^13 + 80*(9*b^6*e - 13*a*b^5*f)*x^11 + 16*(63*b^6*d - 9
9*a*b^5*e + 143*a^2*b^4*f)*x^9 + 48*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e -
 143*a^3*b^3*f)*x^7 - 336*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a
^4*b^2*f)*x^5 - 1050*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*
b*f)*x^3 - 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*
a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*
c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 -
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(35*a^3*b^3*c - 63*a^4*b^2*d + 99
*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7), 1/2520*(280*b^6
*f*x^13 + 40*(9*b^6*e - 13*a*b^5*f)*x^11 + 8*(63*b^6*d - 99*a*b^5*e + 143*a
^2*b^4*f)*x^9 + 24*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e - 143*a^3*b^3*f)*x
^7 - 168*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^5 - 5
25*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^3 + 315*(35
*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*a*b^5*c - 63*a^2*b
^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c - 63*a^3*b^3*d +
 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(
35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8
*x^2 + a^2*b^7)]
```

Sympy [A] time = 16.1898, size = 491, normalized size = 1.71

$$\frac{\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)\log\left(-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16}-\frac{\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)*log(
-b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 - sqrt(
-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)*log(b**7*
sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)/(143
*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 + (x**3*(25*a
**5*b*f - 21*a**4*b**2*e + 17*a**3*b**3*d - 13*a**2*b**4*c) + x*(23*a**6*f
- 19*a**5*b*e + 15*a**4*b**2*d - 11*a**3*b**3*c))/(8*a**2*b**7 + 16*a*b**8*
x**2 + 8*b**9*x**4) + f*x**9/(9*b**3) - x**7*(3*a*f - b*e)/(7*b**4) + x**5*
(6*a**2*f - 3*a*b*e + b**2*d)/(5*b**5) - x**3*(10*a**3*f - 6*a**2*b*e + 3*a
```

```
*b**2*d - b**3*c)/(3*b**6) + x*(15*a**4*f - 10*a**3*b*e + 6*a**2*b**2*d - 3
*a*b**3*c)/b**7
```

Giac [A] time = 1.17982, size = 406, normalized size = 1.41

$$\frac{\left(35\,a^2b^3c - 63\,a^3b^2d - 143\,a^5f + 99\,a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7} - \frac{13\,a^2b^4cx^3 - 17\,a^3b^3dx^3 - 25\,a^5bfx^3 + 21\,a^4b^2x^3e + 11\,a^3b^3c}{8\left(bx^2 + a\right)^2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d - 143*a^5*f + 99*a^4*b*e)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^7) - 1/8*(13*a^2*b^4*c*x^3 - 17*a^3*b^3*d*x^3 - 25*a^5*b
*f*x^3 + 21*a^4*b^2*x^3*e + 11*a^3*b^3*c*x - 15*a^4*b^2*d*x - 23*a^6*f*x +
19*a^5*b*x*e)/((b*x^2 + a)^2*b^7) + 1/315*(35*b^24*f*x^9 - 135*a*b^23*f*x^7
 + 45*b^24*x^7*e + 63*b^24*d*x^5 + 378*a^2*b^22*f*x^5 - 189*a*b^23*x^5*e +
105*b^24*c*x^3 - 315*a*b^23*d*x^3 - 1050*a^3*b^21*f*x^3 + 630*a^2*b^22*x^3*
e - 945*a*b^23*c*x + 1890*a^2*b^22*d*x + 4725*a^4*b^20*f*x - 3150*a^3*b^21*
x*e)/b^27
```

3.134 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=247

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x^3(15a^2be-27a^3f-7ab^2d+3b^3c)}{12ab^5}+\frac{ax(11a^2be-15a^3f-7ab^2d+3b^3c)}{8b^6(a+bx^2)}+\frac{x(13a^2be-21a^3f}{2b}$$

```
[Out] ((3*b^3*c - 7*a*b^2*d + 13*a^2*b*e - 21*a^3*f)*x)/(2*b^6) - ((3*b^3*c - 7*a
*b^2*d + 15*a^2*b*e - 27*a^3*f)*x^3)/(12*a*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^
4) + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(4*a*(a
+ b*x^2)^2) + (a*(3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3*f)*x)/(8*b^6*(a
 + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rubi [A] time = 0.411223, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x^3(15a^2be-27a^3f-7ab^2d+3b^3c)}{12ab^5}+\frac{ax(11a^2be-15a^3f-7ab^2d+3b^3c)}{8b^6(a+bx^2)}+\frac{x(13a^2be-21a^3f}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```



```
[Out] ((3*b^3*c - 7*a*b^2*d + 13*a^2*b*e - 21*a^3*f)*x)/(2*b^6) - ((3*b^3*c - 7*a
*b^2*d + 15*a^2*b*e - 27*a^3*f)*x^3)/(12*a*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^
4) + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(4*a*(a
+ b*x^2)^2) + (a*(3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3*f)*x)/(8*b^6*(a
 + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^5\left(\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^6\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a\left(3b^3c-7ab^2d+11a^2be-15a^3f\right)x}{8b^6\left(a+bx^2\right)} + \int \frac{-a^2\left(3b^3c-7ab^2d+11a^2be-1\right.}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a\left(3b^3c-7ab^2d+11a^2be-15a^3f\right)x}{8b^6\left(a+bx^2\right)} + \int \left(4a\left(3b^3c-7ab^2d+\right.\right.$$

$$= \frac{\left(3b^3c-7ab^2d+13a^2be-21a^3f\right)x}{2b^6} - \frac{\left(3b^3c-7ab^2d+15a^2be-27a^3f\right)x^3}{12ab^5} + \frac{(be-3af)}{5b^4}$$

$$= \frac{\left(3b^3c-7ab^2d+13a^2be-21a^3f\right)x}{2b^6} - \frac{\left(3b^3c-7ab^2d+15a^2be-27a^3f\right)x^3}{12ab^5} + \frac{(be-3af)}{5b^4}$$

Mathematica [A] time = 0.136549, size = 232, normalized size = 0.94

$$\frac{ax\left(17a^2be-21a^3f-13ab^2d+9b^3c\right)}{8b^6\left(a+bx^2\right)} + \frac{a^2x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{4b^6\left(a+bx^2\right)^2} + \frac{x\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{b^6} + \frac{\sqrt{a}\tan^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^7)/(7*b^3) + (a^2*
(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*b^6*(a + b*x^2)^2) + (a*(9*b^3
*c - 13*a*b^2*d + 17*a^2*b*e - 21*a^3*f)*x)/(8*b^6*(a + b*x^2)) + (Sqrt[a]*
(-15*b^3*c + 35*a*b^2*d - 63*a^2*b*e + 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*b^(13/2))
```

Maple [A] time = 0.013, size = 343, normalized size = 1.4

$$\frac{fx^7}{7\,b^3}-\frac{3\,x^5af}{5\,b^4}+\frac{x^5e}{5\,b^3}+2\,\frac{x^3a^2f}{b^5}-\frac{ax^3e}{b^4}+\frac{x^3d}{3\,b^3}-10\,\frac{a^3fx}{b^6}+6\,\frac{a^2ex}{b^5}-3\,\frac{adx}{b^4}+\frac{cx}{b^3}-\frac{21\,a^4x^3f}{8\,b^5\left(bx^2+a\right)^2}+\frac{17\,a^3x^3e}{8\,b^4\left(bx^2+a\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/7*f*x^7/b^3-3/5/b^4*x^5*a*f+1/5/b^3*x^5*e+2/b^5*x^3*a^2*f-1/b^4*x^3*a*e+1
/3/b^3*x^3*d-10/b^6*a^3*f*x+6/b^5*a^2*e*x-3/b^4*a*d*x+1/b^3*c*x-21/8*a^4/b^
5/(b*x^2+a)^2*x^3*f+17/8*a^3/b^4/(b*x^2+a)^2*x^3*e-13/8*a^2/b^3/(b*x^2+a)^2
*x^3*d+9/8*a/b^2/(b*x^2+a)^2*x^3*c-19/8*a^5/b^6/(b*x^2+a)^2*f*x+15/8*a^4/b^
5/(b*x^2+a)^2*e*x-11/8*a^3/b^4/(b*x^2+a)^2*d*x+7/8*a^2/b^3/(b*x^2+a)^2*c*x+
99/8*a^4/b^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f-63/8*a^3/b^5/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*e+35/8*a^2/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
*d-15/8*a/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30561, size = 1517, normalized size = 6.14

$$\left[\frac{240\,b^5fx^{11}+48\left(7\,b^5e-11\,ab^4f\right)x^9+16\left(35\,b^5d-63\,ab^4e+99\,a^2b^3f\right)x^7+112\left(15\,b^5c-35\,ab^4d+63\,a^2b^3e-99\,a^3b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(240*b^5*f*x^11 + 48*(7*b^5*e - 11*a*b^4*f)*x^9 + 16*(35*b^5*d - 63
*a*b^4*e + 99*a^2*b^3*f)*x^7 + 112*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e -
99*a^3*b^2*f)*x^5 + 350*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*
b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b
^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a
^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sq
rt(-a/b) - a)/(b*x^2 + a)) + 210*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e
- 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(120*b^5*f*x^11 + 2
4*(7*b^5*e - 11*a*b^4*f)*x^9 + 8*(35*b^5*d - 63*a*b^4*e + 99*a^2*b^3*f)*x^7
 + 56*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^5 + 175*(15*a
*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^3 - 105*(15*a^2*b^3*c
- 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b^5*c - 35*a*b^4*d + 63*a^2*b^
3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*
a^4*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(15*a^2*b^3*c - 35*a^
3*b^2*d + 63*a^4*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)]
```

Sympy [A] time = 16.2142, size = 311, normalized size = 1.26

$$-\frac{\sqrt{-\frac{a}{b^{13}}}\left(99a^3f-63a^2be+35ab^2d-15b^3c\right)\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}+\frac{\sqrt{-\frac{a}{b^{13}}}\left(99a^3f-63a^2be+35ab^2d-15b^3c\right)\log\left(b^6\sqrt{-\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d - 15*b**3*c)*log(-b*
*6*sqrt(-a/b**13) + x)/16 + sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*
b**2*d - 15*b**3*c)*log(b**6*sqrt(-a/b**13) + x)/16 - (x**3*(21*a**4*b*f -
17*a**3*b**2*e + 13*a**2*b**3*d - 9*a*b**4*c) + x*(19*a**5*f - 15*a**4*b*e
+ 11*a**3*b**2*d - 7*a**2*b**3*c))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x
**4) + f*x**7/(7*b**3) - x**5*(3*a*f - b*e)/(5*b**4) + x**3*(6*a**2*f - 3*a
*b*e + b**2*d)/(3*b**5) - x*(10*a**3*f - 6*a**2*b*e + 3*a*b**2*d - b**3*c)/
b**6
```

Giac [A] time = 1.21404, size = 338, normalized size = 1.37

$$-\frac{\left(15\,ab^{3}c - 35\,a^{2}b^{2}d - 99\,a^{4}f + 63\,a^{3}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^{6}} + \frac{9\,ab^{4}cx^{3} - 13\,a^{2}b^{3}dx^{3} - 21\,a^{4}bfx^{3} + 17\,a^{3}b^{2}x^{3}e + 7\,a^{2}b^{3}cx - }{8\left(bx^{2}+a\right)^{2}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*a*b^3*c - 35*a^2*b^2*d - 99*a^4*f + 63*a^3*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) + 1/8*(9*a*b^4*c*x^3 - 13*a^2*b^3*d*x^3 - 21*a^4*b*f*x^
3 + 17*a^3*b^2*x^3*e + 7*a^2*b^3*c*x - 11*a^3*b^2*d*x - 19*a^5*f*x + 15*a^4
*b*x*e)/((b*x^2 + a)^2*b^6) + 1/105*(15*b^18*f*x^7 - 63*a*b^17*f*x^5 + 21*b
^18*x^5*e + 35*b^18*d*x^3 + 210*a^2*b^16*f*x^3 - 105*a*b^17*x^3*e + 105*b^1
8*c*x - 315*a*b^17*d*x - 1050*a^3*b^15*f*x + 630*a^2*b^16*x*e)/b^21
```

3.135 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=207

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(9a^2be-13a^3f-5ab^2d+b^3c\right)}{8b^5\left(a+bx^2\right)}-\frac{x\left(13a^2be-25a^3f-5ab^2d+b^3c\right)}{4ab^5}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^2be-6\right.}{8\sqrt{a}b}$$

```
[Out] -((b^3*c - 5*a*b^2*d + 13*a^2*b*e - 25*a^3*f)*x)/(4*a*b^5) + ((b*e - 3*a*f)
*x^3)/(3*b^4) + (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^
5)/(4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*x)/(8*
b^5*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rubi [A] time = 0.333674, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(9a^2be-13a^3f-5ab^2d+b^3c\right)}{8b^5\left(a+bx^2\right)}-\frac{x\left(13a^2be-25a^3f-5ab^2d+b^3c\right)}{4ab^5}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^2be-6\right.}{8\sqrt{a}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -((b^3*c - 5*a*b^2*d + 13*a^2*b*e - 25*a^3*f)*x)/(4*a*b^5) + ((b*e - 3*a*f)
*x^3)/(3*b^4) + (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^
5)/(4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*x)/(8*
b^5*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^3\left(\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^4\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)} + \frac{\int \frac{a\left(b^3c-5ab^2d+9a^2be-13a^3f\right)-2b(}{}}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)} + \frac{\int \left(-2\left(b^3c-5ab^2d+13a^2\right.\right.}{}$$

$$= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{(}{}$$

$$= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{(}{}$$

<u>Mathematica</u> [A] time = 0.159392, size = 176, normalized size = 0.85

$$\frac{x\left(a^2b^2\left(225d-875ex^2+504fx^4\right)-525a^3b\left(e-3fx^2\right)+945a^4f-ab^3\left(45c-375dx^2+280ex^4+72fx^6\right)+b^4x^2\left(8\left(15dx\right.\right.\right.}{120b^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*f - 525*a^3*b*(e - 3*f*x^2) + a^2*b^2*(225*d - 875*e*x^2 + 504*
f*x^4) - a*b^3*(45*c - 375*d*x^2 + 280*e*x^4 + 72*f*x^6) + b^4*x^2*(-75*c +
 8*(15*d*x^2 + 5*e*x^4 + 3*f*x^6))))/(120*b^5*(a + b*x^2)^2) + ((3*b^3*c -
15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]
*b^(11/2))
```

Maple [A] time = 0.013, size = 294, normalized size = 1.4

$$\frac{fx^5}{5\,b^3}-\frac{ax^3f}{b^4}+\frac{x^3e}{3\,b^3}+6\,\frac{a^2fx}{b^5}-3\,\frac{aex}{b^4}+\frac{dx}{b^3}+\frac{17\,x^3a^3f}{8\,b^4\left(bx^2+a\right)^2}-\frac{13\,x^3a^2e}{8\,b^3\left(bx^2+a\right)^2}+\frac{9\,ax^3d}{8\,b^2\left(bx^2+a\right)^2}-\frac{5\,x^3c}{8\,b\left(bx^2+a\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/5*f*x^5/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e+6/b^5*a^2*f*x-3/b^4*a*e*x+1/b^3*d
*x+17/8/b^4/(b*x^2+a)^2*x^3*a^3*f-13/8/b^3/(b*x^2+a)^2*x^3*a^2*e+9/8/b^2/(b
*x^2+a)^2*x^3*a*d-5/8/b/(b*x^2+a)^2*x^3*c+15/8/b^5/(b*x^2+a)^2*a^4*f*x-11/8
/b^4/(b*x^2+a)^2*a^3*e*x+7/8/b^3/(b*x^2+a)^2*a^2*d*x-3/8/b^2/(b*x^2+a)^2*a*
c*x-63/8/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^3*f+35/8/b^4/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*a^2*e-15/8/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
*a*d+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3148, size = 1362, normalized size = 6.58

$$\left[\frac{48\,ab^5fx^9+16\left(5\,ab^5e-9\,a^2b^4f\right)x^7+16\left(15\,ab^5d-35\,a^2b^4e+63\,a^3b^3f\right)x^5-50\left(3\,ab^5c-15\,a^2b^4d+35\,a^3b^3e-63\,a^4b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*a*b^5*f*x^9 + 16*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 16*(15*a*b^5*d
- 35*a^2*b^4*e + 63*a^3*b^3*f)*x^5 - 50*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*
b^3*e - 63*a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 6
3*a^5*f + (3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a
*b^4*c - 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x
^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(3*a^2*b^4*c - 15*a^3*b^3*d + 35
*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6), 1/120*(2
4*a*b^5*f*x^9 + 8*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 8*(15*a*b^5*d - 35*a^2*b^
4*e + 63*a^3*b^3*f)*x^5 - 25*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*b^3*e - 63*
a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 63*a^5*f + (
3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c - 15
*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) - 15*(3*a^2*b^4*c - 15*a^3*b^3*d + 35*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x
^4 + 2*a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [A] time = 15.8714, size = 279, normalized size = 1.35

$$\frac{\sqrt{-\frac{1}{ab^{11}}}\left(63a^3f-35a^2be+15ab^2d-3b^3c\right)\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{ab^{11}}}\left(63a^3f-35a^2be+15ab^2d-3b^3c\right)\log\left(ab^5\sqrt{\right.}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**3*c)*log(-
a*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2
*b*e + 15*a*b**2*d - 3*b**3*c)*log(a*b**5*sqrt(-1/(a*b**11)) + x)/16 + (x**
3*(17*a**3*b*f - 13*a**2*b**2*e + 9*a*b**3*d - 5*b**4*c) + x*(15*a**4*f - 1
1*a**3*b*e + 7*a**2*b**2*d - 3*a*b**3*c))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8
*b**7*x**4) + f*x**5/(5*b**3) - x**3*(3*a*f - b*e)/(3*b**4) + x*(6*a**2*f -
 3*a*b*e + b**2*d)/b**5
```

Giac [A] time = 1.20482, size = 270, normalized size = 1.3

$$\frac{\left(3\,b^3c-15\,ab^2d-63\,a^3f+35\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5}-\frac{5\,b^4cx^3-9\,ab^3dx^3-17\,a^3bfx^3+13\,a^2b^2x^3e+3\,ab^3cx-7\,a^2b^2dx-}{8\left(bx^2+a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*c - 15*a*b^2*d - 63*a^3*f + 35*a^2*b*e)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^5) - 1/8*(5*b^4*c*x^3 - 9*a*b^3*d*x^3 - 17*a^3*b*f*x^3 + 13*a^2*
b^2*x^3*e + 3*a*b^3*c*x - 7*a^2*b^2*d*x - 15*a^4*f*x + 11*a^3*b*x*e)/((b*x^
2 + a)^2*b^5) + 1/15*(3*b^12*f*x^5 - 15*a*b^11*f*x^3 + 5*b^12*x^3*e + 15*b^
12*d*x + 90*a^2*b^10*f*x - 45*a*b^11*x*e)/b^15
```

3.136 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=167

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(-7a^2be+11a^3f+3ab^2d+b^3c\right)}{8ab^4\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^2be+35a^3f+3ab^2d+b^3c\right)}{8a^{3/2}b^{9/2}}+\frac{x(be-3a}{b^4}$$

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x^3)/(4*a*(a + b*x^2)^2) - ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f
)*x)/(8*a*b^4*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Rubi [A] time = 0.262115, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1153, 205}

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(-7a^2be+11a^3f+3ab^2d+b^3c\right)}{8ab^4\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^2be+35a^3f+3ab^2d+b^3c\right)}{8a^{3/2}b^{9/2}}+\frac{x(be-3a}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```


```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x^3)/(4*a*(a + b*x^2)^2) - ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f
)*x)/(8*a*b^4*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x\left(-\left(bc+3ad-\frac{3a^2e}{b}+\frac{3a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^2\left(-bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int \frac{b^3c+3ab^2d-7a^2be+11a^3f+8ab(b}{a+bx^2}}{8ab^4}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int \left(8a(be-3af)+8abfx^2\right.}{8a}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\left(b^3c\right.}{}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\left(b^3c\right.}{}$$

<u>Mathematica</u> [A] time = 0.120743, size = 156, normalized size = 0.93

$$\frac{x\left(a^2b^2\left(-9d+75ex^2-56fx^4\right)+5a^3b\left(9e-35fx^2\right)-105a^4f+ab^3\left(-3c-15dx^2+24ex^4+8fx^6\right)+3b^4cx^2\right)}{24ab^4\left(a+bx^2\right)^2} + \frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(-105*a^4*f + 3*b^4*c*x^2 + 5*a^3*b*(9*e - 35*f*x^2) + a^2*b^2*(-9*d + 7
5*e*x^2 - 56*f*x^4) + a*b^3*(-3*c - 15*d*x^2 + 24*e*x^4 + 8*f*x^6)))/(24*a*
b^4*(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Maple [A] time = 0.011, size = 259, normalized size = 1.6

$$\frac{fx^3}{3b^3} - 3\frac{afx}{b^4} + \frac{ex}{b^3} - \frac{13x^3a^2f}{8b^3(bx^2+a)^2} + \frac{9ax^3e}{8b^2(bx^2+a)^2} - \frac{5x^3d}{8b(bx^2+a)^2} + \frac{x^3c}{8(bx^2+a)^2a} - \frac{11a^3fx}{8b^4(bx^2+a)^2} + \frac{7a^2ex}{8b^3(bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/3*f*x^3/b^3-3/b^4*a*f*x+1/b^3*x*e-13/8/b^3/(b*x^2+a)^2*x^3*a^2*f+9/8/b^2/
(b*x^2+a)^2*x^3*a*e-5/8/b/(b*x^2+a)^2*x^3*d+1/8/(b*x^2+a)^2/a*x^3*c-11/8/b^
4/(b*x^2+a)^2*a^3*f*x+7/8/b^3/(b*x^2+a)^2*a^2*e*x-3/8/b^2/(b*x^2+a)^2*a*d*x
-1/8/b/(b*x^2+a)^2*c*x+35/8/b^4*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f-1
5/8/b^3*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e+3/8/b^2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2))*d+1/8/b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5005, size = 1195, normalized size = 7.16

$$\left[\frac{16a^2b^4fx^7 + 16\left(3a^2b^4e - 7a^3b^3f\right)x^5 + 2\left(3ab^5c - 15a^2b^4d + 75a^3b^3e - 175a^4b^2f\right)x^3 - 3\left(a^2b^3c + 3a^3b^2d - 15a^4be\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^2*b^4*f*x^7 + 16*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + 2*(3*a*b^5*c
 - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 - 3*(a^2*b^3*c + 3*a^3*
```

```
b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3*
b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sqr
t(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a^2*b^4*c + 3*a^
3*b^3*d - 15*a^4*b^2*e + 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5), 1/24*(8*a^2*b^4*f*x^7 + 8*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + (3*a*b^5*
c - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 + 3*(a^2*b^3*c + 3*a^3
*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3
*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(a^2*b^4*c + 3*a^3*b^3*d - 15*a^4*b^2*e +
 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [A] time = 12.9172, size = 258, normalized size = 1.54

$$-\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be+3ab^2d+b^3c\right)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be+3ab^2d+b^3c\right)\log\left(a^2b^4\sqrt{\cdot}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(-
a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + sqrt(-1/(a**3*b**9))*(35*a**3*f -
15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/
16 - (x**3*(13*a**3*b*f - 9*a**2*b**2*e + 5*a*b**3*d - b**4*c) + x*(11*a**4
*f - 7*a**3*b*e + 3*a**2*b**2*d + a*b**3*c))/(8*a**3*b**4 + 16*a**2*b**5*x*
*2 + 8*a*b**6*x**4) + f*x**3/(3*b**3) - x*(3*a*f - b*e)/b**4
```

Giac [A] time = 1.22776, size = 234, normalized size = 1.4

$$\frac{\left(b^3c+3\,ab^2d+35\,a^3f-15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4}+\frac{b^4cx^3-5\,ab^3dx^3-13\,a^3bfx^3+9\,a^2b^2x^3e-ab^3cx-3\,a^2b^2dx-11\,a^4fx}{8\left(bx^2+a\right)^2ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*c + 3*a*b^2*d + 35*a^3*f - 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a*b^4) + 1/8*(b^4*c*x^3 - 5*a*b^3*d*x^3 - 13*a^3*b*f*x^3 + 9*a^2*b^2*
```

```
x^3*e - a*b^3*c*x - 3*a^2*b^2*d*x - 11*a^4*f*x + 7*a^3*b*x*e)/((b*x^2 + a)^
2*a*b^4) + 1/3*(b^6*f*x^3 - 9*a*b^5*f*x + 3*b^6*x*e)/b^9
```

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=147

$$\frac{x\left(-5a^2be+9a^3f+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be-15a^3f+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b*x^2)^2) +
 ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b^3*(a + b*x^2)) + ((
3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(7/2))
```

Rubi [A] time = 0.150206, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1814, 1157, 388, 205}

$$\frac{x\left(-5a^2be+9a^3f+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be-15a^3f+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b*x^2)^2) +
 ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b^3*(a + b*x^2)) + ((
3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(7/2))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3}\,dx=\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-\frac{3b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{4a(be-af)x^2}{b^2}-\frac{4afx^4}{b}}{\left(a+bx^2\right)^2}\,dx}{4a}$$

$$=\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\int \frac{\frac{3b^3c+ab^2d+3a^2be-7a^3f}{b^3}+\frac{8a^2fx^2}{b^2}}{a+bx^2}\,dx}{8a^2}$$

$$=\frac{fx}{b^3}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\left(3b^3c+ab^2d+3a^2be-15\cdots\right.}{8a^2b^3}$$

$$=\frac{fx}{b^3}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\left(3b^3c+ab^2d+3a^2be-15\cdots\right.}{8a^{5/2}b^{7/2}\cdots}$$

Mathematica [A] time = 0.114484, size = 141, normalized size = 0.96

$$\frac{x\left(-a^2b^2\left(d+5ex^2-8fx^4\right)+a^3b\left(25fx^2-3e\right)+15a^4f+ab^3\left(5c+dx^2\right)+3b^4cx^2\right)}{8a^2b^3\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be-15a^3f+ab^2d\right.}{8a^{5/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^4*f + 3*b^4*c*x^2 + a*b^3*(5*c + d*x^2) + a^3*b*(-3*e + 25*f*x^2)
- a^2*b^2*(d + 5*e*x^2 - 8*f*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*b^3*c +
 a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^
(7/2))
```

Maple [A] time = 0.01, size = 234, normalized size = 1.6

$$\frac{fx}{b^3}+\frac{9\,ax^3f}{8\,b^2\left(bx^2+a\right)^2}-\frac{5\,x^3e}{8\,b\left(bx^2+a\right)^2}+\frac{x^3d}{8\left(bx^2+a\right)^2a}+\frac{3\,bx^3c}{8\left(bx^2+a\right)^2a^2}+\frac{7\,a^2fx}{8\,b^3\left(bx^2+a\right)^2}-\frac{3\,aex}{8\,b^2\left(bx^2+a\right)^2}-\frac{dx}{8\,b\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] f*x/b^3+9/8/b^2/(b*x^2+a)^2*x^3*a*f-5/8/b/(b*x^2+a)^2*x^3*e+1/8/(b*x^2+a)^2
/a*x^3*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c+7/8/b^3/(b*x^2+a)^2*a^2*f*x-3/8/b^2/(b
*x^2+a)^2*a*e*x-1/8/b/(b*x^2+a)^2*d*x+5/8/(b*x^2+a)^2*x/a*c-15/8/b^3*a/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))*f+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*e+1/8/b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+3/8/a^2/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50816, size = 1062, normalized size = 7.22

$$\left[\frac{16\,a^3b^3fx^5+2\left(3\,ab^5c+a^2b^4d-5\,a^3b^3e+25\,a^4b^2f\right)x^3+\left(3\,a^2b^3c+a^3b^2d+3\,a^4be-15\,a^5f+\left(3\,b^5c+ab^4d+3\,a^2b^3e\right.\right.}{16\left(a^3b^6x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*f*x^5 + 2*(3*a*b^5*c + a^2*b^4*d - 5*a^3*b^3*e + 25*a^4*b
^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*e - 15*a^5*f + (3*b^5*c + a*
b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c + a^2*b^3*d + 3*a^3*
b^2*e - 15*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 2*(5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^6
*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*f*x^5 + (3*a*b^5*c + a^2*b^
4*d - 5*a^3*b^3*e + 25*a^4*b^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*
e - 15*a^5*f + (3*b^5*c + a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*
a*b^4*c + a^2*b^3*d + 3*a^3*b^2*e - 15*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(
a*b)*x/a) + (5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^
6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

Sympy [A] time = 8.28425, size = 243, normalized size = 1.65

$$\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(a^3b^3\sqrt{-\frac{1}{a^5}}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(-a*
*3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*
a**2*b*e - a*b**2*d - 3*b**3*c)*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16
+ (x**3*(9*a**3*b*f - 5*a**2*b**2*e + a*b**3*d + 3*b**4*c) + x*(7*a**4*f -
3*a**3*b*e - a**2*b**2*d + 5*a*b**3*c))/(8*a**4*b**3 + 16*a**3*b**4*x**2 +
```

```
8*a**2*b**5*x**4) + f*x/b**3
```

Giac [A] time = 1.16689, size = 201, normalized size = 1.37

$$\frac{fx}{b^3} + \frac{\left(3\,b^3c + ab^2d - 15\,a^3f + 3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{3\,b^4cx^3 + ab^3dx^3 + 9\,a^3bfx^3 - 5\,a^2b^2x^3e + 5\,ab^3cx - a^2b^2dx + 7\,a}{8\left(bx^2 + a\right)^2 a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d - 15*a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c*x^3 + a*b^3*d*x^3 + 9*a^3*b*f*x^3 -
5*a^2*b^2*x^3*e + 5*a*b^3*c*x - a^2*b^2*d*x + 7*a^4*f*x - 3*a^3*b*x*e)/((b*
x^2 + a)^2*a^2*b^3)
```

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{x\left(-a^2be+5a^3f-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be-3a^3f-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}-\frac{c}{a^3x}$$

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a + b*x^2)^2
) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*x^2)) -
 ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(7/2)*b^(5/2))
```

Rubi [A] time = 0.176184, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1805, 1259, 453, 205}

$$-\frac{x\left(-a^2be+5a^3f-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be-3a^3f-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}-\frac{c}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a + b*x^2)^2
) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*x^2)) -
 ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(7/2)*b^(5/2))
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-4c+\left(\frac{3bc}{a}-3d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{4afx^4}{b}}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}+\frac{\int \frac{8ab^2c-\left(7b^3c-3ab^2d-a^2be-3a^3f\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{8a^3b^2}$$

$$= -\frac{c}{a^3x}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}-\frac{\left(15b^3c-3ab^2d-a^2be-3\right.}{8a^3b^2}$$

$$= -\frac{c}{a^3x}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}-\frac{\left(15b^3c-3ab^2d-a^2be-3\right.}{8a^{7/2}b^{5/2}}$$

Mathematica [A] time = 0.12391, size = 155, normalized size = 1.01

$$-\frac{x\left(-a^2be+5a^3f-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}+\frac{x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{4a^2b^2\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+3a^3f+3ab^2d-15b^3c\right)}{8a^{7/2}b^{5/2}}-\frac{c}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^2*b^2*(a + b
*x^2)^2) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*
x^2)) + ((-15*b^3*c + 3*a*b^2*d + a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(7/2)*b^(5/2))
```

Maple [A] time = 0.013, size = 237, normalized size = 1.6

$$-\frac{c}{a^3x}-\frac{5\,x^3f}{8\,\left(bx^2+a\right)^2b}+\frac{x^3e}{8\,a\left(bx^2+a\right)^2}+\frac{3\,bx^3d}{8\,a^2\left(bx^2+a\right)^2}-\frac{7\,b^2x^3c}{8\,a^3\left(bx^2+a\right)^2}-\frac{3\,axf}{8\,\left(bx^2+a\right)^2b^2}-\frac{ex}{8\,\left(bx^2+a\right)^2b}+\frac{5}{8\,a\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -c/a^3/x-5/8/(b*x^2+a)^2/b*x^3*f+1/8/a/(b*x^2+a)^2*x^3*e+3/8/a^2/(b*x^2+a)^
2*b*x^3*d-7/8/a^3/(b*x^2+a)^2*b^2*x^3*c-3/8*a/(b*x^2+a)^2/b^2*x*f-1/8/(b*x^
2+a)^2/b*x*e+5/8/a/(b*x^2+a)^2*x*d-9/8/a^2/(b*x^2+a)^2*b*x*c+3/8/b^2/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))*f+1/8/a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
*e+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-15/8/a^3*b/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5269, size = 1076, normalized size = 7.03

$$\left[-\frac{16\,a^3b^3c+2\left(15\,ab^5c-3\,a^2b^4d-a^3b^3e+5\,a^4b^2f\right)x^4+2\left(25\,a^2b^4c-5\,a^3b^3d+a^4b^2e+3\,a^5bf\right)x^2-\left(\left(15\,b^5c-3\,ab^4d-\cdots\right.\right.}{16\left(a^4b^5x^5-\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^3*b^3*c + 2*(15*a*b^5*c - 3*a^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*
f)*x^4 + 2*(25*a^2*b^4*c - 5*a^3*b^3*d + a^4*b^2*e + 3*a^5*b*f)*x^2 - ((15*
b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2*b^
3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*e -
3*a^5*f)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*
b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x), -1/8*(8*a^3*b^3*c + (15*a*b^5*c - 3*a
^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*f)*x^4 + (25*a^2*b^4*c - 5*a^3*b^3*d + a^4
*b^2*e + 3*a^5*b*f)*x^2 + ((15*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)
*x^5 + 2*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b
^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x)]
```

Sympy [A] time = 22.8078, size = 250, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(a^4b^2\sqrt{-\cdots}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**7*b**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(-a
**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b**5))*(3*a**3*f + a*
*2*b*e + 3*a*b**2*d - 15*b**3*c)*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16
 - (8*a**2*b**2*c + x**4*(5*a**3*b*f - a**2*b**2*e - 3*a*b**3*d + 15*b**4*c
) + x**2*(3*a**4*f + a**3*b*e - 5*a**2*b**2*d + 25*a*b**3*c))/(8*a**5*b**2*
```

```
x + 16*a**4*b**3*x**3 + 8*a**3*b**4*x**5)
```

Giac [A] time = 1.17095, size = 207, normalized size = 1.35

$$-\frac{c}{a^3x}-\frac{\left(15\,b^3c-3\,ab^2d-3\,a^3f-a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2}-\frac{7\,b^4cx^3-3\,ab^3dx^3+5\,a^3bfx^3-a^2b^2x^3e+9\,ab^3cx-5\,a^2b^2dx}{8\left(bx^2+a\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -c/(a^3*x) - 1/8*(15*b^3*c - 3*a*b^2*d - 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(7*b^4*c*x^3 - 3*a*b^3*d*x^3 + 5*a^3*b*f*x
^3 - a^2*b^2*x^3*e + 9*a*b^3*c*x - 5*a^2*b^2*d*x + 3*a^4*f*x + a^3*b*x*e)/(
(b*x^2 + a)^2*a^3*b^2)
```

3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=168

$$\frac{x\left(3a^2be+a^3f-7ab^2d+11b^3c\right)}{8a^4b\left(a+bx^2\right)}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be+a^3f-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{3bc-ad}{a^4x}-\frac{3a}{}$$

```
[Out] -c/(3*a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d + 3*a^2*b*e + a^3*f)*x
)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Rubi [A] time = 0.242445, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1805, 1259, 1261, 205}

$$\frac{x\left(3a^2be+a^3f-7ab^2d+11b^3c\right)}{8a^4b\left(a+bx^2\right)}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be+a^3f-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{3bc-ad}{a^4x}-\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```


```
[Out] -c/(3*a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d + 3*a^2*b*e + a^3*f)*x
)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{3b^2c}{a^2}+\frac{3bd}{a}-3e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \frac{\int \frac{-8a^2b^2c+8ab^2(2bc-ad)x^2-b\left(11b^3c-7ab\right.}{x^4\left(a+bx^2\right)}}{8a^4b^2}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \frac{\int \left(-\frac{8ab^2c}{x^4}+\frac{8b^2(3bc-ad)}{x^2}-\frac{b\left(35b^3c-1\right.}{}\right.}{8a^4b^2}$$

$$= -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)}+\frac{\left(35b^3c-1\right.}{}$$

$$= -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)}+\frac{\left(35b^3c-1\right.}{}$$

Mathematica [A] time = 0.137575, size = 169, normalized size = 1.01

$$\frac{a^2b^2x^2\left(56c-75dx^2+9ex^4\right)+a^3b\left(3x^2\left(-8d+5ex^2+fx^4\right)-8c\right)-3a^4fx^4+5ab^3x^4\left(35c-9dx^2\right)+105b^4cx^6}{24a^4bx^3\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-3*a^4*f*x^4 + 105*b^4*c*x^6 + 5*a*b^3*x^4*(35*c - 9*d*x^2) + a^2*b^2*x^2*
(56*c - 75*d*x^2 + 9*e*x^4) + a^3*b*(-8*c + 3*x^2*(-8*d + 5*e*x^2 + f*x^4))
)/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Maple [A] time = 0.016, size = 264, normalized size = 1.6

$$-\frac{c}{3\,a^3x^3}-\frac{d}{a^3x}+3\,\frac{bc}{a^4x}+\frac{x^3f}{8\,a\left(bx^2+a\right)^2}+\frac{3\,x^3be}{8\,a^2\left(bx^2+a\right)^2}-\frac{7\,x^3b^2d}{8\,a^3\left(bx^2+a\right)^2}+\frac{11\,x^3b^3c}{8\,a^4\left(bx^2+a\right)^2}-\frac{fx}{8\,\left(bx^2+a\right)^2b}+\frac{5}{8\,a\left(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x)
```

```
[Out] -1/3*c/a^3/x^3-1/a^3/x*d+3/a^4/x*b*c+1/8/a/(b*x^2+a)^2*x^3*f+3/8/a^2/(b*x^2
+a)^2*x^3*b*e-7/8/a^3/(b*x^2+a)^2*x^3*b^2*d+11/8/a^4/(b*x^2+a)^2*x^3*b^3*c-
1/8/(b*x^2+a)^2/b*x*f+5/8/a/(b*x^2+a)^2*x*e-9/8/a^2/(b*x^2+a)^2*b*x*d+13/8/
a^3/(b*x^2+a)^2*b^2*x*c+1/8/a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f+3/8/a
^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*e-15/8/a^3*b/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*d+35/8/a^4*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51377, size = 1219, normalized size = 7.26

$$\left[-\frac{16\,a^4b^2c-6\left(35\,ab^5c-15\,a^2b^4d+3\,a^3b^3e+a^4b^2f\right)x^6-2\left(175\,a^2b^4c-75\,a^3b^3d+15\,a^4b^2e-3\,a^5bf\right)x^4-16\left(7\,a^3b^3c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^4*b^2*c - 6*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^3*e + a^4*b^2
*f)*x^6 - 2*(175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^4 -
 16*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 + 3*((35*b^5*c - 15*a*b^4*d + 3*a^2*b^3
*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e + a^4*b*f)
*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 +
 a^7*b^2*x^3), -1/24*(8*a^4*b^2*c - 3*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^
3*e + a^4*b^2*f)*x^6 - (175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5
```

```
*b*f)*x^4 - 8*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 - 3*((35*b^5*c - 15*a*b^4*d +
 3*a^2*b^3*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e
+ a^4*b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 + a^7*b^2*x^3)]
```

Sympy [A] time = 58.4529, size = 270, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log\left(a^5b\sqrt{-\ldots}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**9*b**3))*(a**3*f + 3*a**2*b*e - 15*a*b**2*d + 35*b**3*c)*log(-
a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + sqrt(-1/(a**9*b**3))*(a**3*f + 3*a**2
*b*e - 15*a*b**2*d + 35*b**3*c)*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + (
-8*a**3*b*c + x**6*(3*a**3*b*f + 9*a**2*b**2*e - 45*a*b**3*d + 105*b**4*c)
+ x**4*(-3*a**4*f + 15*a**3*b*e - 75*a**2*b**2*d + 175*a*b**3*c) + x**2*(-2
4*a**3*b*d + 56*a**2*b**2*c))/(24*a**6*b*x**3 + 48*a**5*b**2*x**5 + 24*a**4
*b**3*x**7)
```

Giac [A] time = 1.16959, size = 230, normalized size = 1.37

$$\frac{\left(35\,b^3c-15\,ab^2d+a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4b}+\frac{11\,b^4cx^3-7\,ab^3dx^3+a^3bfx^3+3\,a^2b^2x^3e+13\,ab^3cx-9\,a^2b^2dx-a^4f\ldots}{8\left(bx^2+a\right)^2a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^3*c - 15*a*b^2*d + a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4*b) + 1/8*(11*b^4*c*x^3 - 7*a*b^3*d*x^3 + a^3*b*f*x^3 + 3*a^2*b^2*
x^3*e + 13*a*b^3*c*x - 9*a^2*b^2*d*x - a^4*f*x + 5*a^3*b*x*e)/((b*x^2 + a)^
2*a^4*b) + 1/3*(9*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^4*x^3)
```

3.140 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=196

$$-\frac{x\left(7a^2be-3a^3f-11ab^2d+15b^3c\right)}{8a^5\left(a+bx^2\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(15a^2be-3a^3f-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}$$

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d + a^2*e)/(a
^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*(a + b*x^2)^2) - ((1
5*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) - ((63*b
^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(11/2)*Sqrt[b])
```

Rubi [A] time = 0.350203, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1802, 205}

$$-\frac{x\left(7a^2be-3a^3f-11ab^2d+15b^3c\right)}{8a^5\left(a+bx^2\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(15a^2be-3a^3f-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```


```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d + a^2*e)/(a
^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*(a + b*x^2)^2) - ((1
5*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) - ((63*b
^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(11/2)*Sqrt[b])
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{3\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)} + \frac{\int \frac{8c-8\left(\frac{2bc}{a}-d\right)x^2+8\left(\frac{3b^2c}{a^2}-\right.}{}}{}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)} + \frac{\int \left(\frac{8c}{ax^6}+\frac{8(-3bc+ad)}{a^2x^4}+\right.}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{3a^4x^3} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+\right.}{8a^5\left(a\right.}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{3a^4x^3} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+\right.}{8a^5\left(a\right.}$$

Mathematica [A] time = 0.110882, size = 196, normalized size = 1.

$$\frac{x\left(-7a^2be+3a^3f+11ab^2d-15b^3c\right)}{8a^5\left(a+bx^2\right)} + \frac{x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{4a^4\left(a+bx^2\right)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^2be+3a^3f+35ab^2d-63b^3c\right)}{8a^{11/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) + (-6*b^2*c + 3*a*b*d - a^2*e)/(
a^5*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^4*(a + b*x^2)^2) +
 ((-15*b^3*c + 11*a*b^2*d - 7*a^2*b*e + 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) + (
(-63*b^3*c + 35*a*b^2*d - 15*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(11/2)*Sqrt[b])
```

Maple [A] time = 0.016, size = 300, normalized size = 1.5

$$-\frac{c}{5\,a^3x^5}-\frac{d}{3\,a^3x^3}+\frac{bc}{a^4x^3}-\frac{e}{a^3x}+3\,\frac{bd}{a^4x}-6\,\frac{b^2c}{a^5x}+\frac{3\,x^3bf}{8\,a^2\left(bx^2+a\right)^2}-\frac{7\,x^3b^2e}{8\,a^3\left(bx^2+a\right)^2}+\frac{11\,x^3b^3d}{8\,a^4\left(bx^2+a\right)^2}-\frac{15\,x^3b^4c}{8\,a^5\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x)
```

```
[Out] -1/5*c/a^3/x^5-1/3/a^3/x^3*d+1/a^4/x^3*b*c-1/a^3/x*e+3/a^4/x*b*d-6/a^5/x*b^
2*c+3/8/a^2/(b*x^2+a)^2*x^3*b*f-7/8/a^3/(b*x^2+a)^2*x^3*b^2*e+11/8/a^4/(b*x
^2+a)^2*x^3*b^3*d-15/8/a^5/(b*x^2+a)^2*x^3*b^4*c+5/8/a/(b*x^2+a)^2*f*x-9/8/
a^2/(b*x^2+a)^2*b*e*x+13/8/a^3/(b*x^2+a)^2*b^2*d*x-17/8/a^4/(b*x^2+a)^2*b^3
*c*x+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f-15/8/a^3/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*b*e+35/8/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*b^2*d
-63/8/a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*b^3*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54559, size = 1386, normalized size = 7.07

$$\left[-\frac{30\left(63\,ab^5c-35\,a^2b^4d+15\,a^3b^3e-3\,a^4b^2f\right)x^8+48\,a^5bc+50\left(63\,a^2b^4c-35\,a^3b^3d+15\,a^4b^2e-3\,a^5bf\right)x^6+16\left(63\,a^3b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
48*a^5*b*c + 50*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 16*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 16*(9*a^4*b^2*c - 5
*a^5*b*d)*x^2 - 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^
9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*
b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5),
 -1/120*(15*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
24*a^5*b*c + 25*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 8*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 8*(9*a^4*b^2*c - 5*a
^5*b*d)*x^2 + 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^9
+ 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*b^
3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5)]
```

Sympy [A] time = 135.007, size = 284, normalized size = 1.45

$$-\frac{\sqrt{-\frac{1}{a^{11}b}}\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^{11}b}}\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)\log\left(a^6\sqrt{\right.}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**11*b))*(3*a**3*f - 15*a**2*b*e + 35*a*b**2*d - 63*b**3*c)*log(
-a**6*sqrt(-1/(a**11*b)) + x)/16 + sqrt(-1/(a**11*b))*(3*a**3*f - 15*a**2*b
*e + 35*a*b**2*d - 63*b**3*c)*log(a**6*sqrt(-1/(a**11*b)) + x)/16 + (-24*a*
*4*c + x**8*(45*a**3*b*f - 225*a**2*b**2*e + 525*a*b**3*d - 945*b**4*c) + x
**6*(75*a**4*f - 375*a**3*b*e + 875*a**2*b**2*d - 1575*a*b**3*c) + x**4*(-1
20*a**4*e + 280*a**3*b*d - 504*a**2*b**2*c) + x**2*(-40*a**4*d + 72*a**3*b*
```

```
c))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5*b**2*x**9)
```

Giac [A] time = 1.23475, size = 267, normalized size = 1.36

$$-\frac{\left(63\,b^3c-35\,ab^2d-3\,a^3f+15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}-\frac{15\,b^4cx^3-11\,ab^3dx^3-3\,a^3bfx^3+7\,a^2b^2x^3e+17\,ab^3cx-13\,a^2b^2}{8\left(bx^2+a\right)^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*b^3*c - 35*a*b^2*d - 3*a^3*f + 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) - 1/8*(15*b^4*c*x^3 - 11*a*b^3*d*x^3 - 3*a^3*b*f*x^3 + 7*a^2
*b^2*x^3*e + 17*a*b^3*c*x - 13*a^2*b^2*d*x - 5*a^4*f*x + 9*a^3*b*x*e)/((b*x
^2 + a)^2*a^5) - 1/15*(90*b^2*c*x^4 - 45*a*b*d*x^4 + 15*a^2*x^4*e - 15*a*b*
c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^5*x^5)
```

3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=234

$$\frac{bx\left(11a^2be-7a^3f-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{a^6x}+\frac{\sqrt{b}\tan^{-1}\left(\right.}{}$$

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15*a
*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c
- 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13
/2))
```

Rubi [A] time = 0.486456, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1802, 205}

$$\frac{bx\left(11a^2be-7a^3f-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{a^6x}+\frac{\sqrt{b}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15*a
*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c
- 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13
/2))
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{3b\left(b^3c-ab^2d+a^2\right.}{a^4}}{x^8\left(a+bx^2\right)^2}}{4a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \frac{\int \frac{8c-8\left(\frac{2bc}{a}-d\right)x^2+8\left(\right.}{}}{}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \frac{\int \left(\frac{8c}{ax^8}+\frac{8(-3bc+aa}{a^2x^6}\right.}{}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2d+\right.}{4a^5\left(a-\right.}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2d+\right.}{4a^5\left(a-\right.}$$

Mathematica [A] time = 0.13587, size = 234, normalized size = 1.

$$\frac{bx\left(11a^2be-7a^3f-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)} + \frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2} + \frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{a^6x} + \frac{\sqrt{b}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15*a
*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c
- 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13
/2))
```

Maple [A] time = 0.016, size = 351, normalized size = 1.5

$$-\frac{c}{7\,a^{3}x^{7}}-\frac{d}{5\,a^{3}x^{5}}+\frac{3\,bc}{5\,a^{4}x^{5}}-\frac{e}{3\,a^{3}x^{3}}+\frac{bd}{a^{4}x^{3}}-2\,\frac{b^{2}c}{a^{5}x^{3}}-\frac{f}{a^{3}x}+3\,\frac{be}{a^{4}x}-6\,\frac{b^{2}d}{a^{5}x}+10\,\frac{b^{3}c}{a^{6}x}-\frac{7\,b^{2}x^{3}f}{8\,a^{3}\left(bx^{2}+a\right)^{2}}+\frac{11\,b^{3}x^{3}e}{8\,a^{4}\left(bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x)
```

```
[Out] -1/7*c/a^3/x^7-1/5/a^3/x^5*d+3/5/a^4/x^5*b*c-1/3/a^3/x^3*e+1/a^4/x^3*b*d-2/
a^5/x^3*b^2*c-1/a^3/x*f+3/a^4/x*b*e-6/a^5/x*b^2*d+10/a^6/x*b^3*c-7/8/a^3*b^
2/(b*x^2+a)^2*x^3*f+11/8/a^4*b^3/(b*x^2+a)^2*x^3*e-15/8/a^5*b^4/(b*x^2+a)^2
*x^3*d+19/8/a^6*b^5/(b*x^2+a)^2*x^3*c-9/8/a^2*b/(b*x^2+a)^2*f*x+13/8/a^3*b^
2/(b*x^2+a)^2*e*x-17/8/a^4*b^3/(b*x^2+a)^2*d*x+21/8/a^5*b^4/(b*x^2+a)^2*c*x
-15/8/a^3*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*f+35/8/a^4*b^2/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))*e-63/8/a^5*b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
d+99/8/a^6*b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46752, size = 1539, normalized size = 6.58

$$\left[\frac{210\left(99\,b^5c - 63\,ab^4d + 35\,a^2b^3e - 15\,a^3b^2f\right)x^{10} + 350\left(99\,ab^4c - 63\,a^2b^3d + 35\,a^3b^2e - 15\,a^4bf\right)x^8 + 112\left(99\,a^2b^3c - 6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(210*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 3
50*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 112*(99*a^
2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 240*a^5*c - 16*(99*a^
3*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 48*(11*a^4*b*c - 7*a^5*d)*x^2 - 105*
((99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c
 - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7), 1/840*(105*(99*
b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175*(99*a*b^4*c -
63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^2*c - 63*a^4*b*
d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2 + 105*((99*b^5*c - 63*a*b
^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c - 63*a^2*b^3*d + 3
5*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e -
 15*a^5*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9
+ a^8*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21301, size = 338, normalized size = 1.44

$$\frac{\left(99\,b^4c-63\,ab^3d-15\,a^3bf+35\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6}+\frac{19\,b^5cx^3-15\,ab^4dx^3-7\,a^3b^2fx^3+11\,a^2b^3x^3e+21\,ab^4cx-17\,a^2}{8\left(bx^2+a\right)^2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(99*b^4*c - 63*a*b^3*d - 15*a^3*b*f + 35*a^2*b^2*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*c*x^3 - 15*a*b^4*d*x^3 - 7*a^3*b^2*f*x^3 +
 11*a^2*b^3*x^3*e + 21*a*b^4*c*x - 17*a^2*b^3*d*x - 9*a^4*b*f*x + 13*a^3*b^
2*x*e)/((b*x^2 + a)^2*a^6) + 1/105*(1050*b^3*c*x^6 - 630*a*b^2*d*x^6 - 105*
a^3*f*x^6 + 315*a^2*b*x^6*e - 210*a*b^2*c*x^4 + 105*a^2*b*d*x^4 - 35*a^3*x^
4*e + 63*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^6*x^7)
```

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=277

$$-\frac{b^2x\left(15a^2be-11a^3f-19ab^2d+23b^3c\right)}{8a^7\left(a+bx^2\right)}-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^6\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}-\frac{b(6a...}{...}$$

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/(5
*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) - (b*(15
*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x)/(4*a^6*(a + b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*d
+ 15*a^2*b*e - 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 99*
a*b^2*d + 63*a^2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2))
```

Rubi [A] time = 0.603401, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1805, 1802, 205}

$$-\frac{b^2x\left(15a^2be-11a^3f-19ab^2d+23b^3c\right)}{8a^7\left(a+bx^2\right)}-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^6\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}-\frac{b(6a...}{...}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```



```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/(5
*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) - (b*(15
*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x)/(4*a^6*(a + b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*d
+ 15*a^2*b*e - 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 99*
a*b^2*d + 63*a^2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2))
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{4b\left(b^3c-ab^2d+a\cdots\right.}{a^4}}{x^{10}\left(a+bx^2\right)^2}}{4a}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2\left(23b^3c-19ab^2d+15a^2be-11a^3f\right)x}{8a^7\left(a+bx^2\right)} + \frac{\int \frac{8c-8\left(\frac{2bc}{a}-d\right)x^2\cdots}{}}{}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2\left(23b^3c-19ab^2d+15a^2be-11a^3f\right)x}{8a^7\left(a+bx^2\right)} + \frac{\int\left(\frac{8c}{ax^{10}}+\frac{8(-3b\cdots}{a^2}\right.}{}$$

$$= -\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}-\frac{b\left(15b^3c-10ab^2\cdots\right.}{a\cdots}$$

$$= -\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}-\frac{b\left(15b^3c-10ab^2\cdots\right.}{a\cdots}$$

Mathematica [A] time = 0.148101, size = 276, normalized size = 1.

$$\frac{b^2x\left(-15a^2be+11a^3f+19ab^2d-23b^3c\right)}{8a^7\left(a+bx^2\right)}+\frac{b^2x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{4a^6\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}+\frac{b\left(-6a\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/(5
*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(-1
5*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x)/(4*a^6*(a + b*x^2)^2) + (b^2*(-23*b^3*c + 19*a*b
^2*d - 15*a^2*b*e + 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-143*b^3*c
 + 99*a*b^2*d - 63*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(1
5/2))
```

Maple [A] time = 0.02, size = 401, normalized size = 1.5

$$-\frac{143\,b^5c}{8\,a^7}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{c}{9\,a^3x^9}-\frac{25\,b^5cx}{8\,a^6\left(bx^2+a\right)^2}+\frac{99\,b^4d}{8\,a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{17\,b^3ex}{8\,a^4\left(bx^2+a\right)^2}+\frac{21\,b\cdots}{8\,a^5\,(bx\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x)
```

```
[Out] -143/8/a^7*b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c-1/9*c/a^3/x^9-25/8/a^6
*b^5/(b*x^2+a)^2*c*x+99/8/a^6*b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d-17/
8/a^4*b^3/(b*x^2+a)^2*e*x+21/8/a^5*b^4/(b*x^2+a)^2*d*x+35/8/a^4*b^2/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))*f-63/8/a^5*b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2))*e+3/7/a^4/x^7*b*c+3/5/a^4/x^5*b*d-6/5/a^5/x^5*b^2*c+1/a^4/x^3*b*e-2/a^
5/x^3*b^2*d+10/3/a^6/x^3*b^3*c+3*b/a^4/x*f-6*b^2/a^5/x*e+10*b^3/a^6/x*d-15*
b^4/a^7/x*c-1/5/a^3/x^5*e-1/3/a^3/x^3*f-1/7/a^3/x^7*d+11/8/a^4*b^3/(b*x^2+a
)^2*x^3*f-23/8/a^7*b^6/(b*x^2+a)^2*x^3*c+13/8/a^3*b^2/(b*x^2+a)^2*f*x+19/8/
a^6*b^5/(b*x^2+a)^2*x^3*d-15/8/a^5*b^4/(b*x^2+a)^2*x^3*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3172, size = 1778, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/5040*(630*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
 1050*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 336
*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 560*a^6*c
 - 48*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 16*(143*
a^4*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 80*(13*a^5*b*c - 9*a^6*d)*x^2 + 31
5*((143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b
^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c -
99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9), -1
/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 + 52
5*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(14
3*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c - 2
4*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4*b
^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2 + 315*((1
43*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b^5*c
- 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c - 99*a^
3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21618, size = 406, normalized size = 1.47

$$-\frac{\left(143\,b^5c-99\,ab^4d-35\,a^3b^2f+63\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^7}-\frac{23\,b^6cx^3-19\,ab^5dx^3-11\,a^3b^3fx^3+15\,a^2b^4x^3e+25\,ab^5cx}{8\left(bx^2+a\right)^2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(143*b^5*c - 99*a*b^4*d - 35*a^3*b^2*f + 63*a^2*b^3*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^7) - 1/8*(23*b^6*c*x^3 - 19*a*b^5*d*x^3 - 11*a^3*b^3*f*
x^3 + 15*a^2*b^4*x^3*e + 25*a*b^5*c*x - 21*a^2*b^4*d*x - 13*a^4*b^2*f*x + 1
7*a^3*b^3*x*e)/((b*x^2 + a)^2*a^7) - 1/315*(4725*b^4*c*x^8 - 3150*a*b^3*d*x
^8 - 945*a^3*b*f*x^8 + 1890*a^2*b^2*x^8*e - 1050*a*b^3*c*x^6 + 630*a^2*b^2*
d*x^6 + 105*a^4*f*x^6 - 315*a^3*b*x^6*e + 378*a^2*b^2*c*x^4 - 189*a^3*b*d*x
^4 + 63*a^4*x^4*e - 135*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^7*x^9)
```

3.143 $\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be+a^3(-f)\right.}{b^6}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rubi [A] time = 0.251575, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1799, 1620}

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be+a^3(-f)\right)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5\sqrt{a+bx}}+\frac{a\left(-2b^3c+3ab^2d-4a^2be+5a^3f\right)\sqrt{}}{b^5}\right.\right.$$
$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^6}-\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)\left(a+bx^2\right)^{3/2}}{3b^6}$$

Mathematica [A] time = 0.165804, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(8a^2b^3\left(231c+99dx^2+66ex^4+50fx^6\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+128a^4b\left(11e+5fx^2\right)-1280a^5f-\right.}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

Maple [A] time = 0.008, size = 193, normalized size = 0.9

$$-\frac{-315\,fx^{10}b^5+350\,ab^4fx^8-385\,b^5ex^8-400\,a^2b^3fx^6+440\,ab^4ex^6-495\,b^5dx^6+480\,a^3b^2fx^4-528\,a^2b^3ex^4+594\,ab}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*e*x^8-400*
a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2*f*x^4-528*a^2*b^3*e
```

```
*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*f*x^2+704*a^3*b^2*e*x^2-792*a^
2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2
*b^3*c)/b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31254, size = 433, normalized size = 2.02

$$\frac{\left(315\,b^5fx^{10}+35\left(11\,b^5e-10\,ab^4f\right)x^8+5\left(99\,b^5d-88\,ab^4e+80\,a^2b^3f\right)x^6+1848\,a^2b^3c-1584\,a^3b^2d+1408\,a^4be-1280}{3465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

Sympy [A] time = 4.4592, size = 442, normalized size = 2.07

$$\begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6}+\frac{128a^4e\sqrt{a+bx^2}}{315b^5}+\frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5}-\frac{16a^3d\sqrt{a+bx^2}}{35b^4}-\frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4}-\frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4}+\frac{8a^2c\sqrt{a+bx^2}}{15b^3}+\frac{8a^2dx^2\sqrt{a+bx^2}}{35b^3}+ \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

Giac [A] time = 1.21487, size = 387, normalized size = 1.81

$$693\left(bx^2+a\right)^{\frac{5}{2}}b^3c - 2310\left(bx^2+a\right)^{\frac{3}{2}}ab^3c + 3465\sqrt{bx^2+a}a^2b^3c + 495\left(bx^2+a\right)^{\frac{7}{2}}b^2d - 2079\left(bx^2+a\right)^{\frac{5}{2}}ab^2d + 3465\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*(693*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 3465
*sqrt(b*x^2 + a)*a^2*b^3*c + 495*(b*x^2 + a)^(7/2)*b^2*d - 2079*(b*x^2 + a)
^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*b^2*d - 3465*sqrt(b*x^2 + a)*a^
3*b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x
^2 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)
*a^4*f - 3465*sqrt(b*x^2 + a)*a^5*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x
^2 + a)^(7/2)*a*b*e + 4158*(b*x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/
2)*a^3*b*e + 3465*sqrt(b*x^2 + a)*a^4*b*e)/b^6
```

3.144 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{(a+bx^2)^{3/2}(3a^2be-4a^3f-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}\,(a^2be+a^3(-f)-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{5/2}(6a^2f-3abe+b^2d)}{5b^5}$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rubi [A] time = 0.194013, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1799, 1620}

$$\frac{(a+bx^2)^{3/2}(3a^2be-4a^3f-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}\,(a^2be+a^3(-f)-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{5/2}(6a^2f-3abe+b^2d)}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4\sqrt{a+bx}} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\sqrt{a+bx}}{b^4}\right.\right.$$
$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^5} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\left(a+bx^2\right)^{3/2}}{3b^5} +$$

Mathematica [A] time = 0.121023, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(24a^2b^2\left(7d+3ex^2+2fx^4\right)-16a^3b\left(9e+4fx^2\right)+128a^4f-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)+b^4x^2\left(10\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

Maple [A] time = 0.007, size = 145, normalized size = 0.9

$$\frac{35\,fx^8b^4-40\,ab^3fx^6+45\,b^4ex^6+48\,a^2b^2fx^4-54\,ab^3ex^4+63\,b^4dx^4-64\,a^3bfx^2+72\,a^2b^2ex^2-84\,ab^3dx^2+105\,b^4cx}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+48*a^2*b^2*
f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2*b^2*e*x^2-84*a*b^3*
d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168*a^2*b^2*d-210*a*b^3*c)/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35754, size = 316, normalized size = 1.89

$$\frac{\left(35\,b^4fx^8+5\left(9\,b^4e-8\,ab^3f\right)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\left(21\,b^4d-18\,ab^3e+16\,a^2b^2f\right)x^4+\left(105\right.}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

Sympy [A] time = 2.7518, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5}-\frac{16a^3e\sqrt{a+bx^2}}{35b^4}-\frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4}+\frac{8a^2d\sqrt{a+bx^2}}{15b^3}+\frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3}+\frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3}-\frac{2ac\sqrt{a+bx^2}}{3b^2}-\frac{4adx^2\sqrt{a+bx^2}}{15b^2}-\frac{6aex^4\sqrt{a+}}{35b^2} \\ \frac{\frac{cx^4}{4}+\frac{dx^6}{6}+\frac{ex^8}{8}+\frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
```

```
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

Giac [A] time = 1.2444, size = 296, normalized size = 1.77

$$\frac{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c-315\sqrt{bx^2+a}ab^3c+63\left(bx^2+a\right)^{\frac{5}{2}}b^2d-210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d+315\sqrt{bx^2+a}a^2b^2d+35\left(bx^2+a\right)^{\frac{9}{2}}f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(105*(b*x^2 + a)^(3/2)*b^3*c - 315*sqrt(b*x^2 + a)*a*b^3*c + 63*(b*x^
2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)*a*b^2*d + 315*sqrt(b*x^2 + a)*a^
2*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2 +
 a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 315*sqrt(b*x^2 + a)*a^4*f +
 45*(b*x^2 + a)^(7/2)*b*e - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(
3/2)*a^2*b*e - 315*sqrt(b*x^2 + a)*a^3*b*e)/b^5
```

3.145 $\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.133137, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1799, 1850}

$$\frac{\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}} + \frac{\left(b^2d-2abe+3a^2f\right)\sqrt{a+bx}}{b^3} + \frac{(be-3af)(a}{b^3}\right.\right.$$
$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^4} + \frac{\left(b^2d-2abe+3a^2f\right)\left(a+bx^2\right)^{3/2}}{3b^4} + \frac{(be-3af)\left(\right.}{5b}$$

Mathematica [A] time = 0.0823981, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(8a^2b\left(7e+3fx^2\right)-48a^3f-2ab^2\left(35d+14ex^2+9fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

Maple [A] time = 0.005, size = 99, normalized size = 0.8

$$-\frac{-15\,fx^6b^3+18\,ab^2fx^4-21\,b^3ex^4-24\,a^2bfx^2+28\,ab^2ex^2-35\,b^3dx^2+48\,a^3f-56\,a^2be+70\,ab^2d-105\,b^3c}{105\,b^4}\sqrt{bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4-24*a^2*b*
f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)
/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37243, size = 221, normalized size = 1.83

$$\frac{\left(15\,b^3 f x^6 + 3\left(7\,b^3 e - 6\,ab^2 f\right)x^4 + 105\,b^3 c - 70\,ab^2 d + 56\,a^2 be - 48\,a^3 f + \left(35\,b^3 d - 28\,ab^2 e + 24\,a^2 bf\right)x^2\right)\sqrt{bx^2 + a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

Sympy [A] time = 1.6298, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3 f\sqrt{a+bx^2}}{35b^4} + \frac{8a^2 e\sqrt{a+bx^2}}{15b^3} + \frac{8a^2 fx^2\sqrt{a+bx^2}}{35b^3} - \frac{2ad\sqrt{a+bx^2}}{3b^2} - \frac{4aex^2\sqrt{a+bx^2}}{15b^2} - \frac{6afx^4\sqrt{a+bx^2}}{35b^2} + \frac{c\sqrt{a+bx^2}}{b} + \frac{dx^2\sqrt{a+bx^2}}{3b} + \frac{ex^4\sqrt{a+bx^2}}{5b} + \frac{fx^6\sqrt{a+bx^2}}{7b} & \text{for}\: b \neq 0 \\ \frac{\frac{cx^2}{2} + \frac{dx^4}{4} + \frac{ex^6}{6} + \frac{fx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

Giac [A] time = 1.17078, size = 207, normalized size = 1.71

$$\frac{105\sqrt{bx^2+a}b^3c + 35\left(bx^2+a\right)^{\frac{3}{2}}b^2d - 105\sqrt{bx^2+a}ab^2d + 15\left(bx^2+a\right)^{\frac{7}{2}}f - 63\left(bx^2+a\right)^{\frac{5}{2}}af + 105\left(bx^2+a\right)^{\frac{3}{2}}a^2f - 10}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(105*sqrt(b*x^2 + a)*b^3*c + 35*(b*x^2 + a)^(3/2)*b^2*d - 105*sqrt(b*
x^2 + a)*a*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f - 105*sqrt(b*x^2 + a)*a^3*f + 21*(b*x^2 + a)^(5/2)*
b*e - 70*(b*x^2 + a)^(3/2)*a*b*e + 105*sqrt(b*x^2 + a)*a^2*b*e)/b^4
```

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{a+bx^2}\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a + b*x^2)^
(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.140266, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1799, 1620, 63, 208}

$$\frac{\sqrt{a+bx^2}\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a + b*x^2)^
(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^2\sqrt{a+bx}}+\frac{c}{x\sqrt{a+bx}}+\frac{(be-2af)\sqrt{a+bx}}{b^2}+\frac{f(a+bx)^{3/2}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{c\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.124084, size = 86, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\left(8a^2f-2ab\left(5e+2fx^2\right)+b^2\left(15d+5ex^2+3fx^4\right)\right)}{15b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*f - 2*a*b*(5*e + 2*f*x^2) + b^2*(15*d + 5*e*x^2 + 3
*f*x^4)))/(15*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.008, size = 134, normalized size = 1.3

$$\frac{fx^4}{5b}\sqrt{bx^2+a}-\frac{4afx^2}{15b^2}\sqrt{bx^2+a}+\frac{8a^2f}{15b^3}\sqrt{bx^2+a}+\frac{ex^2}{3b}\sqrt{bx^2+a}-\frac{2ae}{3b^2}\sqrt{bx^2+a}+\frac{d}{b}\sqrt{bx^2+a}-c\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*f*x^4/b*(b*x^2+a)^(1/2)-4/15*f/b^2*a*x^2*(b*x^2+a)^(1/2)+8/15*f/b^3*a^2
*(b*x^2+a)^(1/2)+1/3*e*x^2/b*(b*x^2+a)^(1/2)-2/3*e*a/b^2*(b*x^2+a)^(1/2)+d/
b*(b*x^2+a)^(1/2)-c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39378, size = 483, normalized size = 4.69

$$\left[\frac{15\sqrt{a}b^3c\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(3ab^2fx^4+15ab^2d-10a^2be+8a^3f+\left(5ab^2e-4a^2bf\right)x^2\right)\sqrt{bx^2+a}}{30ab^3},\frac{15\sqrt{-a}b^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^3*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f + (5*a*b^2*e - 4*a^
2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3), 1/15*(15*sqrt(-a)*b^3*c*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f +
(5*a*b^2*e - 4*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3)]
```

Sympy [A] time = 24.9387, size = 102, normalized size = 0.99

$$\frac{f\left(a+bx^2\right)^{\frac{5}{2}}}{5b^3}-\frac{\left(a+bx^2\right)^{\frac{3}{2}}\left(2af-be\right)}{3b^3}+\frac{\sqrt{a+bx^2}\left(a^2f-abe+b^2d\right)}{b^3}+\frac{c\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(5/2)/(5*b**3) - (a + b*x**2)**(3/2)*(2*a*f - b*e)/(3*b**3)
 + sqrt(a + b*x**2)*(a**2*f - a*b*e + b**2*d)/b**3 + c*atan(1/(sqrt(-1/a)*s
qrt(a + b*x**2)))/(a*sqrt(-1/a))
```

Giac [A] time = 1.22864, size = 171, normalized size = 1.66

$$\frac{c\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\sqrt{bx^2+a}ab^{14}d+3\left(bx^2+a\right)^{\frac{5}{2}}b^{12}f-10\left(bx^2+a\right)^{\frac{3}{2}}ab^{12}f+15\sqrt{bx^2+a}a^2b^{12}f+5\left(bx^2+a\right)^{\frac{3}{2}}b^{13}}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(15*sqrt(b*x^2 + a)*b^14
*d + 3*(b*x^2 + a)^(5/2)*b^12*f - 10*(b*x^2 + a)^(3/2)*a*b^12*f + 15*sqrt(b
*x^2 + a)*a^2*b^12*f + 5*(b*x^2 + a)^(3/2)*b^13*e - 15*sqrt(b*x^2 + a)*a*b^
13*e)/b^15
```

3.147 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2) + (f*(a +
 b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*a^(3/2))
```

Rubi [A] time = 0.202996, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1799, 1621, 897, 1153, 208}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2) + (f*(a +
 b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*a^(3/2))
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-aex-afx^2}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\operatorname{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(bc-2ad)+a^2be-a^3f}{b^2} - \frac{(abe-2a^2f)x^2}{b^2} - \frac{afx^4}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\operatorname{Subst}\left(\int \left(-a\left(e-\frac{af}{b}\right) - \frac{afx^2}{b} + \frac{bc-2ad}{2\left(-\frac{a}{b}+\frac{x^2}{b}\right)}\right)dx, x, \sqrt{a+bx^2}\right)}{ab}$$

$$= \frac{(be-af)\sqrt{a+bx^2}}{b^2} - \frac{c\sqrt{a+bx^2}}{2ax^2} + \frac{f\left(a+bx^2\right)^{3/2}}{3b^2} - \frac{(bc-2ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{2ab}$$

$$= \frac{(be-af)\sqrt{a+bx^2}}{b^2} - \frac{c\sqrt{a+bx^2}}{2ax^2} + \frac{f\left(a+bx^2\right)^{3/2}}{3b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.382209, size = 131, normalized size = 1.31

$$\frac{3b^3cx^2\sqrt{\frac{bx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right) - \left(a+bx^2\right)\left(4a^2fx^2-2abx^2\left(3e+fx^2\right)+3b^2c\right)}{6ab^2x^2\sqrt{a+bx^2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -((d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]) + (-((a + b*x^2)*(3*b^2*c +
 4*a^2*f*x^2 - 2*a*b*x^2*(3*e + f*x^2))) + 3*b^3*c*x^2*Sqrt[1 + (b*x^2)/a]*
ArcTanh[Sqrt[1 + (b*x^2)/a]])/(6*a*b^2*x^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.009, size = 127, normalized size = 1.3

$$\frac{fx^2}{3b}\sqrt{bx^2+a} - \frac{2af}{3b^2}\sqrt{bx^2+a} + \frac{e}{b}\sqrt{bx^2+a} - d\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}} - \frac{c}{2ax^2}\sqrt{bx^2+a} + \frac{bc}{2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*f*x^2/b*(b*x^2+a)^(1/2)-2/3*f*a/b^2*(b*x^2+a)^(1/2)+e/b*(b*x^2+a)^(1/2)
-d/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*c*(b*x^2+a)^(1/2)/a/x^
2+1/2*c*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46024, size = 482, normalized size = 4.82

$$\left[-\frac{3\left(b^{3}c-2\,ab^{2}d\right)\sqrt{a}x^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)-2\left(2\,a^{2}bfx^{4}-3\,ab^{2}c+2\left(3\,a^{2}be-2\,a^{3}f\right)x^{2}\right)\sqrt{bx^{2}+a}}{12\,a^{2}b^{2}x^{2}},-\frac{3\left(b^{3}c-2\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c - 2*a*b^2*d)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 2*(2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^2*b*e - 2*a^3*f)
*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2), -1/6*(3*(b^3*c - 2*a*b^2*d)*sqrt(-a)*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^
2*b*e - 2*a^3*f)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2)]
```

Sympy [A] time = 40.1251, size = 138, normalized size = 1.38

$$e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)+f\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + f*P
iecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne(
b, 0)), (x**4/(4*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*x)
- d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*
a**(3/2))
```

Giac [A] time = 1.23002, size = 154, normalized size = 1.54

$$-\frac{\frac{3\left(b^2c-2\,abd\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{3\sqrt{bx^2+a}bc}{ax^2}-\frac{2\left(\left(bx^2+a\right)^{\frac{3}{2}}b^2f-3\sqrt{bx^2+a}ab^2f+3\sqrt{bx^2+a}b^3e\right)}{b^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(b^2*c - 2*a*b*d)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 3
*sqrt(b*x^2 + a)*b*c/(a*x^2) - 2*((b*x^2 + a)^(3/2)*b^2*f - 3*sqrt(b*x^2 +
a)*a*b^2*f + 3*sqrt(b*x^2 + a)*b^3*e)/b^3)/b
```

3.148 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=114

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}(3bc-4ad)}{8a^2x^2}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c - 4*a*d)*Sq
rt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^2*e)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] time = 0.232688, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1799, 1621, 897, 1157, 388, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}(3bc-4ad)}{8a^2x^2}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c - 4*a*d)*Sq
rt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^2*e)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc-4ad)-2aex-2afx^2}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(3bc-4ad)+2a^2be-2a^3f}{b^2}-\frac{(2abe-4a^2f)x^2}{b^2}-\frac{2afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \sqrt{a+bx^2}\right)}{2ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3bc+4ad-\frac{8a^2e}{b}+\frac{8a^3f}{b^2}\right)-\frac{4a^2fx^2}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right.}{4a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} + \frac{\left(3bc-4ad+\frac{8a^2e}{b}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{8a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{\left(3b^2c-4abd+8a^2e\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Mathematica [C] time = 0.345416, size = 141, normalized size = 1.24

$$-\frac{b^2c\sqrt{a+bx^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3} - \frac{bd\sqrt{a+bx^2}\left(\frac{a}{bx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (e*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] - (b*d
*Sqrt[a + b*x^2]*(a/(b*x^2) - ArcTanh[Sqrt[1 + (b*x^2)/a]]/Sqrt[1 + (b*x^2)
/a]))/(2*a^2) - (b^2*c*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (
b*x^2)/a])/a^3
```

Maple [A] time = 0.01, size = 162, normalized size = 1.4

$$\frac{f}{b}\sqrt{bx^2+a}-e\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{c}{4ax^4}\sqrt{bx^2+a}+\frac{3bc}{8a^2x^2}\sqrt{bx^2+a}-\frac{3b^2c}{8}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] f*(b*x^2+a)^(1/2)/b-e/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/4*c*(
b*x^2+a)^(1/2)/a/x^4+3/8*c*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*c*b^2/a^(5/2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*d/a/x^2*(b*x^2+a)^(1/2)+1/2*d*b/a^(3/
2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47811, size = 501, normalized size = 4.39

$$\left[\frac{\left(3b^3c-4ab^2d+8a^2be\right)\sqrt{a}x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(8a^3fx^4-2a^2bc+\left(3ab^2c-4a^2bd\right)x^2\right)\sqrt{bx^2+a}}{16a^3bx^4},\frac{\left(3b^3c-4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*a^3*f*x^4 - 2*a^2*b*c + (3*a*b^2*c - 4
*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4), 1/8*((3*b^3*c - 4*a*b^2*d + 8*
a^2*b*e)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*a^3*f*x^4 - 2*a
```

```
^2*b*c + (3*a*b^2*c - 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4)]
```

Sympy [A] time = 78.6774, size = 194, normalized size = 1.7

$$f\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)-\frac{c}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}c}{8ax^3\sqrt{\frac{a}{bx^2}+1}}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{3b^{\frac{3}{2}}c}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{a}}+\frac{bd\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] f*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) - c/(
4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(8*a*x**3*sqrt(a/(b*x**2)
+ 1)) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*a*x) + 3*b**(3/2)*c/(8*a**2*x*sqr
t(a/(b*x**2) + 1)) - e*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*d*asinh(sqrt(
a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5
/2))
```

Giac [A] time = 1.16988, size = 190, normalized size = 1.67

$$\frac{8\sqrt{bx^2+a}f+\frac{\left(3b^3c-4ab^2d+8a^2be\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}b^3c-5\sqrt{bx^2+a}ab^3c-4\left(bx^2+a\right)^{\frac{3}{2}}ab^2d+4\sqrt{bx^2+a}a^2b^2d}{a^2b^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*f + (3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*arctan(sqrt(b*
x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(3/2)*b^3*c - 5*sqrt(b*x
^2 + a)*a*b^3*c - 4*(b*x^2 + a)^(3/2)*a*b^2*d + 4*sqrt(b*x^2 + a)*a^2*b^2*d
)/(a^2*b^2*x^4))/b
```

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2be-16a^3f-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{\sqrt{a+bx^2}\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\sqrt{a+bx^2}(5bc-6ad)}{24a^2x^4}-\frac{c\sqrt{a+bx^2}}{6ax^6}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2])/(24*a^2*
x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/(16*a^3*x^2) + ((5*b
^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(16*a^(7/2))
```

Rubi [A] time = 0.276366, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1799, 1621, 897, 1157, 385, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2be-16a^3f-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{\sqrt{a+bx^2}\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\sqrt{a+bx^2}(5bc-6ad)}{24a^2x^4}-\frac{c\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2])/(24*a^2*
x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/(16*a^3*x^2) + ((5*b
^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(16*a^(7/2))
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
```

```
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc-6ad)-3aex-3afx^2}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(5bc-6ad)+3a^2be-3a^3f}{b^2} - \frac{(3abe-6a^2f)x^2}{b^2} - \frac{3afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx, x, \sqrt{a+bx^2}\right)}{3ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}\left(5bc-6ad+\frac{8a^2e}{b}-\frac{8a^3f}{b^2}\right)-\frac{12a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \sqrt{a+bx^2}\right.}{12a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + \frac{\left(b^2\left(\frac{12a^3f}{b^3} - \frac{3\left(5bc-\right.}{}\right.\right.}{}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + \frac{\left(5b^3c-6ab^2d+8a\right.}{}$$

<u>Mathematica</u> [C] time = 0.962059, size = 162, normalized size = 1.11

$$\frac{b^3c\sqrt{a+bx^2}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^4} - \frac{b^2d\sqrt{a+bx^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3} - \frac{be\sqrt{a+bx^2}\left(\frac{a}{bx^2} - \frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a^2} - \frac{f\tanh^{-1}\left(\right.}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -((f*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]) - (b*e*Sqrt[a + b*x^2]*(a/(
b*x^2) - ArcTanh[Sqrt[1 + (b*x^2)/a]]/Sqrt[1 + (b*x^2)/a]))/(2*a^2) - (b^2*
d*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^2)/a])/a^3 + (b^3
*c*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x^2)/a])/a^4
```

Maple [A] time = 0.01, size = 238, normalized size = 1.6

$$-f\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{d}{4\,ax^4}\sqrt{bx^2+a}+\frac{3\,bd}{8\,a^2x^2}\sqrt{bx^2+a}-\frac{3\,b^2d}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}-\frac{e}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x)
```

```
[Out] -f/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/4*d/a/x^4*(b*x^2+a)^(1/2
)+3/8*d*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*d*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^
2+a)^(1/2))/x)-1/2*e/a/x^2*(b*x^2+a)^(1/2)+1/2*e*b/a^(3/2)*ln((2*a+2*a^(1/2
)*(b*x^2+a)^(1/2))/x)-1/6*c*(b*x^2+a)^(1/2)/a/x^6+5/24*c*b/a^2/x^4*(b*x^2+a
)^(1/2)-5/16*c*b^2/a^3/x^2*(b*x^2+a)^(1/2)+5/16*c*b^3/a^(7/2)*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62761, size = 605, normalized size = 4.14

$$\left[-\frac{3\left(5\,b^3c-6\,ab^2d+8\,a^2be-16\,a^3f\right)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(5\,ab^2c-6\,a^2bd+8\,a^3e\right)x^4+8\,a^3c-2\left(5\,a^2b\right.\right.}{96\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(a)*x^6*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(5*a*b^2*c - 6*a^2*b*d +
```

```
8*a^3*e)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^
4*x^6), -1/48*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(-a)*x^6*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(5*a*b^2*c - 6*a^2*b*d + 8*a^3*e)*x^4
 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6)]
```

Sympy [B] time = 110.921, size = 303, normalized size = 2.08

$$-\frac{c}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{d}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}c}{24ax^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}d}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{5b^{\frac{3}{2}}c}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{3}{2}}d}{8a^2x\sqrt{\frac{a}{bx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2
) + 1)) + sqrt(b)*c/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*d/(8*a*x**3*
sqrt(a/(b*x**2) + 1)) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(2*a*x) - 5*b**(3/2)
*c/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*d/(8*a**2*x*sqrt(a/(b*x
**2) + 1)) - 5*b**(5/2)*c/(16*a**3*x*sqrt(a/(b*x**2) + 1)) - f*asinh(sqrt(a
)/(sqrt(b)*x))/sqrt(a) + b*e*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b*
*2*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*b**3*c*asinh(sqrt(a)/(sqrt
(b)*x))/(16*a**(7/2))
```

Giac [A] time = 1.21274, size = 313, normalized size = 2.14

$$-\frac{\frac{3\left(5\,b^4c-6\,ab^3d-16\,a^3bf+8\,a^2b^2e\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{15\left(bx^2+a\right)^{\frac{5}{2}}b^4c-40\left(bx^2+a\right)^{\frac{3}{2}}ab^4c+33\,\sqrt{bx^2+a}a^2b^4c-18\left(bx^2+a\right)^{\frac{5}{2}}ab^3d+48\left(bx^2+a\right)^{\frac{3}{2}}a^2b^3d-30\,\sqrt{bx^2+}}{a^3b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*b^4*c - 6*a*b^3*d - 16*a^3*b*f + 8*a^2*b^2*e)*arctan(sqrt(b*x^2
 + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x^2 + a)^(5/2)*b^4*c - 40*(b*x^2 +
a)^(3/2)*a*b^4*c + 33*sqrt(b*x^2 + a)*a^2*b^4*c - 18*(b*x^2 + a)^(5/2)*a*b^
3*d + 48*(b*x^2 + a)^(3/2)*a^2*b^3*d - 30*sqrt(b*x^2 + a)*a^3*b^3*d + 24*(b
*x^2 + a)^(5/2)*a^2*b^2*e - 48*(b*x^2 + a)^(3/2)*a^3*b^2*e + 24*sqrt(b*x^2
```

```
+ a)*a^4*b^2*e)/(a^3*b^3*x^6))/b
```

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{a+bx^2}\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^4x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^{9/2}}-\frac{\sqrt{a+bx^2}\left(48a\right.}{19}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(8*a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2])/(48*a^2*
x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2])/(192*a^3*x^4) + (
(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*Sqrt[a + b*x^2])/(128*a^4*x
^2) - (b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(9/2))
```

Rubi [A] time = 0.350011, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1799, 1621, 897, 1157, 385, 199, 208}

$$\frac{\sqrt{a+bx^2}\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^4x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^{9/2}}-\frac{\sqrt{a+bx^2}\left(48a\right.}{19}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(8*a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2])/(48*a^2*
x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2])/(192*a^3*x^4) + (
(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*Sqrt[a + b*x^2])/(128*a^4*x
^2) - (b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(9/2))
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
```

```
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(7bc-8ad)-4aex-4afx^2}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)}{8a}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(7bc-8ad)+4a^2be-4a^3f}{b^2}-\frac{\left(4abe-8a^2f\right)x^2}{b^2}-\frac{4afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^4}\,dx, x, \sqrt{a+bx^2}\right)}{4ab}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-35bc+40ad-\frac{48a^2e}{b}+\frac{48a^3f}{b^2}\right)-\frac{24a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx, x, \sqrt{a+\cdots}\right.}{24a^2}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \frac{\left(b^2\left(\frac{24a^3f}{b^3}+\frac{3(\cdots}{\cdots}\right.\right.}{\cdots}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \frac{\left(35b^3c-40ab^2\cdots\right.}{\cdots}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \frac{\left(35b^3c-40ab^2\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.328437, size = 140, normalized size = 0.72

$$
\frac{b\sqrt{a+bx^2}\left(-2a^2be\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)-\frac{a^4f}{bx^2}+\frac{a^3f\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}-2b^3c\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{bx^2}{a}+1\right)+2ab^2d\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx^2}{a}+\cdots\right.\right.}{2a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-((a^4*f)/(b*x^2)) + (a^3*f*ArcTanh[Sqrt[1 + (b*x^2)/a]
])/Sqrt[1 + (b*x^2)/a] - 2*a^2*b*e*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^
2)/a] + 2*a*b^2*d*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x^2)/a] - 2*b^3*c*H
ypergeometric2F1[1/2, 5, 3/2, 1 + (b*x^2)/a]))/(2*a^5)
```

Maple [A] time = 0.014, size = 320, normalized size = 1.6

$$-\frac{c}{8\,a x^8}\sqrt{b x^2+a}+\frac{7\,bc}{48\,a^2x^6}\sqrt{b x^2+a}-\frac{35\,b^2c}{192\,a^3x^4}\sqrt{b x^2+a}+\frac{35\,b^3c}{128\,a^4x^2}\sqrt{b x^2+a}-\frac{35\,cb^4}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/8*c*(b*x^2+a)^(1/2)/a/x^8+7/48*c*b/a^2/x^6*(b*x^2+a)^(1/2)-35/192*c*b^2/
a^3/x^4*(b*x^2+a)^(1/2)+35/128*c*b^3/a^4/x^2*(b*x^2+a)^(1/2)-35/128*c*b^4/a
^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/4*e/a/x^4*(b*x^2+a)^(1/2)+3/
8*e*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*e*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)
^(1/2))/x)-1/2*f/a/x^2*(b*x^2+a)^(1/2)+1/2*f*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)-1/6*d/a/x^6*(b*x^2+a)^(1/2)+5/24*d*b/a^2/x^4*(b*x^2+a)^(1
/2)-5/16*d*b^2/a^3/x^2*(b*x^2+a)^(1/2)+5/16*d*b^3/a^(7/2)*ln((2*a+2*a^(1/2)
*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05362, size = 803, normalized size = 4.12

$$\left[-\frac{3\left(35\,b^4c-40\,ab^3d+48\,a^2b^2e-64\,a^3bf\right)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(3\left(35\,ab^3c-40\,a^2b^2d+48\,a^3be-64\,a^4f\right.\right.}{768\,a^5x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(a)*x^8*
log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3*(35*a*b^3*c - 40
*a^2*b^2*d + 48*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a
^3*b*d + 48*a^4*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5
*x^8), 1/384*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(-a
)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(35*a*b^3*c - 40*a^2*b^2*d + 48
*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a^3*b*d + 48*a^4
*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5*x^8)]
```

Sympy [B] time = 176.308, size = 444, normalized size = 2.28

$$-\frac{c}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{d}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{e}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}c}{48ax^7\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}d}{24ax^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}e}{8ax^3\sqrt{\frac{a}{bx^2}+1}}-\frac{\sqrt{b}f\sqrt{\frac{a}{bx}}}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - d/(6*sqrt(b)*x**7*sqrt(a/(b*x**2
) + 1)) - e/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(48*a*x**7*sq
rt(a/(b*x**2) + 1)) + sqrt(b)*d/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*
e/(8*a*x**3*sqrt(a/(b*x**2) + 1)) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(2*a*x)
- 7*b**(3/2)*c/(192*a**2*x**5*sqrt(a/(b*x**2) + 1)) - 5*b**(3/2)*d/(48*a**2
*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*e/(8*a**2*x*sqrt(a/(b*x**2) + 1))
+ 35*b**(5/2)*c/(384*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(5/2)*d/(16*a**
3*x*sqrt(a/(b*x**2) + 1)) + 35*b**(7/2)*c/(128*a**4*x*sqrt(a/(b*x**2) + 1))
 + b*f*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*e*asinh(sqrt(a)/(sq
rt(b)*x))/(8*a**(5/2)) + 5*b**3*d*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(7/2))
- 35*b**4*c*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(9/2))
```

Giac [B] time = 1.22429, size = 487, normalized size = 2.5

$$\frac{\frac{3\left(35\,b^5c-40\,ab^4d-64\,a^3b^2f+48\,a^2b^3e\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{105\left(bx^2+a\right)^{\frac{7}{2}}b^5c-385\left(bx^2+a\right)^{\frac{5}{2}}ab^5c+511\left(bx^2+a\right)^{\frac{3}{2}}a^2b^5c-279\sqrt{bx^2+a}a^3b^5c-120\left(bx^2+a\right)^{\frac{7}{2}}ab^4d+4}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(3*(35*b^5*c - 40*a*b^4*d - 64*a^3*b^2*f + 48*a^2*b^3*e)*arctan(sqrt(
b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x^2 + a)^(7/2)*b^5*c - 385*(b
*x^2 + a)^(5/2)*a*b^5*c + 511*(b*x^2 + a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x^2
+ a)*a^3*b^5*c - 120*(b*x^2 + a)^(7/2)*a*b^4*d + 440*(b*x^2 + a)^(5/2)*a^2*
b^4*d - 584*(b*x^2 + a)^(3/2)*a^3*b^4*d + 264*sqrt(b*x^2 + a)*a^4*b^4*d - 1
92*(b*x^2 + a)^(7/2)*a^3*b^2*f + 576*(b*x^2 + a)^(5/2)*a^4*b^2*f - 576*(b*x
^2 + a)^(3/2)*a^5*b^2*f + 192*sqrt(b*x^2 + a)*a^6*b^2*f + 144*(b*x^2 + a)^(
7/2)*a^2*b^3*e - 528*(b*x^2 + a)^(5/2)*a^3*b^3*e + 624*(b*x^2 + a)^(3/2)*a^
4*b^3*e - 240*sqrt(b*x^2 + a)*a^5*b^3*e)/(a^4*b^4*x^8))/b
```

3.151 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=245

$$\frac{x^3\sqrt{a+bx^2}\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{384b^4}-\frac{ax\sqrt{a+bx^2}\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{256b^5}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{}$$

```
[Out] -(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*x^2])/(256
*b^5) + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x^3*Sqrt[a + b*x^2
])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2*f)*x^5*Sqrt[a + b*x^2])/(480*
b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*
x^2])/(10*b) + (a^2*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(11/2))
```

Rubi [A] time = 0.258489, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1809, 1267, 459, 321, 217, 206}

$$\frac{x^3\sqrt{a+bx^2}\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{384b^4}-\frac{ax\sqrt{a+bx^2}\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{256b^5}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*x^2])/(256
*b^5) + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x^3*Sqrt[a + b*x^2
])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2*f)*x^5*Sqrt[a + b*x^2])/(480*
b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*
x^2])/(10*b) + (a^2*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(11/2))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4 \left(c + dx^2 + ex^4 + fx^6\right)}{\sqrt{a + bx^2}} dx = \frac{fx^9\sqrt{a + bx^2}}{10b} + \frac{\int \frac{x^4\left(10bc+10bdx^2+(10be-9af)x^4\right)}{\sqrt{a+bx^2}} dx}{10b}$$

$$= \frac{(10be - 9af)x^7\sqrt{a + bx^2}}{80b^2} + \frac{fx^9\sqrt{a + bx^2}}{10b} + \frac{\int \frac{x^4\left(80b^2c+\left(80b^2d-70abe+63a^2f\right)x^2\right)}{\sqrt{a+bx^2}} dx}{80b^2}$$

$$= \frac{\left(80b^2d - 70abe + 63a^2f\right)x^5\sqrt{a + bx^2}}{480b^3} + \frac{(10be - 9af)x^7\sqrt{a + bx^2}}{80b^2} + \frac{fx^9\sqrt{a + bx^2}}{10b} + \frac{(9}{}$$

$$= \frac{\left(96b^3c - 80ab^2d + 70a^2be - 63a^3f\right)x^3\sqrt{a + bx^2}}{384b^4} + \frac{\left(80b^2d - 70abe + 63a^2f\right)x^5\sqrt{a + bx}}{480b^3}$$

$$= -\frac{a\left(96b^3c - 80ab^2d + 70a^2be - 63a^3f\right)x\sqrt{a + bx^2}}{256b^5} + \frac{\left(96b^3c - 80ab^2d + 70a^2be - 63a^3\right.}{384b^4}$$

$$= -\frac{a\left(96b^3c - 80ab^2d + 70a^2be - 63a^3f\right)x\sqrt{a + bx^2}}{256b^5} + \frac{\left(96b^3c - 80ab^2d + 70a^2be - 63a^3\right.}{384b^4}$$

$$= -\frac{a\left(96b^3c - 80ab^2d + 70a^2be - 63a^3f\right)x\sqrt{a + bx^2}}{256b^5} + \frac{\left(96b^3c - 80ab^2d + 70a^2be - 63a^3\right.}{384b^4}$$

Mathematica [A] time = 0.223898, size = 184, normalized size = 0.75

$$\frac{\sqrt{b}x\sqrt{a + bx^2}\left(4a^2b^2\left(300d + 175ex^2 + 126fx^4\right) - 210a^3b\left(5e + 3fx^2\right) + 945a^4f - 16ab^3\left(90c + 50dx^2 + 35ex^4 + 27fx^6\right)}{3840b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(945*a^4*f - 210*a^3*b*(5*e + 3*f*x^2) + 4*a^2*b
^2*(300*d + 175*e*x^2 + 126*f*x^4) + 32*b^4*x^2*(30*c + 20*d*x^2 + 15*e*x^4
 + 12*f*x^6) - 16*a*b^3*(90*c + 50*d*x^2 + 35*e*x^4 + 27*f*x^6)) - 15*a^2*(
-96*b^3*c + 80*a*b^2*d - 70*a^2*b*e + 63*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(3840*b^(11/2))
```

Maple [A] time = 0.022, size = 368, normalized size = 1.5

$$\frac{fx^9}{10\,b}\sqrt{bx^2 + a} - \frac{9\,afx^7}{80\,b^2}\sqrt{bx^2 + a} + \frac{21\,a^2fx^5}{160\,b^3}\sqrt{bx^2 + a} - \frac{21\,a^3fx^3}{128\,b^4}\sqrt{bx^2 + a} + \frac{63\,fa^4x}{256\,b^5}\sqrt{bx^2 + a} - \frac{63\,fa^5}{256}\ln\left(x\sqrt{b} + \sqrt{bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/10*f*x^9*(b*x^2+a)^(1/2)/b-9/80*f/b^2*a*x^7*(b*x^2+a)^(1/2)+21/160*f/b^3*
a^2*x^5*(b*x^2+a)^(1/2)-21/128*f/b^4*a^3*x^3*(b*x^2+a)^(1/2)+63/256*f/b^5*a
^4*x*(b*x^2+a)^(1/2)-63/256*f/b^(11/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/
8*e*x^7/b*(b*x^2+a)^(1/2)-7/48*e/b^2*a*x^5*(b*x^2+a)^(1/2)+35/192*e/b^3*a^2
*x^3*(b*x^2+a)^(1/2)-35/128*e/b^4*a^3*x*(b*x^2+a)^(1/2)+35/128*e/b^(9/2)*a^
4*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*d*x^5/b*(b*x^2+a)^(1/2)-5/24*d/b^2*a*x^
3*(b*x^2+a)^(1/2)+5/16*d/b^3*a^2*x*(b*x^2+a)^(1/2)-5/16*d/b^(7/2)*a^3*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))+1/4*c*x^3/b*(b*x^2+a)^(1/2)-3/8*c/b^2*a*x*(b*x^2+a
)^(1/2)+3/8*c/b^(5/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0697, size = 983, normalized size = 4.01

$$\left[-\frac{15\left(96\,a^{2}b^{3}c-80\,a^{3}b^{2}d+70\,a^{4}be-63\,a^{5}f\right)\sqrt{b}\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\left(384\,b^{5}fx^{9}+48\left(10\,b^{5}e-9\,ab^{4}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*b^5*f*x^9 + 48*(10
*b^5*e - 9*a*b^4*f)*x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10
*(96*b^5*c - 80*a*b^4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c
 - 80*a^2*b^3*d + 70*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6, -1/38
```

```
40*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*f*x^9 + 48*(10*b^5*e - 9*a*b^4*f)*
x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10*(96*b^5*c - 80*a*b^
4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c - 80*a^2*b^3*d + 70
*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6]
```

Sympy [B] time = 33.212, size = 586, normalized size = 2.39

$$\frac{63a^{\frac{9}{2}}fx}{256b^5\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{7}{2}}ex}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{21a^{\frac{7}{2}}fx^3}{256b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}dx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}ex^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^{\frac{5}{2}}fx^5}{640b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}cx}{8b^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 63*a**(9/2)*f*x/(256*b**5*sqrt(1 + b*x**2/a)) - 35*a**(7/2)*e*x/(128*b**4*s
qrt(1 + b*x**2/a)) + 21*a**(7/2)*f*x**3/(256*b**4*sqrt(1 + b*x**2/a)) + 5*a
**(5/2)*d*x/(16*b**3*sqrt(1 + b*x**2/a)) - 35*a**(5/2)*e*x**3/(384*b**3*sqr
t(1 + b*x**2/a)) - 21*a**(5/2)*f*x**5/(640*b**3*sqrt(1 + b*x**2/a)) - 3*a**
(3/2)*c*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*d*x**3/(48*b**2*sqrt(1 +
 b*x**2/a)) + 7*a**(3/2)*e*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + 3*a**(3/2)*
f*x**7/(160*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*c*x**3/(8*b*sqrt(1 + b*x**2/
a)) - sqrt(a)*d*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*e*x**7/(48*b*sqrt(
1 + b*x**2/a)) - sqrt(a)*f*x**9/(80*b*sqrt(1 + b*x**2/a)) - 63*a**5*f*asinh
(sqrt(b)*x/sqrt(a))/(256*b**(11/2)) + 35*a**4*e*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(9/2)) - 5*a**3*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*c*a
sinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + c*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)
) + d*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + e*x**9/(8*sqrt(a)*sqrt(1 + b*x*
*2/a)) + f*x**11/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.18716, size = 302, normalized size = 1.23

$$\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8fx^2}{b}-\frac{9ab^7f-10b^8e}{b^9}\right)x^2+\frac{80b^8d+63a^2b^6f-70ab^7e}{b^9}\right)x^2+\frac{5\left(96b^8c-80ab^7d-63a^3b^5f+70a^2b^6e\right.}{b^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*f*x^2/b - (9*a*b^7*f - 10*b^8*e)/b^9)*x^2 + (80*b^8*d +
63*a^2*b^6*f - 70*a*b^7*e)/b^9)*x^2 + 5*(96*b^8*c - 80*a*b^7*d - 63*a^3*b^5
*f + 70*a^2*b^6*e)/b^9)*x^2 - 15*(96*a*b^7*c - 80*a^2*b^6*d - 63*a^4*b^4*f
+ 70*a^3*b^5*e)/b^9)*sqrt(b*x^2 + a)*x - 1/256*(96*a^2*b^3*c - 80*a^3*b^2*d
 - 63*a^5*f + 70*a^4*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

3.152 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{x\sqrt{a+bx^2}\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^4}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x^3\sqrt{a+bx^2}\,(}{}$$

```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2])/(128*b^
4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x^2])/(192*b^3) + ((8
*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) + (f*x^7*Sqrt[a + b*x^2])/(8*b)
 - (a*(64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(128*b^(9/2))
```

Rubi [A] time = 0.207644, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1809, 1267, 459, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^4}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x^3\sqrt{a+bx^2}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```


```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2])/(128*b^
4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x^2])/(192*b^3) + ((8
*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) + (f*x^7*Sqrt[a + b*x^2])/(8*b)
 - (a*(64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(128*b^(9/2))
```


Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{fx^7\sqrt{a+bx^2}}{8b}+\frac{\int \frac{x^2\left(8bc+8bdx^2+(8be-7af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{8b}$$

$$= \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}+\frac{\int \frac{x^2\left(48b^2c+\left(48b^2d-40abe+35a^2f\right)x^2\right)}{\sqrt{a+bx^2}}\,dx}{48b^2}$$

$$= \frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}-\frac{1}{64}\Big($$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-}{}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-}{}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-}{}$$

Mathematica [A] time = 0.158357, size = 149, normalized size = 0.77

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(10a^2b\left(12e+7fx^2\right)-105a^3f-8ab^2\left(18d+10ex^2+7fx^4\right)+16b^3\left(12c+6dx^2+4ex^4+3fx^6\right)\right)+3a\tanh}{384b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*f + 10*a^2*b*(12*e + 7*f*x^2) - 8*a*b^
2*(18*d + 10*e*x^2 + 7*f*x^4) + 16*b^3*(12*c + 6*d*x^2 + 4*e*x^4 + 3*f*x^6)
) + 3*a*(-64*b^3*c + 48*a*b^2*d - 40*a^2*b*e + 35*a^3*f)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(384*b^(9/2))
```

Maple [A] time = 0.008, size = 284, normalized size = 1.5

$$\frac{fx^7}{8b}\sqrt{bx^2+a}-\frac{7afx^5}{48b^2}\sqrt{bx^2+a}+\frac{35a^2fx^3}{192b^3}\sqrt{bx^2+a}-\frac{35a^3fx}{128b^4}\sqrt{bx^2+a}+\frac{35fa^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}+\frac{ex^5}{6b}\sqrt{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*f*x^7*(b*x^2+a)^(1/2)/b-7/48*f/b^2*a*x^5*(b*x^2+a)^(1/2)+35/192*f/b^3*a
^2*x^3*(b*x^2+a)^(1/2)-35/128*f/b^4*a^3*x*(b*x^2+a)^(1/2)+35/128*f/b^(9/2)*
a^4*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*e*x^5/b*(b*x^2+a)^(1/2)-5/24*e/b^2*a*
x^3*(b*x^2+a)^(1/2)+5/16*e/b^3*a^2*x*(b*x^2+a)^(1/2)-5/16*e/b^(7/2)*a^3*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*d*x^3/b*(b*x^2+a)^(1/2)-3/8*d/b^2*a*x*(b*x^2
+a)^(1/2)+3/8*d/b^(5/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*c*x/b*(b*x^2+
a)^(1/2)-1/2*c*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71605, size = 779, normalized size = 4.02

$$\left[-\frac{3\left(64\,ab^3c-48\,a^2b^2d+40\,a^3be-35\,a^4f\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,b^4fx^7+8\left(8\,b^4e-7\,ab^3f\right)x^5\right.}{768\,b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(b)*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*f*x^7 + 8*(8*b^4*e
- 7*a*b^3*f)*x^5 + 2*(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4
*c - 48*a*b^3*d + 40*a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5, 1/384
*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*f*x^7 + 8*(8*b^4*e - 7*a*b^3*f)*x^5 + 2*
(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4*c - 48*a*b^3*d + 40*
a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5]
```

Sympy [B] time = 21.593, size = 444, normalized size = 2.29

$$-\frac{35a^{\frac{7}{2}}fx}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}ex}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}fx^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}dx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}ex^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{7a^{\frac{3}{2}}fx^5}{192b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}cx\sqrt{1+\frac{bx^2}{a}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**(7/2)*f*x/(128*b**4*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*e*x/(16*b**3*sq
rt(1 + b*x**2/a)) - 35*a**(5/2)*f*x**3/(384*b**3*sqrt(1 + b*x**2/a)) - 3*a*
*(3/2)*d*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*e*x**3/(48*b**2*sqrt(1
+ b*x**2/a)) + 7*a**(3/2)*f*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*d*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
(a)*e*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*f*x**7/(48*b*sqrt(1 + b*x**2
/a)) + 35*a**4*f*asinh(sqrt(b)*x/sqrt(a))/(128*b**(9/2)) - 5*a**3*e*asinh(s
qrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*d*asinh(sqrt(b)*x/sqrt(a))/(8*b**(
5/2)) - a*c*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*x**5/(4*sqrt(a)*sqrt(
1 + b*x**2/a)) + e*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**9/(8*sqrt(a)*
sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.22415, size = 236, normalized size = 1.22

$$\frac{1}{384}\left(2\left(4\left(\frac{6fx^2}{b}-\frac{7ab^5f-8b^6e}{b^7}\right)x^2+\frac{48b^6d+35a^2b^4f-40ab^5e}{b^7}\right)x^2+\frac{3\left(64b^6c-48ab^5d-35a^3b^3f+40a^2b^4e\right)}{b^7}\right)\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*f*x^2/b - (7*a*b^5*f - 8*b^6*e)/b^7)*x^2 + (48*b^6*d + 35*a^
2*b^4*f - 40*a*b^5*e)/b^7)*x^2 + 3*(64*b^6*c - 48*a*b^5*d - 35*a^3*b^3*f +
40*a^2*b^4*e)/b^7)*sqrt(b*x^2 + a)*x + 1/128*(64*a*b^3*c - 48*a^2*b^2*d - 3
5*a^4*f + 40*a^3*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=145

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6a^2be-5a^3f-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{24b^2}+fx^5$$

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*b*e - 5*a*
f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^2])/(6*b) + ((16*b^3
*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(16*b^(7/2))
```

Rubi [A] time = 0.119289, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1815, 1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6a^2be-5a^3f-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{24b^2}+fx^5$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*b*e - 5*a*
f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^2])/(6*b) + ((16*b^3
*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(16*b^(7/2))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
```

```
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \frac{fx^5\sqrt{a+bx^2}}{6b}+\frac{\int \frac{6bc+6bdx^2+(6be-5af)x^4}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}+\frac{\int \frac{24b^2c+3\left(8b^2d-6abe+5a^2f\right)x^2}{\sqrt{a+bx^2}}\,dx}{24b^2}$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3}+\frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}-\frac{1}{16}\left(-16c+\frac{a}{\ldots}\right.$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3}+\frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}-\frac{1}{16}\left(-16c+\frac{a}{\ldots}\right.$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3}+\frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}+\frac{\left(16c-\frac{a\left(8b^2d-\ldots\right.}{\ldots}\right.}{\ldots}$$

Mathematica [A] time = 0.108642, size = 118, normalized size = 0.81

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6a^2be-5a^3f-8ab^2d+16b^3c\right)+\sqrt{b}x\sqrt{a+bx^2}\left(15a^2f-2ab\left(9e+5fx^2\right)+4b^2\left(6d+3ex^2+2fx^4\right)\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^2*f - 2*a*b*(9*e + 5*f*x^2) + 4*b^2*(6*d +
 3*e*x^2 + 2*f*x^4)) + 3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(48*b^(7/2))
```

Maple [A] time = 0.006, size = 203, normalized size = 1.4

$$\frac{fx^5}{6\,b}\sqrt{bx^2+a}-\frac{5\,afx^3}{24\,b^2}\sqrt{bx^2+a}+\frac{5\,a^2fx}{16\,b^3}\sqrt{bx^2+a}-\frac{5\,a^3f}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{ex^3}{4\,b}\sqrt{bx^2+a}-\frac{3\,aex}{8\,b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*f*x^5*(b*x^2+a)^(1/2)/b-5/24*f/b^2*a*x^3*(b*x^2+a)^(1/2)+5/16*f/b^3*a^2
*x*(b*x^2+a)^(1/2)-5/16*f/b^(7/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*e*x
^3/b*(b*x^2+a)^(1/2)-3/8*e/b^2*a*x*(b*x^2+a)^(1/2)+3/8*e/b^(5/2)*a^2*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))+1/2*d*x/b*(b*x^2+a)^(1/2)-1/2*d*a/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))+c*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52251, size = 581, normalized size = 4.01

$$\left[-\frac{3\left(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,b^3fx^5+2\left(6\,b^3e-5\,ab^2f\right)x^3+3\left(8\,b^3d\right.\right.}{96\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(b)*log(-2*b*x^2
 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2
*f)*x^3 + 3*(8*b^3*d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4, -1/4
8*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) - (8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2*f)*x^3 + 3*(8*b^3*
d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] time = 12.1099, size = 362, normalized size = 2.5

$$\frac{5a^{\frac{5}{2}}fx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}ex}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}fx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}ex^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}fx^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^3f\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*f*x/(16*b**3*sqrt(1 + b*x**2/a)) - 3*a**(3/2)*e*x/(8*b**2*sqrt(1
 + b*x**2/a)) + 5*a**(3/2)*f*x**3/(48*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*d*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*e*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
(a)*f*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*f*asinh(sqrt(b)*x/sqrt(a))/(1
6*b**(7/2)) + 3*a**2*e*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*d*asinh(sq
rt(b)*x/sqrt(a))/(2*b**(3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/
sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0)
 & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
+ e*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**7/(6*sqrt(a)*sqrt(1 + b*x**2
```

```
/a))
```

Giac [A] time = 1.19111, size = 174, normalized size = 1.2

$$\frac{1}{48}\left(2\left(\frac{4\,fx^2}{b}-\frac{5\,ab^3f-6\,b^4e}{b^5}\right)x^2+\frac{3\left(8\,b^4d+5\,a^2b^2f-6\,ab^3e\right)}{b^5}\right)\sqrt{bx^2+ax}-\frac{\left(16\,b^3c-8\,ab^2d-5\,a^3f+6\,a^2be\right)\log\left(\left|-\right.\right.}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*f*x^2/b - (5*a*b^3*f - 6*b^4*e)/b^5)*x^2 + 3*(8*b^4*d + 5*a^2*b^
2*f - 6*a*b^3*e)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*c - 8*a*b^2*d - 5*a^
3*f + 6*a^2*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2])/(8*b^2)
+ (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.135977, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2])/(8*b^2)
+ (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{ax} - \frac{\int \frac{-adx-aex^3-afx^5}{x\sqrt{a+bx^2}}\,dx}{a}$$

$$= -\frac{c\sqrt{a+bx^2}}{ax} - \frac{\int \frac{-ad-aex^2-afx^4}{\sqrt{a+bx^2}}\,dx}{a}$$

$$= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{fx^3\sqrt{a+bx^2}}{4b} - \frac{\int \frac{-4abd-a(4be-3af)x^2}{\sqrt{a+bx^2}}\,dx}{4ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} + \frac{\left(8ab^2d-a^2(4be-3af)\right)\int \frac{1}{\sqrt{a+bx^2}}\,d}{8ab^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} + \frac{\left(8ab^2d-a^2(4be-3af)\right)\text{Subst}\left(\int\right.}{8ab^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} + \frac{\left(8b^2d-4abe+3a^2f\right)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+}}\right.}{8b^{5/2}}$$

Mathematica [A] time = 0.110432, size = 103, normalized size = 0.88

$$\frac{\frac{\sqrt{b}\sqrt{a+bx^2}\left(-3a^2fx^2+2abx^2\left(2e+fx^2\right)-8b^2c\right)}{ax}+\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[a + b*x^2]*(-8*b^2*c - 3*a^2*f*x^2 + 2*a*b*x^2*(2*e + f*x^2)
))/(a*x) + (8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(8*b^(5/2))
```

Maple [A] time = 0.007, size = 140, normalized size = 1.2

$$\frac{fx^3}{4b}\sqrt{bx^2+a} - \frac{3afx}{8b^2}\sqrt{bx^2+a} + \frac{3a^2f}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}} + \frac{ex}{2b}\sqrt{bx^2+a} - \frac{ae}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} + d\ln\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*f*x^3*(b*x^2+a)^(1/2)/b-3/8*f/b^2*a*x*(b*x^2+a)^(1/2)+3/8*f/b^(5/2)*a^2
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*e*x/b*(b*x^2+a)^(1/2)-1/2*e*a/b^(3/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))+d*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-c*(b*x^
2+a)^(1/2)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41081, size = 489, normalized size = 4.18

$$\left[\frac{\left(8\,ab^2d - 4\,a^2be + 3\,a^3f\right)\sqrt{b}x\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\left(2\,ab^2fx^4 - 8\,b^3c + \left(4\,ab^2e - 3\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{16\,ab^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(2*a*b^2*f*x^4 - 8*b^3*c + (4*a*b^2*e - 3*a^2*b
*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x), -1/8*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f
)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^2*f*x^4 - 8*b^3*c
+ (4*a*b^2*e - 3*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x)]
```

Sympy [A] time = 6.77758, size = 250, normalized size = 2.14

$$-\frac{3a^{\frac{3}{2}}fx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}ex\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}fx^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0 \\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0 \\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*e*x*sqrt(1 + b*x**2/a
)/(2*b) - sqrt(a)*f*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a**2*f*asinh(sqrt(b)*
x/sqrt(a))/(8*b**(5/2)) - a*e*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*Pie
cewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/
b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt
(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a +
f*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.2104, size = 163, normalized size = 1.39

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2fx^2}{b}-\frac{3abf-4b^2e}{b^3}\right)x+\frac{2\sqrt{b}c}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}-\frac{\left(8b^{\frac{5}{2}}d+3a^2\sqrt{b}f-4ab^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*f*x^2/b - (3*a*b*f - 4*b^2*e)/b^3)*x + 2*sqrt(b)*c/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(8*b^(5/2)*d + 3*a^2*sqrt(b)*f
- 4*a*b^(3/2)*e)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^3
```

3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}} dx$

Optimal. Leaf size=110

$$\frac{\sqrt{a+bx^2}(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2])/(3*a^2*x
) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.127105, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1265, 388, 217, 206}

$$\frac{\sqrt{a+bx^2}(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2])/(3*a^2*x
) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(3/2))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{3ax^3} - \frac{\int \frac{(2bc-3ad)x-3aex^3-3afx^5}{x^3\sqrt{a+bx^2}}\,dx}{3a}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} - \frac{\int \frac{2bc-3ad-3aex^2-3afx^4}{x^2\sqrt{a+bx^2}}\,dx}{3a}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{\int \frac{3a^2e+3a^2fx^2}{\sqrt{a+bx^2}}\,dx}{3a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{fx\sqrt{a+bx^2}}{2b} + \frac{(2be-af)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{fx\sqrt{a+bx^2}}{2b} + \frac{(2be-af)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{fx\sqrt{a+bx^2}}{2b} + \frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.105422, size = 93, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(3a^2fx^4-2ab\left(c+3dx^2\right)+4b^2cx^2\right)}{6a^2bx^3} + \frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*b^2*c*x^2 + 3*a^2*f*x^4 - 2*a*b*(c + 3*d*x^2)))/(6*a^2*
b*x^3) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Maple [A] time = 0.009, size = 117, normalized size = 1.1

$$\frac{fx}{2b}\sqrt{bx^2+a} - \frac{af}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} + e\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} - \frac{c}{3ax^3}\sqrt{bx^2+a} + \frac{2bc}{3a^2x}\sqrt{bx^2+a} - \frac{d}{ax}\sqrt{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(b*x^2+a)^(1/2)/b-1/2*f*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+e*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-1/3*c*(b*x^2+a)^(1/2)/a/x^3+2/3*c*b/a^
2/x*(b*x^2+a)^(1/2)-d/a/x*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46964, size = 479, normalized size = 4.35

$$\left[-\frac{3\left(2\,a^2be-a^3f\right)\sqrt{b}x^3\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(3\,a^2bfx^4-2\,ab^2c+2\left(2\,b^3c-3\,ab^2d\right)x^2\right)\sqrt{bx^2+a}}{12\,a^2b^2x^3},-\frac{3\,(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*b*e - a^3*f)*sqrt(b)*x^3*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*c - 3*a*b^2*d)*x^2
)*sqrt(b*x^2 + a))/(a^2*b^2*x^3), -1/6*(3*(2*a^2*b*e - a^3*f)*sqrt(-b)*x^3*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*
c - 3*a*b^2*d)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3)]
```

Sympy [A] time = 3.97899, size = 197, normalized size = 1.79

$$\frac{\sqrt{a}fx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+e\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3ax^2}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{2b^{\frac{3}{2}}c\sqrt{\right.}{3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + b*x**2/a)/(2*b) - a*f*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + e*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)
*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x*
*2) + 1)/(3*a*x**2) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*c*sqrt(
a/(b*x**2) + 1)/(3*a**2)
```

Giac [A] time = 1.20394, size = 238, normalized size = 2.16

$$\frac{\sqrt{bx^2+a}\,fx}{2\,b} + \frac{\left(a\sqrt{b}f - 2\,b^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right)}{4\,b^2} + \frac{2\left(3\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^4\sqrt{b}d + 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}c - 6\cdots\right.}{3\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + 1/4*(a*sqrt(b)*f - 2*b^(3/2)*e)*log((sqrt(b)*x
- sqrt(b*x^2 + a))^2)/b^2 + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*sqrt(b)*
d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c - 6*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a*sqrt(b)*d - 2*a*b^(3/2)*c + 3*a^2*sqrt(b)*d)/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^3
```

3.156 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{a+bx^2}\left(15a^2e-10abd+8b^2c\right)}{15a^3x}+\frac{\sqrt{a+bx^2}(4bc-5ad)}{15a^2x^3}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2])/(15*a^2*
x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2])/(15*a^3*x) + (f*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.133001, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1265, 451, 217, 206}

$$-\frac{\sqrt{a+bx^2}\left(15a^2e-10abd+8b^2c\right)}{15a^3x}+\frac{\sqrt{a+bx^2}(4bc-5ad)}{15a^2x^3}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2])/(15*a^2*
x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2])/(15*a^3*x) + (f*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{(4bc-5ad)x-5aex^3-5afx^5}{x^5\sqrt{a+bx^2}}\,dx}{5a}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{4bc-5ad-5aex^2-5afx^4}{x^4\sqrt{a+bx^2}}\,dx}{5a}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} + \frac{\int \frac{8b^2c-10abd+15a^2e+15a^2fx^2}{x^2\sqrt{a+bx^2}}\,dx}{15a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{\left(8b^2c-10abd+15a^2e\right)\sqrt{a+bx^2}}{15a^3x} + f\int \frac{1}{\sqrt{a+bx^2}}\,d$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{\left(8b^2c-10abd+15a^2e\right)\sqrt{a+bx^2}}{15a^3x} + f\,\mathrm{Subst}\left(\int \frac{1}{1-$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{\left(8b^2c-10abd+15a^2e\right)\sqrt{a+bx^2}}{15a^3x} + \frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx}}\right.}{\sqrt{b}}$$

Mathematica [A] time = 0.106839, size = 95, normalized size = 0.81

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{\sqrt{a+bx^2}\left(a^2\left(3c+5dx^2+15ex^4\right)-2abx^2\left(2c+5dx^2\right)+8b^2cx^4\right)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(8*b^2*c*x^4 - 2*a*b*x^2*(2*c + 5*d*x^2) + a^2*(3*c + 5*d
*x^2 + 15*e*x^4)))/(15*a^3*x^5) + (f*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
Sqrt[b]
```

Maple [A] time = 0.007, size = 136, normalized size = 1.2

$$f\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} - \frac{d}{3\,ax^3}\sqrt{bx^2+a} + \frac{2\,bd}{3\,a^2x}\sqrt{bx^2+a} - \frac{c}{5\,ax^5}\sqrt{bx^2+a} + \frac{4\,bc}{15\,x^3a^2}\sqrt{bx^2+a} - \frac{8\,b^2c}{15\,a^3x}\sqrt{bx^2+a} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x)
```

```
[Out] f*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-1/3*d/a/x^3*(b*x^2+a)^(1/2)+2/3*d*b
/a^2/x*(b*x^2+a)^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x^5+4/15*c*b/a^2/x^3*(b*x^2+
a)^(1/2)-8/15*c*b^2/a^3/x*(b*x^2+a)^(1/2)-e/a/x*(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40456, size = 509, normalized size = 4.31

$$\left[\frac{15\,a^3\sqrt{b}fx^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(\left(8\,b^3c-10\,ab^2d+15\,a^2be\right)x^4+3\,a^2bc-\left(4\,ab^2c-5\,a^2bd\right)x^2\right)\sqrt{bx^2}}{30\,a^3bx^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^3*sqrt(b)*f*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4 + 3*a^2*b*c - (4*a*b^2*c - 5*
a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5), -1/15*(15*a^3*sqrt(-b)*f*x^5*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4
 + 3*a^2*b*c - (4*a*b^2*c - 5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5)]
```

Sympy [A] time = 3.20046, size = 456, normalized size = 3.86

$$-\frac{3a^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2a^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3a^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{12a}{15a^5b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*a**2*b**(13/2)
*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*a*b**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*b**(17/2)*c*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + f*
Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt
(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*s
qrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(
3*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*d*sqrt(a/(b*x**2)
 + 1)/(3*a**2)
```

Giac [B] time = 1.29502, size = 437, normalized size = 3.7

$$-\frac{f\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}}+\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8\sqrt{b}e+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{3}{2}}d-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 a\sqrt{b}e\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*f*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*sqrt(b)*e + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(3/2)*
d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(b)*e + 80*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*b^(5/2)*c - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d + 9
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*e - 40*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a*b^(5/2)*c + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*d -
60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*e + 8*a^2*b^(5/2)*c - 10*a^3
*b^(3/2)*d + 15*a^4*sqrt(b)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{a+bx^2}\left(70a^2be-105a^3f-56ab^2d+48b^3c\right)}{105a^4x}-\frac{\sqrt{a+bx^2}\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}(6bc-7ad)}{35a^2x^5}-\frac{c\sqrt{a+b}}{7ax^7}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2])/(35*a^2*
x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (
(48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(105*a^4*
x)
```

Rubi [A] time = 0.184285, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1803, 12, 264}

$$\frac{\sqrt{a+bx^2}\left(70a^2be-105a^3f-56ab^2d+48b^3c\right)}{105a^4x}-\frac{\sqrt{a+bx^2}\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}(6bc-7ad)}{35a^2x^5}-\frac{c\sqrt{a+b}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2])/(35*a^2*
x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (
(48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(105*a^4*
x)
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{7ax^7} - \frac{\int \frac{6bc-7a\left(d+ex^2+fx^4\right)}{x^6\sqrt{a+bx^2}}\,dx}{7a}$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} + \frac{\int \frac{4b(6bc-7ad)-5a\left(-7ae-7afx^2\right)}{x^4\sqrt{a+bx^2}}\,dx}{35a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} - \frac{\int \frac{2b\left(24b^2c-28abd\cdots\right.}{x^2\sqrt{a\cdots}}}{10\cdots}$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} - \frac{\left(48b^3c-56ab^2\cdots\right.}{\cdots}$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} + \frac{\left(48b^3c-56ab^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.0782557, size = 103, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(2a^2bx^2\left(9c+14dx^2+35ex^4\right)-a^3\left(15c+21dx^2+35x^4\left(e+3fx^2\right)\right)-8ab^2x^4\left(3c+7dx^2\right)+48b^3cx^6\right)}{105a^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*b^3*c*x^6 - 8*a*b^2*x^4*(3*c + 7*d*x^2) + 2*a^2*b*x^2*
(9*c + 14*d*x^2 + 35*e*x^4) - a^3*(15*c + 21*d*x^2 + 35*x^4*(e + 3*f*x^2)))
)/(105*a^4*x^7)
```

Maple [A] time = 0.005, size = 111, normalized size = 0.8

$$-\frac{105\,a^3fx^6-70\,a^2bex^6+56\,ab^2dx^6-48\,b^3cx^6+35\,a^3ex^4-28\,a^2bdx^4+24\,ab^2cx^4+21\,a^3dx^2-18\,a^2bcx^2+15\,ca^3}{105\,x^7a^4}\sqrt{bx^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(105*a^3*f*x^6-70*a^2*b*e*x^6+56*a*b^2*d*x^6-48*b^3*
c*x^6+35*a^3*e*x^4-28*a^2*b*d*x^4+24*a*b^2*c*x^4+21*a^3*d*x^2-18*a^2*b*c*x^
2+15*a^3*c)/x^7/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66397, size = 232, normalized size = 1.66

$$\frac{\left(\left(48\,b^3c-56\,ab^2d+70\,a^2be-105\,a^3f\right)x^6-\left(24\,ab^2c-28\,a^2bd+35\,a^3e\right)x^4-15\,a^3c+3\left(6\,a^2bc-7\,a^3d\right)x^2\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*x^6 - (24*a*b^2*c -
 28*a^2*b*d + 35*a^3*e)*x^4 - 15*a^3*c + 3*(6*a^2*b*c - 7*a^3*d)*x^2)*sqrt(
b*x^2 + a)/(a^4*x^7)
```

Sympy [B] time = 4.74422, size = 891, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*a**6*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*a**5*b**(21/2)*c*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*a**4*b**(23/2)*c*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4
*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 5*a**3*b**(25/2)*c*x**6*sq
rt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**1
1*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*d*x**2*sqrt(a/(b*x**2) +
1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 30*a**2*b*
*(27/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*a**2*b**(13/2)*d*x**4*s
qrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x
**8) + 40*a*b**(29/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105
*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(1
5/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) + 16*b**(31/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9
*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) -
 8*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqr
t(b)*f*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*e*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

Giac [B] time = 1.22979, size = 748, normalized size = 5.34

$$\frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}\sqrt{b}f-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a\sqrt{b}f+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{3}{2}}e+560\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*sqrt(b)*f - 630*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*a*sqrt(b)*f + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(3/2
)*e + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)*d + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^2*sqrt(b)*f - 910*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(3/
2)*e + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)*c - 1400*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a*b^(5/2)*d - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*sq
```

```
rt(b)*f + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*e - 1008*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c + 1176*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^2*b^(5/2)*d + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*f - 1260*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(3/2)*e + 336*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^2*b^(7/2)*c - 392*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2)*d
- 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*f + 490*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^4*b^(3/2)*e - 48*a^3*b^(7/2)*c + 56*a^4*b^(5/2)*d + 105*a^
6*sqrt(b)*f - 70*a^5*b^(3/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$-\frac{2b\sqrt{a+bx^2}\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^5x}+\frac{\sqrt{a+bx^2}\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^4x^3}-\frac{\sqrt{a+bx^2}\left(21a^2e\right.}{105a}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(9*a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2])/(63*a^2*
x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^5) + (
(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^4*
x^3) - (2*b*(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2
])/(315*a^5*x)
```

Rubi [A] time = 0.254315, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1803, 12, 271, 264}

$$-\frac{2b\sqrt{a+bx^2}\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^5x}+\frac{\sqrt{a+bx^2}\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^4x^3}-\frac{\sqrt{a+bx^2}\left(21a^2e\right.}{105a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```


```
[Out] -(c*Sqrt[a + b*x^2])/(9*a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2])/(63*a^2*
x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^5) + (
(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^4*
x^3) - (2*b*(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2
])/(315*a^5*x)
```


Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{9ax^9} - \frac{\int \frac{8bc-9a\left(d+ex^2+fx^4\right)}{x^8\sqrt{a+bx^2}}\,dx}{9a}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} + \frac{\int \frac{6b(8bc-9ad)-7a\left(-9ae-9afx^2\right)}{x^6\sqrt{a+bx^2}}\,dx}{63a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} - \frac{\int \frac{4b\left(48b^2c-54ab\ldots\right.}{x^4\sqrt{\ldots}}}{3\ldots}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} - \frac{\left(64b^3c-72ab\ldots\right.}{\ldots}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} + \frac{\left(64b^3c-72ab\ldots\right.}{\ldots}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} + \frac{\left(64b^3c-72ab\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.0855189, size = 134, normalized size = 0.71

$$-\frac{\sqrt{a+bx^2}\left(24a^2b^2x^4\left(2c+3dx^2+7ex^4\right)-2a^3bx^2\left(20c+27dx^2+42ex^4+105fx^6\right)+a^4\left(35c+45dx^2+63ex^4+105fx\ldots\right.\right.}{315a^5x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(128*b^4*c*x^8 - 16*a*b^3*x^6*(4*c + 9*d*x^2) + 24*a^2*b^
2*x^4*(2*c + 3*d*x^2 + 7*e*x^4) - 2*a^3*b*x^2*(20*c + 27*d*x^2 + 42*e*x^4 +
 105*f*x^6) + a^4*(35*c + 45*d*x^2 + 63*e*x^4 + 105*f*x^6)))/(315*a^5*x^9)
```

Maple [A] time = 0.006, size = 157, normalized size = 0.8

$$-\frac{-210\,a^3bfx^8+168\,a^2b^2ex^8-144\,ab^3dx^8+128\,b^4cx^8+105\,a^4fx^6-84\,a^3bex^6+72\,a^2b^2dx^6-64\,ab^3cx^6+63\,a^4ex^4-54}{315\,x^9a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/315*(b*x^2+a)^(1/2)*(-210*a^3*b*f*x^8+168*a^2*b^2*e*x^8-144*a*b^3*d*x^8+
128*b^4*c*x^8+105*a^4*f*x^6-84*a^3*b*e*x^6+72*a^2*b^2*d*x^6-64*a*b^3*c*x^6+
63*a^4*e*x^4-54*a^3*b*d*x^4+48*a^2*b^2*c*x^4+45*a^4*d*x^2-40*a^3*b*c*x^2+35
*a^4*c)/x^9/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26911, size = 327, normalized size = 1.73

$$-\frac{\left(2\left(64\,b^4c-72\,ab^3d+84\,a^2b^2e-105\,a^3bf\right)x^8-\left(64\,ab^3c-72\,a^2b^2d+84\,a^3be-105\,a^4f\right)x^6+35\,a^4c+3\left(16\,a^2b^2c-18\right.}{315\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/315*(2*(64*b^4*c - 72*a*b^3*d + 84*a^2*b^2*e - 105*a^3*b*f)*x^8 - (64*a*
b^3*c - 72*a^2*b^2*d + 84*a^3*b*e - 105*a^4*f)*x^6 + 35*a^4*c + 3*(16*a^2*b
^2*c - 18*a^3*b*d + 21*a^4*e)*x^4 - 5*(8*a^3*b*c - 9*a^4*d)*x^2)*sqrt(b*x^2
 + a)/(a^5*x^9)
```

Sympy [B] time = 6.78238, size = 1642, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**8*b**(33/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*
b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**2
0*x**16) - 100*a**7*b**(35/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x
**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14
 + 315*a**5*b**20*x**16) - 98*a**6*b**(37/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(3
15*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a
**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**6*b**(19/2)*d*sqrt(a/(b*x**2
) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35
*a**4*b**12*x**12) - 28*a**5*b**(39/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(315*a**
9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b*
*19*x**14 + 315*a**5*b**20*x**16) - 9*a**5*b**(21/2)*d*x**2*sqrt(a/(b*x**2)
 + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*
a**4*b**12*x**12) - 35*a**4*b**(41/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(315*a**9
*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**
19*x**14 + 315*a**5*b**20*x**16) - 5*a**4*b**(23/2)*d*x**4*sqrt(a/(b*x**2)
+ 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a
**4*b**12*x**12) - 3*a**4*b**(9/2)*e*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**
4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 280*a**3*b**(43/2)*c*x**10*sqr
t(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*
b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 5*a**3*b**(25
/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 +
105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*e*x**2*sqrt(
a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
 - 560*a**2*b**(45/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1
260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*
a**5*b**20*x**16) + 30*a**2*b**(27/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*
b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**1
```

```
2) - 3*a**2*b**(13/2)*e*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a
**4*b**5*x**6 + 15*a**3*b**6*x**8) - 448*a*b**(47/2)*c*x**14*sqrt(a/(b*x**2
) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12
 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 40*a*b**(29/2)*d*x**10*s
qrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**
11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(15/2)*e*x**6*sqrt(a/(b*x**2) + 1
)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 128*b**(49/
2)*c*x**16*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**1
0 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) +
 16*b**(31/2)*d*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b*
*10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 8*b**(17/2)*e*x**8
*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6
*x**8) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*f*sqrt(a/(b
*x**2) + 1)/(3*a**2)
```

Giac [B] time = 1.24343, size = 900, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(3/2)*f - 1995*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*a*b^(3/2)*f + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(5/
2)*e + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d + 5355*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^2*b^(3/2)*f - 3780*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
*b^(5/2)*e + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c - 6552*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^3*b^(3/2)*f + 6804*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*e - 5376*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c + 6048*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*d + 6825*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*f -
 6216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*e + 2304*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a^2*b^(9/2)*c - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(7/2)*d - 3465*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*f + 3024*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*e - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^3*b^(9/2)*c + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)*d + 945*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*f - 756*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^5*b^(5/2)*e + 64*a^4*b^(9/2)*c - 72*a^5*b^(7/2)*d - 105*a^7*b^(3/
2)*f + 84*a^6*b^(5/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.159 $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=381

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{210a^2b^4\left(a+bx^2\right)^{3/2}}-$$

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2)) - ((2*A
*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^7)/(210*a^2*b^4*
(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(
24*b^2*B - 66*a*b*C + 143*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*
A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7
) + ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b*x^2]
)/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429*a^3*D)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rubi [A] time = 0.662335, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 459, 288, 321, 217, 206}

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{210a^2b^4\left(a+bx^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2)) - ((2*A
*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^7)/(210*a^2*b^4*
(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(
24*b^2*B - 66*a*b*C + 143*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*
A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7
) + ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b*x^2]
)/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429*a^3*D)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
```

```
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^7\left(\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^8\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^7\left(\left(8Ab-\frac{9a\left(4b^2\cdots\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^8\left(8Ab-\frac{9a\left(4b^2B\cdots\right.}{\cdots}\right.}{\left(\cdots\right.}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3-72ab^2\cdots\right.}{210\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3-72ab^2\cdots\right.}{210\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3-72ab^2\cdots\right.}{210\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3-72ab^2\cdots\right.}{210\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3-72ab^2\cdots\right.}{210\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3-72ab^2\cdots\right.}{210\cdots}$$

Mathematica [A] time = 0.533611, size = 273, normalized size = 0.72

$$x\left(-12a^3b^3\left(140A-2100Bx^2+6699Cx^4-6292Dx^6\right)+a^2b^4x^2\left(-5600A+29232Bx^2-34848Cx^4+5005Dx^6\right)+42a^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(45045*a^6*D - 2310*a^5*b*(9*C - 65*D*x^2) + 42*a^4*b^2*(180*B - 1650*C*
x^2 + 4147*D*x^4) - 12*a^3*b^3*(140*A - 2100*B*x^2 + 6699*C*x^4 - 6292*D*x^
6) - 2*a*b^5*x^4*(3248*A - 6336*B*x^2 + 1155*C*x^4 + 455*D*x^6) + a^2*b^4*x
^2*(-5600*A + 29232*B*x^2 - 34848*C*x^4 + 5005*D*x^6) + 4*b^6*x^6*(-704*A +
 35*(6*B*x^2 + 3*C*x^4 + 2*D*x^6))))/(1680*b^7*(a + b*x^2)^(7/2)) + ((16*A*
b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[
b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(15/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.301, size = 517, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] A/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*C*x^11/b/(b*x^2+a)^(7/2)+99/8*C
/b^(13/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/7*A*x^7/b/(b*x^2+a)^(7/2)-1/5
*A/b^2*x^5/(b*x^2+a)^(5/2)-1/3*A/b^3*x^3/(b*x^2+a)^(3/2)-A/b^4*x/(b*x^2+a)^
(1/2)+1/6*D*x^13/b/(b*x^2+a)^(7/2)-429/16*D/b^(15/2)*a^3*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))+1/2*B*x^9/b/(b*x^2+a)^(7/2)-9/2*B/b^(11/2)*a*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))+429/16*D/b^7*a^3*x/(b*x^2+a)^(1/2)-11/8*C/b^2*a*x^9/(b*x^2+a)^(
7/2)-99/56*C/b^3*a^2*x^7/(b*x^2+a)^(7/2)-99/40*C/b^4*a^2*x^5/(b*x^2+a)^(5/2
)-33/8*C/b^5*a^2*x^3/(b*x^2+a)^(3/2)-99/8*C/b^6*a^2*x/(b*x^2+a)^(1/2)+9/14*
B/b^2*a*x^7/(b*x^2+a)^(7/2)+9/10*B/b^3*a*x^5/(b*x^2+a)^(5/2)+3/2*B/b^4*a*x^
3/(b*x^2+a)^(3/2)+9/2*B/b^5*a*x/(b*x^2+a)^(1/2)+429/112*D/b^4*a^3*x^7/(b*x^
2+a)^(7/2)+429/80*D/b^5*a^3*x^5/(b*x^2+a)^(5/2)+143/16*D/b^6*a^3*x^3/(b*x^2
+a)^(3/2)-13/24*D/b^2*a*x^11/(b*x^2+a)^(7/2)+143/48*D/b^3*a^2*x^9/(b*x^2+a)
^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24753, size = 462, normalized size = 1.21

$$\frac{\left(\left(\left(\left(35\left(2\left(\frac{4\,Dx^2}{b} - \frac{13\,Da^4b^{11} - 6\,Ca^3b^{12}}{a^3b^{13}}\right)x^2 + \frac{143\,Da^5b^{10} - 66\,Ca^4b^{11} + 24\,Ba^3b^{12}}{a^3b^{13}}\right)x^2 + \frac{176\left(429\,Da^6b^9 - 198\,Ca^5b^{10} + 72\,Ba^4b^{11} - 16\,Aa^3b^{12}\right)}{a^3b^{13}}\right)x^2 + 40}{1680}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/1680*((((35*(2*(4*D*x^2/b - (13*D*a^4*b^11 - 6*C*a^3*b^12)/(a^3*b^13))*x^
2 + (143*D*a^5*b^10 - 66*C*a^4*b^11 + 24*B*a^3*b^12)/(a^3*b^13))*x^2 + 176*
(429*D*a^6*b^9 - 198*C*a^5*b^10 + 72*B*a^4*b^11 - 16*A*a^3*b^12)/(a^3*b^13)
)*x^2 + 406*(429*D*a^7*b^8 - 198*C*a^6*b^9 + 72*B*a^5*b^10 - 16*A*a^4*b^11)
/(a^3*b^13))*x^2 + 350*(429*D*a^8*b^7 - 198*C*a^7*b^8 + 72*B*a^6*b^9 - 16*A
*a^5*b^10)/(a^3*b^13))*x^2 + 105*(429*D*a^9*b^6 - 198*C*a^8*b^7 + 72*B*a^7*
b^8 - 16*A*a^6*b^9)/(a^3*b^13))*x/(b*x^2 + a)^(7/2) + 1/16*(429*D*a^3 - 198
*C*a^2*b + 72*B*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(15/2)
```

3.160 $\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}} dx$

Optimal. Leaf size=279

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(3a^2D-2abC+b^2B\right)}{5ab^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(99a^2D-36abC+8b^2B\right)}{60ab^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(99a^2D-36abC+8b^2B\right)}{12ab^5\sqrt{a+bx^2}}-\frac{x\sqrt{}}{}$$

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + ((b^2
*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)) + ((8*b^2*B - 36*a
*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(3/2)) + (D*x^7)/(4*b^3*(a + b*
x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^
2]) - ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b
^2*B - 36*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/
2))
```

Rubi [A] time = 0.45491, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 459, 288, 321, 217, 206}

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(3a^2D-2abC+b^2B\right)}{5ab^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(99a^2D-36abC+8b^2B\right)}{60ab^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(99a^2D-36abC+8b^2B\right)}{12ab^5\sqrt{a+bx^2}}-\frac{x\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + ((b^2
*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)) + ((8*b^2*B - 36*a
*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(3/2)) + (D*x^7)/(4*b^3*(a + b*
x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^
2]) - ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b
^2*B - 36*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/
2))
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^5\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^6\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^5\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right)x+\frac{35a^2D\cdots}{b}\right.}{\left(a+bx^2\right)^{5/2}}}{35a^2b}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^6\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right)+\frac{35a^2Dx^2\cdots}{b}\right.}{\left(a+bx^2\right)^{5/2}}}{35a^2b}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{Dx^7}{4b^3\left(a+bx^2\right)^{3/2}} - \frac{\left(8b^2B-3\cdots\right.}{}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2D\right)x^5}{60ab^4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2D\right)x^5}{60ab^4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2D\right)x^5}{60ab^4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2D\right)x^5}{60ab^4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2D\right)x^5}{60ab^4\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.430943, size = 229, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(99a^2D-36abC+8b^2B\right)}{8\sqrt{a}b^{13/2}\sqrt{\frac{bx^2}{a}+1}}-\frac{x\left(42a^4b^2\left(20B-300Cx^2+957Dx^4\right)+8a^3b^3x^2\left(350B-1827Cx^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x*(10395*a^6*D - 120*A*b^6*x^6 - 630*a^5*b*(6*C - 55*D*x^2) + 42*a^4*b^2*
(20*B - 300*C*x^2 + 957*D*x^4) + a^2*b^4*x^4*(3248*B - 6336*C*x^2 + 1155*D*
x^4) + 8*a^3*b^3*x^2*(350*B - 1827*C*x^2 + 2178*D*x^4) + 2*a*b^5*x^6*(704*B
 - 105*(2*C*x^2 + D*x^4))))/(840*a*b^6*(a + b*x^2)^(7/2)) + ((8*b^2*B - 36*
a*b*C + 99*a^2*D)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*
b^(13/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.008, size = 460, normalized size = 1.7

$$\frac{Dx^{11}}{4b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{11\,aDx^9}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{99\,a^2Dx^7}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{99\,a^2Dx^5}{40\,b^4}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{33\,Dx^3a^2}{8\,b^5}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{99}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/4*D*x^11/b/(b*x^2+a)^(7/2)-11/8*D/b^2*a*x^9/(b*x^2+a)^(7/2)-99/56*D/b^3*a
^2*x^7/(b*x^2+a)^(7/2)-99/40*D/b^4*a^2*x^5/(b*x^2+a)^(5/2)-33/8*D/b^5*a^2*x
^3/(b*x^2+a)^(3/2)-99/8*D/b^6*a^2*x/(b*x^2+a)^(1/2)+99/8*D/b^(13/2)*a^2*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*C*x^9/b/(b*x^2+a)^(7/2)+9/14*C/b^2*a*x^7/(b*
x^2+a)^(7/2)+9/10*C/b^3*a*x^5/(b*x^2+a)^(5/2)+3/2*C/b^4*a*x^3/(b*x^2+a)^(3/
2)+9/2*C/b^5*a*x/(b*x^2+a)^(1/2)-9/2*C/b^(11/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))-1/7*B*x^7/b/(b*x^2+a)^(7/2)-1/5*B/b^2*x^5/(b*x^2+a)^(5/2)-1/3*B/b^3*x^
3/(b*x^2+a)^(3/2)-B/b^4*x/(b*x^2+a)^(1/2)+B/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))-1/2*A*x^5/b/(b*x^2+a)^(7/2)-5/8*A/b^2*a*x^3/(b*x^2+a)^(7/2)-15/56*A/
b^3*a^2*x/(b*x^2+a)^(7/2)+3/56*A/b^3*a*x/(b*x^2+a)^(5/2)+1/14*A/b^3*x/(b*x^
2+a)^(3/2)+1/7*A/b^3/a*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20426, size = 358, normalized size = 1.28

$$\frac{((((105(\frac{2Dx^2}{b} - \frac{11Da^4b^9 - 4Ca^3b^{10}}{a^3b^{11}})x^2 - \frac{8(2178Da^5b^8 - 792Ca^4b^9 + 176Ba^3b^{10} - 15Aa^2b^{11})}{a^3b^{11}})x^2 - \frac{406(99Da^6b^7 - 36Ca^5b^8 + 8Ba^4b^9)}{a^3b^{11}})x^2 - \frac{350 \cdots}{\cdots}}{840(bx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/840*((((105*(2*D*x^2/b - (11*D*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^11))*x^2 -
8*(2178*D*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15*A*a^2*b^11)/(a^3*b^
11))*x^2 - 406*(99*D*a^6*b^7 - 36*C*a^5*b^8 + 8*B*a^4*b^9)/(a^3*b^11))*x^2
- 350*(99*D*a^7*b^6 - 36*C*a^6*b^7 + 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105*(99
*D*a^8*b^5 - 36*C*a^7*b^6 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/2) -
1/8*(99*D*a^2 - 36*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(13/2)
```

3.161 $\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=210

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}+\frac{ax(bC-3aD)}{3b^5\left(a+bx^2\right)^{3/2}}+\frac{(2bC-9aD)}{}$$

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + ((2*A
*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(3/2)) - ((4*b*C - 15*a*D)*x)/(
3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rubi [A] time = 0.387121, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 455, 1157, 388, 217, 206}

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}+\frac{ax(bC-3aD)}{3b^5\left(a+bx^2\right)^{3/2}}+\frac{(2bC-9aD)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```


```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + ((2*A
*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(3/2)) - ((4*b*C - 15*a*D)*x)/(
3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```


Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^3\left(-\left(2Ab+\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^4\left(-2Ab-\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^3\left(\frac{35a^2(bC-2aD)x}{b^2}\cdots\right.}{\left(a+bx^2\right)\cdots}}{35a^2\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^4\left(\frac{35a^2(bC-2aD)}{b^2}\cdots\right.}{\left(a+bx^2\right)\cdots}}{35a^2\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x\cdots}{3b^5\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x\cdots}{3b^5\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x\cdots}{3b^5\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x\cdots}{3b^5\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x\cdots}{3b^5\left(a+bx^2\right)^{3/\cdots}}$$

Mathematica [A] time = 0.440886, size = 194, normalized size = 0.92

$$\frac{\sqrt{b}x\left(14a^4b^2x^2\left(261Dx^2-50C\right)+4a^3b^3x^4\left(396Dx^2-203C\right)+a^2b^4x^6\left(105Dx^2-352C\right)-210a^5b\left(C-15Dx^2\right)+945a}{210a^2b^{11/2}\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(945*a^6*D + 12*A*b^6*x^6 + 6*a*b^5*x^4*(7*A + 5*B*x^2) - 210*a^
5*b*(C - 15*D*x^2) + a^2*b^4*x^6*(-352*C + 105*D*x^2) + 14*a^4*b^2*x^2*(-50
*C + 261*D*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*D*x^2)) + 105*a^(5/2)*(2*b*C
- 9*a*D)*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2
10*a^2*b^(11/2)*(a + b*x^2)^(7/2))
```

Maple [B] time = 0.012, size = 405, normalized size = 1.9

$$\frac{Dx^9}{2b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{9\,aDx^7}{14\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{9\,aDx^5}{10\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{3\,Dx^3a}{2\,b^4}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{9\,aDx}{2\,b^5}\frac{1}{\sqrt{bx^2+a}}-\frac{9\,aD}{2}\ln\left(x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2*D*x^9/b/(b*x^2+a)^(7/2)+9/14*D/b^2*a*x^7/(b*x^2+a)^(7/2)+9/10*D/b^3*a*x
^5/(b*x^2+a)^(5/2)+3/2*D/b^4*a*x^3/(b*x^2+a)^(3/2)+9/2*D/b^5*a*x/(b*x^2+a)^
(1/2)-9/2*D/b^(11/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/7*C*x^7/b/(b*x^2+a)^
(7/2)-1/5*C/b^2*x^5/(b*x^2+a)^(5/2)-1/3*C/b^3*x^3/(b*x^2+a)^(3/2)-C/b^4*x/(
b*x^2+a)^(1/2)+C/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/2*B*x^5/b/(b*x^2+a
)^(7/2)-5/8*B/b^2*a*x^3/(b*x^2+a)^(7/2)-15/56*B/b^3*a^2*x/(b*x^2+a)^(7/2)+3
/56*B/b^3*a*x/(b*x^2+a)^(5/2)+1/14*B/b^3*x/(b*x^2+a)^(3/2)+1/7*B/b^3/a*x/(b
*x^2+a)^(1/2)-1/4*A*x^3/b/(b*x^2+a)^(7/2)-3/28*A/b^2*a*x/(b*x^2+a)^(7/2)+3/
140*A/b^2*x/(b*x^2+a)^(5/2)+1/35*A/b^2/a*x/(b*x^2+a)^(3/2)+2/35*A/b^2/a^2*x
/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20251, size = 274, normalized size = 1.3

$$\frac{\left(\left(\left(\left(\frac{105\,Dx^2}{b} + \frac{2\left(792\,Da^4b^7 - 176\,Ca^3b^8 + 15\,Ba^2b^9 + 6\,Aab^{10}\right)}{a^3b^9}\right)x^2 + \frac{14\left(261\,Da^5b^6 - 58\,Ca^4b^7 + 3\,Aa^2b^9\right)}{a^3b^9}\right)x^2 + \frac{350\left(9\,Da^6b^5 - 2\,Ca^5b^6\right)}{a^3b^9}\right)x^2 + \frac{105\left(9\,Da^7b \cdots}{a^3 \cdots}\right. \cdots}{210\left(bx^2 + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/210*((((105*D*x^2/b + 2*(792*D*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a^2*b^9 + 6
*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*D*a^5*b^6 - 58*C*a^4*b^7 + 3*A*a^2*b^9)
/(a^3*b^9))*x^2 + 350*(9*D*a^6*b^5 - 2*C*a^5*b^6)/(a^3*b^9))*x^2 + 105*(9*D
*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9*D*a - 2*C*b
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x^7\left(15a^2bC-176a^3D+6ab^2B+8Ab^3\right)}{105a^3b\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(-58a^3D+3ab^2B+4Ab^3\right)}{15a^2b^2\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(Ab^3-10a^3D\right)}{3ab^3\left(a+bx^2\right)^{7/2}}-\frac{a^3Dx}{b^4\left(a+bx^2\right)^{7/2}}+\frac{D\tan}{}$$

```
[Out] -((a^3*D*x)/(b^4*(a + b*x^2)^(7/2))) + ((A*b^3 - 10*a^3*D)*x^3)/(3*a*b^3*(a
 + b*x^2)^(7/2)) + ((4*A*b^3 + 3*a*b^2*B - 58*a^3*D)*x^5)/(15*a^2*b^2*(a +
b*x^2)^(7/2)) + ((8*A*b^3 + 6*a*b^2*B + 15*a^2*b*C - 176*a^3*D)*x^7)/(105*a
^3*b*(a + b*x^2)^(7/2)) + (D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.313859, antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1804, 1585, 1263, 1584, 452, 288, 217, 206}

$$\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{105a^3b^3\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(17a^2D-10abC+3b^2B\right)+4Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{Dx}{b^4\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2)) + ((4*A
*b^3 + a*(3*b^2*B - 10*a*b*C + 17*a^2*D))*x^3)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + ((8*A*b^3 + a*(6*b^2*B + 15*a*b*C - 71*a^2*D))*x^3)/(105*a^3*b^3*(a +
b*x^2)^(3/2)) - (D*x)/(b^4*Sqrt[a + b*x^2]) + (D*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/b^(9/2)
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(4Ab+\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x\left(\left(8Ab+\frac{3a(2b\cdots}{\cdots}\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^2\left(8Ab+\frac{3a(2b\cdots}{\cdots}\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3+a\left(6\cdots\right.\right.}{105\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3+a\left(6\cdots\right.\right.}{105\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3+a\left(6\cdots\right.\right.}{105\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3+a\left(6\cdots\right.\right.}{105\cdots}$$

<u>Mathematica</u> [A] time = 0.410325, size = 168, normalized size = 0.94

$$\frac{a^2b^4x^3\left(35A+21Bx^2+15Cx^4\right)-176a^3b^3Dx^7-406a^4b^2Dx^5-350a^5bDx^3-105a^6Dx+2ab^5x^5\left(14A+3Bx^2\right)+8Ab^6\cdots}{105a^3b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^6*D*x - 350*a^5*b*D*x^3 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7 - 176*a^3
*b^3*D*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3*(35*A + 21*B*x^2 +
15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) + (Sqrt[a]*D*Sqrt[1 + (b*x^2)/a]
*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.008, size = 363, normalized size = 2.

$$-\frac{Dx^7}{7b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Dx^5}{5b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Dx^3}{3b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Dx}{b^4}\frac{1}{\sqrt{bx^2+a}}+D\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}-\frac{Cx^5}{2b}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*D*x^7/b/(b*x^2+a)^(7/2)-1/5*D/b^2*x^5/(b*x^2+a)^(5/2)-1/3*D/b^3*x^3/(b
*x^2+a)^(3/2)-D*x/b^4/(b*x^2+a)^(1/2)+D/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))-1/2*C*x^5/b/(b*x^2+a)^(7/2)-5/8*C/b^2*a*x^3/(b*x^2+a)^(7/2)-15/56*C/b^3*
a^2*x/(b*x^2+a)^(7/2)+3/56*C/b^3*a*x/(b*x^2+a)^(5/2)+1/14*C/b^3*x/(b*x^2+a)
^(3/2)+1/7*C/b^3/a*x/(b*x^2+a)^(1/2)-1/4*B*x^3/b/(b*x^2+a)^(7/2)-3/28*B/b^2
*a*x/(b*x^2+a)^(7/2)+3/140*B/b^2*x/(b*x^2+a)^(5/2)+1/35*B/b^2/a*x/(b*x^2+a)
^(3/2)+2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)-1/7*A/b*x/(b*x^2+a)^(7/2)+1/35*A/b/
a*x/(b*x^2+a)^(5/2)+4/105*A/b/a^2*x/(b*x^2+a)^(3/2)+8/105*A/b/a^3*x/(b*x^2+
a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34302, size = 216, normalized size = 1.21

$$-\frac{\left(\left(x^2\left(\frac{\left(176\,Da^3b^6-15\,Ca^2b^7-6\,Bab^8-8\,Ab^9\right)x^2}{a^3b^7}+\frac{7\left(58\,Da^4b^5-3\,Ba^2b^7-4\,Aab^8\right)}{a^3b^7}\right)+\frac{35\left(10\,Da^5b^4-Aa^2b^7\right)}{a^3b^7}\right)x^2+\frac{105\,Da^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{D\log\left(\left|-\sqrt{b}x+\right.\right.}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*D*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9)*x^2/(a^3
*b^7) + 7*(58*D*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3*b^7)) + 35*(10*D*a^
5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*D*a^3/b^4)*x/(b*x^2 + a)^(7/2) - D*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=134

$$\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x^2)^(7/2))
 + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*x^2)^(7/2)) + ((48*A
*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)/(105*a^4*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.21044, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1813, 1803, 12, 264}

$$\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x^2)^(7/2))
 + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*x^2)^(7/2)) + ((48*A
*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)/(105*a^4*(a + b*x^2)^(7/2))
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
```

```
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^2\left(6Ab+a\left(B+Cx^2+Dx^4\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^4\left(4b(6Ab+aB)+3a\left(aC+aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{\left(2b\left(24Ab^2+4abB+3a^2C\right)\right.}{\left(a+bx^2\right)^{9/2}}}{15a^3}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(48Ab^3+a\left(8b^2B+\right.\right.}{}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(48Ab^3+a\left(8b^2B+\right.\right.}{105a^4\left(a+\right.}$$

Mathematica [A] time = 0.100952, size = 98, normalized size = 0.73

$$\frac{2a^2bx^3\left(105A+14Bx^2+3Cx^4\right)+a^3\left(105Ax+35Bx^3+21Cx^5+15Dx^7\right)+8ab^2x^5\left(21A+Bx^2\right)+48Ab^3x^7}{105a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
+ 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(105*a^4*(a +
b*x^2)^(7/2))
```

Maple [A] time = 0.005, size = 109, normalized size = 0.8

$$\frac{x\left(48\,Ax^6b^3+8\,Bx^6ab^2+6\,a^2bCx^6+15\,Da^3x^6+168\,Ax^4ab^2+28\,Bx^4a^2b+21\,a^3Cx^4+210\,Ax^2a^2b+35\,Bx^2a^3+105\,A}{105\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*x*(48*A*b^3*x^6+8*B*a*b^2*x^6+6*C*a^2*b*x^6+15*D*a^3*x^6+168*A*a*b^2*
x^4+28*B*a^2*b*x^4+21*C*a^3*x^4+210*A*a^2*b*x^2+35*B*a^3*x^2+105*A*a^3)/(b*
x^2+a)^(7/2)/a^4
```

Maxima [B] time = 1.05759, size = 452, normalized size = 3.37

$$-\frac{Dx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Dax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Cx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4}+\frac{8\,Ax}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^
2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b
```

```
) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20181, size = 177, normalized size = 1.32

$$\frac{\left(\left(x^2\left(\frac{\left(15\,Da^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6\right)x^2}{a^4b^3}+\frac{7\left(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5\right)}{a^4b^3}\right)+\frac{35\left(Ba^3b^3+6\,Aa^2b^4\right)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*b
^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b^3
 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2bx^7\left(4b\left(48Ab^2-a(aC+6bB)\right)-3a^3D\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(4b\left(48Ab^2-a(aC+6bB)\right)-3a^3D\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(}{a^2}$$

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((4*b*(48*A*
b^2 - a*(6*b*B + a*C)) - 3*a^3*D)*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - (2*b*(4
*b*(48*A*b^2 - a*(6*b*B + a*C)) - 3*a^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.24635, antiderivative size = 179, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 1813, 12, 271, 264}

$$-\frac{2bx^7\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - (2*b*(19
2*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4\right)}{\left(a+bx^2\right)^{9/2}}dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(6b(8Ab-aB)+a\left(-aC-aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}dx}{a^2}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{\left(4b\left(48Ab^2-6abB-a^2C\right)-3a}{\left(a+bx^2\right)^{9/2}}}{3a^3}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-4ab(6bB+\right.}{}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-4ab(6bB+\right.}{15a^4\left(a+bx\right.}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-4ab(6bB+\right.}{15a^4\left(a+bx\right.}$$

Mathematica [A] time = 0.152244, size = 133, normalized size = 0.72

$$\frac{8a^2b^2x^4\left(-210A+21Bx^2+Cx^4\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)-7a^4\left(15A-15Bx^2-5Cx^4-3Dx^6\right)+4}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) - 7*a^4*(15*A - 15*B*x^2 - 5*C*x^4 - 3*D*x^6) + 2*a^3*b*x^2
*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6))/(105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 157, normalized size = 0.9

$$-\frac{384\,Ab^4x^8-48\,Bab^3x^8-8\,Ca^2b^2x^8-6\,Da^3bx^8+1344\,Aab^3x^6-168\,Ba^2b^2x^6-28\,Ca^3bx^6-21\,Da^4x^6+1680\,Aa^2b^2x^4}{105\,xa^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^8+1344*A*a
*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-21*D*a^4*x^6+1680*A*a^2*b^2*x^4-2
10*B*a^3*b*x^4-35*C*a^4*x^4+840*A*a^3*b*x^2-105*B*a^4*x^2+105*A*a^4)/x/(b*x
^2+a)^(7/2)/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

[Out] Timed out

Giac [A] time = 1.21629, size = 285, normalized size = 1.54

$$\frac{\left(\left(x^2\left(\frac{\left(6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5\right)}{a^{14}b^3}\right)x^2+\frac{105\left(B\right.}{}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A*a^9*b^7)*
x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11*b^5 - 132*A*a^1
0*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^14*b
^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7/2)
+ 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=242

$$\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{15a^5\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(160Ab^3-a\left(a^2(-\right.\right.}{3a^4\left(a+\right.}$$

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b*x^2)^(7/2
)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x^2)^(7/2)) + ((160*A
*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(3*a^4*(a + b*x^2)^(7/2)) + (4*
b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(7/
2)) + (8*b^2*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(a
+ b*x^2)^(7/2))
```

Rubi [A] time = 0.321803, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 1813, 12, 271, 264}

$$\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{15a^5\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(160Ab^3-a\left(a^2(-\right.\right.}{3a^4\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```


```
[Out] -A/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b*x^2)^(7/2
)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x^2)^(7/2)) + ((160*A
*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(3*a^4*(a + b*x^2)^(7/2)) + (4*
b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(7/
2)) + (8*b^2*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(a
+ b*x^2)^(7/2))
```


Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
```

```
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{10Ab-3a\left(B+Cx^2+Dx^4\right)}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{8b(10Ab-3aB)-a\left(-3aC-3aDx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{\left(6b\left(80Ab^2-24abB\cdots\right.\right.}{\left(a+b\cdots\right.}}{3\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3-a\left(48\cdots\right.\right.}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3-a\left(48\cdots\right.\right.}{3a^4\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3-a\left(48\cdots\right.\right.}{3a^4\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3-a\left(48\cdots\right.\right.}{3a^4\cdots}$$

Mathematica [A] time = 0.129439, size = 165, normalized size = 0.68

$$\frac{8a^3b^2x^4\left(350A-210Bx^2+21Cx^4+Dx^6\right)+16a^2b^3x^6\left(350A-84Bx^2+3Cx^4\right)+14a^4bx^2\left(25A-60Bx^2+15Cx^4+2D\cdots\right.}{105a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A -
 84*B*x^2 + 3*C*x^4) - 35*a^5*(A + 3*B*x^2 - 3*C*x^4 - D*x^6) + 8*a^3*b^2*x
^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^6) + 14*a^4*b*x^2*(25*A - 60*B*x^2 +
 15*C*x^4 + 2*D*x^6))/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.007, size = 205, normalized size = 0.9

$$-\frac{-1280\,Ab^5x^{10}+384\,Bab^4x^{10}-48\,Ca^2b^3x^{10}-8\,Da^3b^2x^{10}-4480\,Aab^4x^8+1344\,Ba^2b^3x^8-168\,Ca^3b^2x^8-28\,Da^4bx^8-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-1280*A*b^5*x^10+384*B*a*b^4*x^10-48*C*a^2*b^3*x^10-8*D*a^3*b^2*x^1
0-4480*A*a*b^4*x^8+1344*B*a^2*b^3*x^8-168*C*a^3*b^2*x^8-28*D*a^4*b*x^8-5600
*A*a^2*b^3*x^6+1680*B*a^3*b^2*x^6-210*C*a^4*b*x^6-35*D*a^5*x^6-2800*A*a^3*b
^2*x^4+840*B*a^4*b*x^4-105*C*a^5*x^4-350*A*a^4*b*x^2+105*B*a^5*x^2+35*A*a^5
)/x^3/(b*x^2+a)^(7/2)/a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21572, size = 471, normalized size = 1.95

$$\frac{\left(\left(x^2\left(\frac{\left(8\,Da^{15}b^5+48\,Ca^{14}b^6-279\,Ba^{13}b^7+790\,Aa^{12}b^8\right)x^2}{a^{18}b^3}+\frac{7\left(4\,Da^{16}b^4+24\,Ca^{15}b^5-132\,Ba^{14}b^6+365\,Aa^{13}b^7\right)}{a^{18}b^3}\right)+\frac{35\left(Da^{17}b^3+6\,Ca^{16}b^4-30\,Ba^{15}b^5+80\,Aa^{1}\right.}{a^{18}b^3}\right.\right.}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790*A*a^12*b^
8)*x^2/(a^18*b^3) + 7*(4*D*a^16*b^4 + 24*C*a^15*b^5 - 132*B*a^14*b^6 + 365*
A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 + 6*C*a^16*b^4 - 30*B*a^15*b^5 + 8
0*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a^15*
b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A
*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3*a^5)
```

3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{16x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x\left(192Ab^3-a\left(3a^2D-24\right.\right.}{35a^5\left(a+bx^2\right)}$$

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^(
7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b*x^2)^(7/2)) - ((192
*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(21*a^4*(a + b*x^2)^(7/2)) -
 (2*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(35*a^5*(a + b*x^2)^
(5/2)) - (8*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a
+ b*x^2)^(3/2)) - (16*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(1
05*a^7*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.428996, antiderivative size = 275, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1803, 12, 192, 191}

$$-\frac{16x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x\left(-3a^3D-8ab(10bB-3a\right.}{35a^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^(
7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b*x^2)^(7/2)) - ((192
*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(21*a^4*(a + b*x^2)^(7/2)) -
(2*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(35*a^5*(a + b*x^2)^(5
/2)) - (8*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a +
b*x^2)^(3/2)) - (16*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(105*
a^7*Sqrt[a + b*x^2])
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
```

```
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{12Ab-5a\left(B+Cx^2+Dx^4\right)}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{10b(12Ab-5aB)-3a\left(-5aC-5aDx^2\right)}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{15a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8b\left(120Ab^2-50abB+1\ldots\right.}{\left(a+bx^2\right)^{9\ldots}}}{15a^3\ldots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-8ab(10\ldots\right.}{\ldots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-8ab(10\ldots\right.}{21a^4\left(a\ldots\right.}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-8ab(10\ldots\right.}{21a^4\left(a\ldots\right.}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-8ab(10\ldots\right.}{21a^4\left(a\ldots\right.}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-8ab(10\ldots\right.}{21a^4\left(a\ldots\right.}$$

Mathematica [A] time = 0.146291, size = 202, normalized size = 0.72

$$\frac{16a^3b^3x^6\left(-420A+350Bx^2-84Cx^4+3Dx^6\right)+56a^4b^2x^4\left(-15A+50Bx^2-30Cx^4+3Dx^6\right)-128a^2b^4x^8\left(105A-35Bx^2\ldots\right.}{105\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(105
*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350*B*x^2 - 84*C*x^4 +
```

```
3*D*x^6) + 56*a^4*b^2*x^4*(-15*A + 50*B*x^2 - 30*C*x^4 + 3*D*x^6) + 14*a^5*
b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 + 15*D*x^6) - 7*a^6*(3*A + 5*x^2*(B + 3*C*
x^2 - 3*D*x^4)))/(105*a^7*x^5*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.007, size = 253, normalized size = 0.9

$$-\frac{3072\,Ab^{6}x^{12}-1280\,Bab^{5}x^{12}+384\,Ca^{2}b^{4}x^{12}-48\,Da^{3}b^{3}x^{12}+10752\,Aab^{5}x^{10}-4480\,Ba^{2}b^{4}x^{10}+1344\,Ca^{3}b^{3}x^{10}-168}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(3072*A*b^6*x^12-1280*B*a*b^5*x^12+384*C*a^2*b^4*x^12-48*D*a^3*b^3*x
^12+10752*A*a*b^5*x^10-4480*B*a^2*b^4*x^10+1344*C*a^3*b^3*x^10-168*D*a^4*b^
2*x^10+13440*A*a^2*b^4*x^8-5600*B*a^3*b^3*x^8+1680*C*a^4*b^2*x^8-210*D*a^5*
b*x^8+6720*A*a^3*b^3*x^6-2800*B*a^4*b^2*x^6+840*C*a^5*b*x^6-105*D*a^6*x^6+8
40*A*a^4*b^2*x^4-350*B*a^5*b*x^4+105*C*a^6*x^4-84*A*a^5*b*x^2+35*B*a^6*x^2+
21*A*a^6)/x^5/(b*x^2+a)^(7/2)/a^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25856, size = 799, normalized size = 2.84

$$\frac{\left(\left(x^2\left(\frac{(48\,Da^{18}b^6-279\,Ca^{17}b^7+790\,Ba^{16}b^8-1686\,Aa^{15}b^9)x^2}{a^{22}b^3}+\frac{7\left(24\,Da^{19}b^5-132\,Ca^{18}b^6+365\,Ba^{17}b^7-768\,Aa^{16}b^8\right)}{a^{22}b^3}\right)+\frac{35\left(6\,Da^{20}b^4-30\,Ca^{19}b^5+80\,Ba^{18}b^6-\right.}{a^{22}b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1686*A*a^15
*b^9)*x^2/(a^22*b^3) + 7*(24*D*a^19*b^5 - 132*C*a^18*b^6 + 365*B*a^17*b^7 -
 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*D*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a^18
*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4 + 1
0*B*a^19*b^5 - 20*A*a^18*b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(15*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^3*b^(3/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2)
- 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(
5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2))/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=334

$$\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^7\left(a+bx^2\right)^{3/2}}+\frac{16bx\left(48Ab^3-a\left(3a^2D-\right.\right.}{35a^6\left(a+b\right.}$$

```
[Out] -A/(7*a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*x^2)^(7/2)
) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*x^2)^(7/2)) + (48*A*
b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a^4*x*(a + b*x^2)^(7/2)) + (8*b
*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7/2
)) + (16*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a + b
*x^2)^(5/2)) + (64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(105
*a^7*(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2
*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.480311, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 12, 271, 192, 191}

$$\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^7\left(a+bx^2\right)^{3/2}}+\frac{16bx\left(48Ab^3-a\left(3a^2D-\right.\right.}{35a^6\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```


```
[Out] -A/(7*a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*x^2)^(7/2)
) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*x^2)^(7/2)) + (48*A*
b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a^4*x*(a + b*x^2)^(7/2)) + (8*b
*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7/2
)) + (16*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a + b
*x^2)^(5/2)) + (64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(105
*a^7*(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2
*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```


Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
```

```
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{14Ab-7a\left(B+Cx^2+Dx^4\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{12b(14Ab-7aB)-5a\left(-7aC-7aDx^2\right)}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{35a^2}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{10b\left(168Ab^2-84abB\ldots\right.}{x^2\left(a+bx\ldots\right.}}{105\ldots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^3-a\left(24b^2\ldots\right.\right.}{\ldots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(24b^2B\ldots\right.}{3a^4x\left(a\ldots\right.}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(24b^2B\ldots\right.}{3a^4x\left(a\ldots\right.}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(24b^2B\ldots\right.}{3a^4x\left(a\ldots\right.}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(24b^2B\ldots\right.}{3a^4x\left(a\ldots\right.}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(24b^2B\ldots\right.}{3a^4x\left(a\ldots\right.}$$

Mathematica [A] time = 0.146353, size = 234, normalized size = 0.7

$$\frac{128a^3b^4x^8\left(105A-105Bx^2+35Cx^4-3Dx^6\right)+112a^4b^3x^6\left(15A-60Bx^2+50Cx^4-12Dx^6\right)-56a^5b^2x^4\left(3A+15Bx^2-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] (6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x^10*(105*A
 - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 + 25*C*x^4 - 60*D*x^6)
 + 112*a^4*b^3*x^6*(15*A - 60*B*x^2 + 50*C*x^4 - 12*D*x^6) + 128*a^3*b^4*x^
8*(105*A - 105*B*x^2 + 35*C*x^4 - 3*D*x^6) - 56*a^5*b^2*x^4*(3*A + 15*B*x^2
 - 50*C*x^4 + 30*D*x^6) - a^7*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)))/(10
5*a^8*x^7*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.008, size = 301, normalized size = 0.9

$$-\frac{-6144\,Ab^{7}x^{14}+3072\,Bab^{6}x^{14}-1280\,Ca^{2}b^{5}x^{14}+384\,Da^{3}b^{4}x^{14}-21504\,Aab^{6}x^{12}+10752\,Ba^{2}b^{5}x^{12}-4480\,Ca^{3}b^{4}x^{12}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6144*A*b^7*x^14+3072*B*a*b^6*x^14-1280*C*a^2*b^5*x^14+384*D*a^3*b^
4*x^14-21504*A*a*b^6*x^12+10752*B*a^2*b^5*x^12-4480*C*a^3*b^4*x^12+1344*D*a
^4*b^3*x^12-26880*A*a^2*b^5*x^10+13440*B*a^3*b^4*x^10-5600*C*a^4*b^3*x^10+1
680*D*a^5*b^2*x^10-13440*A*a^3*b^4*x^8+6720*B*a^4*b^3*x^8-2800*C*a^5*b^2*x^
8+840*D*a^6*b*x^8-1680*A*a^4*b^3*x^6+840*B*a^5*b^2*x^6-350*C*a^6*b*x^6+105*
D*a^7*x^6+168*A*a^5*b^2*x^4-84*B*a^6*b*x^4+35*C*a^7*x^4-42*A*a^6*b*x^2+21*B
*a^7*x^2+15*A*a^7)/x^7/(b*x^2+a)^(7/2)/a^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33665, size = 1266, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((279*D*a^21*b^7 - 790*C*a^20*b^8 + 1686*B*a^19*b^9 - 3072*A*a
^18*b^10)*x^2/(a^26*b^3) + 7*(132*D*a^22*b^6 - 365*C*a^21*b^7 + 768*B*a^20*
b^8 - 1386*A*a^19*b^9)/(a^26*b^3)) + 35*(30*D*a^23*b^5 - 80*C*a^22*b^6 + 16
5*B*a^21*b^7 - 294*A*a^20*b^8)/(a^26*b^3))*x^2 + 105*(4*D*a^24*b^4 - 10*C*a
^23*b^5 + 20*B*a^22*b^6 - 35*A*a^21*b^7)/(a^26*b^3))*x/(b*x^2 + a)^(7/2) +
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*sqrt(b) - 420*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*C*a^2*b^(3/2) + 1050*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*B*a*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 630*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*D*a^4*sqrt(b) + 2730*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*C*a^3*b^(3/2) - 7140*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) +
 14700*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt(b)*x - sqr
```

```
t(b*x^2 + a))^8*D*a^5*sqrt(b) - 7210*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^4*
b^(3/2) + 19950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) - 42840*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*D*a^6*sqrt(b) + 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^5*b^(3/2) - 2
8560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 64680*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*
sqrt(b) - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3/2) + 21966*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 49812*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*sqrt(b) + 30
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^7*b^(3/2) - 8652*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^6*b^(5/2) + 19404*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^
(7/2) + 105*D*a^9*sqrt(b) - 490*C*a^8*b^(3/2) + 1386*B*a^7*b^(5/2) - 3072*A
*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^7)
```

3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=392

$$-\frac{256b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{315a^8\left(a+bx^2\right)^{3/2}}-\frac{32b^2x\left(128Ab^3\right.}{1}$$

```
[Out] -A/(9*a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a + b*x^2)^(
7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a + b*x^2)^(7/2)) + (12
8*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5*a^2*D))/(45*a^4*x^3*(a + b*x^2)^(7/2
)) - (2*b*(128*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5*a^2*D)))/(9*a^5*x*(a +
b*x^2)^(7/2)) - (16*b^2*(128*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5*a^2*D))*x
)/(63*a^6*(a + b*x^2)^(7/2)) - (32*b^2*(128*A*b^3 - 3*a*(24*b^2*B - 12*a*b*
C + 5*a^2*D))*x)/(105*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 3*a*(2
4*b^2*B - 12*a*b*C + 5*a^2*D))*x)/(315*a^8*(a + b*x^2)^(3/2)) - (256*b^2*(1
28*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5*a^2*D))*x)/(315*a^9*Sqrt[a + b*x^2]
)
```

Rubi [A] time = 0.546361, antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 12, 271, 192, 191}

$$-\frac{256b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^8\left(a+bx^2\right)^{3/2}}-\frac{32b^2x\left(-15a^3D-\right.}{105a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(9*a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a + b*x^2)^(
7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a + b*x^2)^(7/2)) + (12
8*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)/(45*a^4*x^3*(a + b*x^2)^(7/2)) -
 (2*b*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D))/(9*a^5*x*(a + b*x^2)^(
7/2)) - (16*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(63*a^6*(a
 + b*x^2)^(7/2)) - (32*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)
/(105*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) -
 15*a^3*D)*x)/(315*a^8*(a + b*x^2)^(3/2)) - (256*b^2*(128*A*b^3 - 36*a*b*(2
*b*B - a*C) - 15*a^3*D)*x)/(315*a^9*Sqrt[a + b*x^2])
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{16Ab-9a\left(B+Cx^2+Dx^4\right)}{x^8\left(a+bx^2\right)^{9/2}}\,dx}{9a}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{14b(16Ab-9aB)-7a\left(-9aC-9aDx^2\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{63a^2}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{12b\left(224Ab^2-126ab\right.}{x^4\left(a+b\right.}}{315}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} - \frac{\left(128Ab^3-36ab(2\right.}{}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3-36ab(2}{45a^4x^3\left(a\right.}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3-36ab(2}{45a^4x^3\left(a\right.}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3-36ab(2}{45a^4x^3\left(a\right.}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3-36ab(2}{45a^4x^3\left(a\right.}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3-36ab(2}{45a^4x^3\left(a\right.}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3-36ab(2}{45a^4x^3\left(a\right.}$$

Mathematica [A] time = 0.162759, size = 270, normalized size = 0.69

$$\frac{256a^3b^5x^{10}\left(-280A+315Bx^2-126Cx^4+15Dx^6\right)+4480a^4b^4x^8\left(-2A+9Bx^2-9Cx^4+3Dx^6\right)+112a^5b^3x^6\left(8A+45B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2*b^6*x^12*
(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*B*x^2 + 45*C*x^4 - 1
50*D*x^6) + 4480*a^4*b^4*x^8*(-2*A + 9*B*x^2 - 9*C*x^4 + 3*D*x^6) + 256*a^3
*b^5*x^10*(-280*A + 315*B*x^2 - 126*C*x^4 + 15*D*x^6) - a^8*(35*A + 45*B*x^
2 + 63*C*x^4 + 105*D*x^6) + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^4 + 1
50*D*x^6) + 2*a^7*b*x^2*(40*A + 21*(3*B*x^2 + 6*C*x^4 + 25*D*x^6)))/(315*a^
9*x^9*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.008, size = 349, normalized size = 0.9

$$-\frac{32768\,Ab^{8}x^{16}-18432\,Bab^{7}x^{16}+9216\,Ca^{2}b^{6}x^{16}-3840\,Da^{3}b^{5}x^{16}+114688\,Aab^{7}x^{14}-64512\,Ba^{2}b^{6}x^{14}+32256\,Ca^{3}b^{5}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/315*(32768*A*b^8*x^16-18432*B*a*b^7*x^16+9216*C*a^2*b^6*x^16-3840*D*a^3*
b^5*x^16+114688*A*a*b^7*x^14-64512*B*a^2*b^6*x^14+32256*C*a^3*b^5*x^14-1344
0*D*a^4*b^4*x^14+143360*A*a^2*b^6*x^12-80640*B*a^3*b^5*x^12+40320*C*a^4*b^4
*x^12-16800*D*a^5*b^3*x^12+71680*A*a^3*b^5*x^10-40320*B*a^4*b^4*x^10+20160*
C*a^5*b^3*x^10-8400*D*a^6*b^2*x^10+8960*A*a^4*b^4*x^8-5040*B*a^5*b^3*x^8+25
20*C*a^6*b^2*x^8-1050*D*a^7*b*x^8-896*A*a^5*b^3*x^6+504*B*a^6*b^2*x^6-252*C
*a^7*b*x^6+105*D*a^8*x^6+224*A*a^6*b^2*x^4-126*B*a^7*b*x^4+63*C*a^8*x^4-80*
A*a^7*b*x^2+45*B*a^8*x^2+35*A*a^8)/x^9/(b*x^2+a)^(7/2)/a^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32324, size = 1569, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((790*D*a^24*b^8 - 1686*C*a^23*b^9 + 3072*B*a^22*b^10 - 5053*A*
a^21*b^11)*x^2/(a^30*b^3) + 7*(365*D*a^25*b^7 - 768*C*a^24*b^8 + 1386*B*a^2
3*b^9 - 2264*A*a^22*b^10)/(a^30*b^3)) + 35*(80*D*a^26*b^6 - 165*C*a^25*b^7
+ 294*B*a^24*b^8 - 476*A*a^23*b^9)/(a^30*b^3))*x^2 + 105*(10*D*a^27*b^5 - 2
0*C*a^26*b^6 + 35*B*a^25*b^7 - 56*A*a^24*b^8)/(a^30*b^3))*x/(b*x^2 + a)^(7/
2) - 2/315*(1260*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(3/2) - 3150*(sqr
t(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(5/2) + 6300*(sqrt(b)*x - sqrt(b*x^2 +
 a))^16*B*a*b^(7/2) - 11025*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9/2) - 10
710*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(3/2) + 27720*(sqrt(b)*x - sqr
```

```
t(b*x^2 + a))^14*C*a^3*b^(5/2) - 56700*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a
^2*b^(7/2) + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2) + 39270*(s
qrt(b)*x - sqrt(b*x^2 + a))^12*D*a^5*b^(3/2) - 105840*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*C*a^4*b^(5/2) + 223020*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b
^(7/2) - 405300*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2) - 81270*(sqr
t(b)*x - sqrt(b*x^2 + a))^10*D*a^6*b^(3/2) + 226800*(sqrt(b)*x - sqrt(b*x^2
 + a))^10*C*a^5*b^(5/2) - 495180*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(
7/2) + 927360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/2) + 103950*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*D*a^7*b^(3/2) - 297108*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*C*a^6*b^(5/2) + 666036*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^5*b^(7/2)
 - 1291374*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2) - 84210*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*D*a^8*b^(3/2) + 243432*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*C*a^7*b^(5/2) - 551124*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^6*b^(7/2) + 10
73856*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^5*b^(9/2) + 42210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*D*a^9*b^(3/2) - 121968*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a
^8*b^(5/2) + 275076*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^7*b^(7/2) - 533124*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^6*b^(9/2) - 11970*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*D*a^10*b^(3/2) + 34272*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^9*b^(
5/2) - 76644*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^8*b^(7/2) + 147456*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^7*b^(9/2) + 1470*D*a^11*b^(3/2) - 4158*C*a^10*
b^(5/2) + 9216*B*a^9*b^(7/2) - 17609*A*a^8*b^(9/2))/(((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^9*a^8)
```

3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be+a^3(-\right.}{b^6}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rubi [A] time = 0.222303, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1811, 1799, 1850}

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be+a^3(-\right.}{b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```


```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```


Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
```

```
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx &= \int \frac{x\left(cx^4+dx^6+ex^8+fx^{10}\right)}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{cx^2+dx^3+ex^4+fx^5}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5\sqrt{a+bx}}+\frac{a\left(-2b^3c+3ab^2d-4a^2be+5a^3f\right)\sqrt{a+}}{b^5}\right.\right.\\ &= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^6}-\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)\left(a+bx^2\right)^{3/2}}{3b^6}+\end{aligned}$$

Mathematica [A] time = 0.179052, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(8a^2b^3\left(231c+99dx^2+66ex^4+50fx^6\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+128a^4b\left(11e+5fx^2\right)-1280a^5f-2\right.}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

Maple [A] time = 0.007, size = 193, normalized size = 0.9

$$-\frac{-315\,fx^{10}b^5+350\,ab^4fx^8-385\,b^5ex^8-400\,a^2b^3fx^6+440\,ab^4ex^6-495\,b^5dx^6+480\,a^3b^2fx^4-528\,a^2b^3ex^4+594\,ab^4}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*e*x^8-400*
a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2*f*x^4-528*a^2*b^3*e
*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*f*x^2+704*a^3*b^2*e*x^2-792*a^
2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2
*b^3*c)/b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42306, size = 433, normalized size = 2.02

$$\frac{\left(315\,b^5fx^{10} + 35\left(11\,b^5e - 10\,ab^4f\right)x^8 + 5\left(99\,b^5d - 88\,ab^4e + 80\,a^2b^3f\right)x^6 + 1848\,a^2b^3c - 1584\,a^3b^2d + 1408\,a^4be - 128\right.}{346}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

Sympy [A] time = 4.55471, size = 442, normalized size = 2.07

$$\begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} + \frac{8a^2dx^2\sqrt{a+bx^2}}{35b^3} + \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

Giac [A] time = 1.22787, size = 387, normalized size = 1.81

$$\frac{693\left(bx^2+a\right)^{\frac{5}{2}}b^3c - 2310\left(bx^2+a\right)^{\frac{3}{2}}ab^3c + 3465\sqrt{bx^2+a}a^2b^3c + 495\left(bx^2+a\right)^{\frac{7}{2}}b^2d - 2079\left(bx^2+a\right)^{\frac{5}{2}}ab^2d + 3465\left(bx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*(693*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 3465
*sqrt(b*x^2 + a)*a^2*b^3*c + 495*(b*x^2 + a)^(7/2)*b^2*d - 2079*(b*x^2 + a)
^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*b^2*d - 3465*sqrt(b*x^2 + a)*a^
3*b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x
^2 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)
*a^4*f - 3465*sqrt(b*x^2 + a)*a^5*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x
^2 + a)^(7/2)*a*b*e + 4158*(b*x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/
2)*a^3*b*e + 3465*sqrt(b*x^2 + a)*a^4*b*e)/b^6
```

3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{(a+bx^2)^{3/2}(3a^2be-4a^3f-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}(a^2be+a^3(-f)-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{5/2}(6a^2f-3abe+b^2d}{5b^5}$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rubi [A] time = 0.174734, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1811, 1799, 1850}

$$\frac{(a+bx^2)^{3/2}(3a^2be-4a^3f-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}(a^2be+a^3(-f)-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{5/2}(6a^2f-3abe+b^2d}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx = \int \frac{x\left(cx^2+dx^4+ex^6+fx^8\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{cx+dx^2+ex^3+fx^4}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4\sqrt{a+bx}} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\sqrt{a+bx}}{b^4} + \right.\right.$$
$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^5} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\left(a+bx^2\right)^{3/2}}{3b^5} +$$

Mathematica [A] time = 0.125844, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(24a^2b^2\left(7d+3ex^2+2fx^4\right)-16a^3b\left(9e+4fx^2\right)+128a^4f-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)+b^4x^2\left(105\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

Maple [A] time = 0.006, size = 145, normalized size = 0.9

$$\frac{35\,fx^8b^4-40\,ab^3fx^6+45\,b^4ex^6+48\,a^2b^2fx^4-54\,ab^3ex^4+63\,b^4dx^4-64\,a^3bfx^2+72\,a^2b^2ex^2-84\,ab^3dx^2+105\,b^4cx^2}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+48*a^2*b^2*
f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2*b^2*e*x^2-84*a*b^3*
d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168*a^2*b^2*d-210*a*b^3*c)/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29102, size = 316, normalized size = 1.89

$$\frac{\left(35\,b^4fx^8+5\left(9\,b^4e-8\,ab^3f\right)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\left(21\,b^4d-18\,ab^3e+16\,a^2b^2f\right)x^4+\left(10\right.}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

Sympy [A] time = 2.78539, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4adx^2\sqrt{a+bx^2}}{15b^2} - \frac{6aex^4\sqrt{}}{35b} \\ \frac{\frac{cx^4}{4}+\frac{dx^6}{6}+\frac{ex^8}{8}+\frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

Giac [A] time = 1.17954, size = 296, normalized size = 1.77

$$\underline{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c - 315\sqrt{bx^2+a}ab^3c + 63\left(bx^2+a\right)^{\frac{5}{2}}b^2d - 210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d + 315\sqrt{bx^2+a}a^2b^2d + 35\left(bx^2+a\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(105*(b*x^2 + a)^(3/2)*b^3*c - 315*sqrt(b*x^2 + a)*a*b^3*c + 63*(b*x^
2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)*a*b^2*d + 315*sqrt(b*x^2 + a)*a^
2*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2 +
 a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 315*sqrt(b*x^2 + a)*a^4*f +
 45*(b*x^2 + a)^(7/2)*b*e - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(
3/2)*a^2*b*e - 315*sqrt(b*x^2 + a)*a^3*b*e)/b^5
```

3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.150072, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1811, 1799, 1850}

$$\frac{\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx &= \int \frac{x\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{\left(b^2d-2abe+3a^2f\right)\sqrt{a+bx}}{b^3}+\frac{(be-3af)(a+bx\right.\right.\right.\\ &= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^4}+\frac{\left(b^2d-2abe+3a^2f\right)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{(be-3af)\left(a+\right.}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.0880218, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(8a^2b\left(7e+3fx^2\right)-48a^3f-2ab^2\left(35d+14ex^2+9fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

Maple [A] time = 0.004, size = 99, normalized size = 0.8

$$-\frac{-15\,fx^6b^3+18\,ab^2fx^4-21\,b^3ex^4-24\,a^2bfx^2+28\,ab^2ex^2-35\,b^3dx^2+48\,a^3f-56\,a^2be+70\,ab^2d-105\,b^3c}{105\,b^4}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4-24*a^2*b*
f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)
/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36614, size = 221, normalized size = 1.83

$$\frac{\left(15\,b^3fx^6+3\left(7\,b^3e-6\,ab^2f\right)x^4+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+\left(35\,b^3d-28\,ab^2e+24\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

Sympy [A] time = 1.61059, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b}+\frac{ex^4\sqrt{a+bx^2}}{5b}+\frac{fx \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

Giac [A] time = 1.16157, size = 207, normalized size = 1.71

$$\frac{105\,\sqrt{bx^2+a}ab^3c+35\left(bx^2+a\right)^{\frac{3}{2}}b^2d-105\,\sqrt{bx^2+a}aab^2d+15\left(bx^2+a\right)^{\frac{7}{2}}f-63\left(bx^2+a\right)^{\frac{5}{2}}af+105\left(bx^2+a\right)^{\frac{3}{2}}a^2f-105}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(105*sqrt(b*x^2 + a)*b^3*c + 35*(b*x^2 + a)^(3/2)*b^2*d - 105*sqrt(b*
x^2 + a)*a*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f - 105*sqrt(b*x^2 + a)*a^3*f + 21*(b*x^2 + a)^(5/2)*
b*e - 70*(b*x^2 + a)^(3/2)*a*b*e + 105*sqrt(b*x^2 + a)*a^2*b*e)/b^4
```

3.172 $\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=261

$$\frac{x^3\left(a\left(-71a^2bD+162a^3F+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(17a^2bD-24a^3F-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(Ab^4\right.}{}$$

```
[Out] ((A*b^4 - a*(b^3*B - a*b^2*C + a^2*b*D - a^3*F))*x^3)/(7*a*b^4*(a + b*x^2)^
(7/2)) + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))*x^3)
/(35*a^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 + a*(6*b^3*B + 15*a*b^2*C - 71*
a^2*b*D + 162*a^3*F))*x^3)/(105*a^3*b^4*(a + b*x^2)^(3/2)) - ((b*D - 4*a*F)
*x)/(b^5*Sqrt[a + b*x^2]) + (F*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rubi [A] time = 0.716416, antiderivative size = 257, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {1804, 1800, 1585, 1263, 1584, 455, 388, 217, 206}

$$\frac{x^3\left(a\left(-71a^2bD+162a^3F+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(17a^2bD-24a^3F-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(\frac{A}{a}-\right.}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x^2)^(7/2))
 + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))*x^3)/(35*a
^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 + a*(6*b^3*B + 15*a*b^2*C - 71*a^2*b*
D + 162*a^3*F))*x^3)/(105*a^3*b^4*(a + b*x^2)^(3/2)) - ((b*D - 4*a*F)*x)/(b
^5*Sqrt[a + b*x^2]) + (F*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
```

```
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1800

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/c, Int[(c*x)^(m + 1)*PolynomialQuotient[Pq, x, x]*(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && EqQ[Coeff[Pq, x, 0], 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(4Ab+\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}\right)x-\frac{7a\left(b^2C-abD+a^2F\right)x^3}{b^2}-7a\left(\cdots\right.}{\left(a+bx^2\right)^{7/2}}\cdots}{7ab}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}-\frac{7a\left(b^2C-abD+a^2F\right)x^2}{b^2}-7a\left(D\cdots\right.\right.}{\left(a+bx^2\right)^{7/2}}\cdots}{7ab}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-24a^3F\right)\right)\cdots}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

Mathematica [A] time = 0.504825, size = 221, normalized size = 0.85

$$\frac{\sqrt{b}x\left(2a^2b^5x^2\left(35A+21Bx^2+15Cx^4\right)+14a^5b^2x^2\left(261Fx^2-50D\right)+4a^4b^3x^4\left(396Fx^2-203D\right)+a^3b^4x^6\left(105Fx^2-35\right.}{210a^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210*a
^6*b*(D - 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5*b^2*x^2*(-5
0*D + 261*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2) + 2*a^2*b^5*x^2*(35*A
 + 21*B*x^2 + 15*C*x^4)) + 105*a^(7/2)*(2*b*D - 9*a*F)*(a + b*x^2)^3*Sqrt[1
 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(210*a^3*b^(11/2)*(a + b*x^2)^(
7/2))
```

Maple [B] time = 0.01, size = 478, normalized size = 1.8

$$-\frac{5\,aCx^3}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{15\,a^2Cx}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,Fax^3}{2\,b^4}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{9\,Fax}{2\,b^5}\frac{1}{\sqrt{bx^2+a}}-\frac{Ax}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,aCx}{56\,b^3}\left(bx^2+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -5/8*C/b^2*a*x^3/(b*x^2+a)^(7/2)-15/56*C/b^3*a^2*x/(b*x^2+a)^(7/2)+3/2*F/b^
4*a*x^3/(b*x^2+a)^(3/2)+9/2*F/b^5*a*x/(b*x^2+a)^(1/2)-1/7*A/b*x/(b*x^2+a)^(
7/2)+3/56*C/b^3*a*x/(b*x^2+a)^(5/2)+1/35*A/b/a*x/(b*x^2+a)^(5/2)+4/105*A/b/
a^2*x/(b*x^2+a)^(3/2)+8/105*A/b/a^3*x/(b*x^2+a)^(1/2)+1/7*C/b^3/a*x/(b*x^2+
a)^(1/2)-3/28*B/b^2*a*x/(b*x^2+a)^(7/2)+1/35*B/b^2/a*x/(b*x^2+a)^(3/2)+D/b^
(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)+1/2*F*
x^9/b/(b*x^2+a)^(7/2)-9/2*F/b^(11/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/2*C*
x^5/b/(b*x^2+a)^(7/2)+1/14*C/b^3*x/(b*x^2+a)^(3/2)-1/4*B*x^3/b/(b*x^2+a)^(7
/2)+3/140*B/b^2*x/(b*x^2+a)^(5/2)-1/7*D*x^7/b/(b*x^2+a)^(7/2)-1/5*D/b^2*x^5
/(b*x^2+a)^(5/2)-1/3*D/b^3*x^3/(b*x^2+a)^(3/2)+9/14*F/b^2*a*x^7/(b*x^2+a)^(
7/2)+9/10*F/b^3*a*x^5/(b*x^2+a)^(5/2)-D*x/b^4/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4274, size = 302, normalized size = 1.16

$$\frac{\left(\left(\left(\left(\frac{105\,Fx^2}{b} + \frac{2\left(792\,Fa^4b^7 - 176\,Da^3b^8 + 15\,Ca^2b^9 + 6\,Bab^{10} + 8\,Ab^{11}\right)}{a^3b^9}\right)x^2 + \frac{14\left(261\,Fa^5b^6 - 58\,Da^4b^7 + 3\,Ba^2b^9 + 4\,Aab^{10}\right)}{a^3b^9}\right)x^2 + \frac{70\left(45\,Fa^6b^5 - 10\,Da^5b^6 + Aa\right.}{a^3b^9}}{210\left(bx^2 + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a^2*b^9 + 6
*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^6 - 58*D*a^4*b^7 + 3
*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*(45*F*a^6*b^5 - 10*D*a^5*b^6 +
 A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))*x/(
b*x^2 + a)^(7/2) + 1/2*(9*F*a - 2*D*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)
))/b^(11/2)
```

3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=214

$$\frac{x^7\left(a\left(15a^2bD-176a^3F+6ab^2C+8b^3B\right)+48Ab^4\right)}{105a^4b\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(a\left(-58a^3F+3ab^2C+4b^3B\right)+24Ab^4\right)}{15a^3b^2\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(-10a^4F+ab^3B+\cdots\right.}{3a^2b^3\left(a+bx^2\right.}$$

```
[Out] ((A*b^4 - a^4*F)*x)/(a*b^4*(a + b*x^2)^(7/2)) + ((6*A*b^4 + a*b^3*B - 10*a^
4*F)*x^3)/(3*a^2*b^3*(a + b*x^2)^(7/2)) + ((24*A*b^4 + a*(4*b^3*B + 3*a*b^2
*C - 58*a^3*F))*x^5)/(15*a^3*b^2*(a + b*x^2)^(7/2)) + ((48*A*b^4 + a*(8*b^3
*B + 6*a*b^2*C + 15*a^2*b*D - 176*a^3*F))*x^7)/(105*a^4*b*(a + b*x^2)^(7/2)
) + (F*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.409623, antiderivative size = 250, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1814, 1157, 385, 217, 206}

$$\frac{x\left(\frac{15a^2bD-176a^3F+6ab^2C+8b^3B}{b^4}+\frac{48A}{a}\right)}{105a^3\sqrt{a+bx^2}}+\frac{x\left(a\left(-45a^2bD+122a^3F+3ab^2C+4b^3B\right)+24Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x\left(\frac{15a^2bD-22a^3F-8ab^2C+b^3B}{b^4}+\cdots\right.}{35a\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2)^(7/2)) +
 (((6*A)/a + (b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3*F)/b^4)*x)/(35*a*(a +
 b*x^2)^(5/2)) + ((24*A*b^4 + a*(4*b^3*B + 3*a*b^2*C - 45*a^2*b*D + 122*a^3
*F))*x)/(105*a^3*b^4*(a + b*x^2)^(3/2)) + (((48*A)/a + (8*b^3*B + 6*a*b^2*C
 + 15*a^2*b*D - 176*a^3*F)/b^4)*x)/(105*a^3*Sqrt[a + b*x^2]) + (F*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-6A-\frac{a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^4}-\frac{7a\left(b^2C-abD+a^2F\right)x^2}{b^3}-\frac{7a(bD-aF)x^4}{b^2}-\frac{7aFx^6}{b}}{\left(a+bx^2\right)^{7/2}}}{7a}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{24Ab^4+4ab^3B+3a^2b^2C-1}{b^4}}{}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\left(4b^3B+\right.\right.}{105a}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\left(4b^3B+\right.\right.}{105a}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\left(4b^3B+\right.\right.}{105a}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\left(4b^3B+\right.\right.}{105a}$$

Mathematica [A] time = 0.445792, size = 197, normalized size = 0.92

$$\frac{x\left(a^3b^4\left(105A+35Bx^2+21Cx^4+15Dx^6\right)+2a^2b^5x^2\left(105A+14Bx^2+3Cx^4\right)-406a^5b^2Fx^4-176a^4b^3Fx^6-350a^6bFx^2\right.}{105a^4b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x^6 - 176*a
^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x^2*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6)))/(105*a^4*b
^4*(a + b*x^2)^(7/2)) + (Sqrt[a]*F*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/
Sqrt[a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.008, size = 427, normalized size = 2.

$$-\frac{Fx^7}{7b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Fx^5}{5b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Fx^3}{3b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Fx}{b^4}\frac{1}{\sqrt{bx^2+a}}+F\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}-\frac{Dx^5}{2b}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*F*x^7/b/(b*x^2+a)^(7/2)-1/5*F/b^2*x^5/(b*x^2+a)^(5/2)-1/3*F/b^3*x^3/(b
*x^2+a)^(3/2)-F/b^4*x/(b*x^2+a)^(1/2)+F/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))-1/2*D*x^5/b/(b*x^2+a)^(7/2)-5/8*D/b^2*a*x^3/(b*x^2+a)^(7/2)-15/56*D/b^3*
a^2*x/(b*x^2+a)^(7/2)+3/56*D/b^3*a*x/(b*x^2+a)^(5/2)+1/14*D/b^3*x/(b*x^2+a)
^(3/2)+1/7*D/b^3/a*x/(b*x^2+a)^(1/2)-1/4*C*x^3/b/(b*x^2+a)^(7/2)-3/28*C/b^2
*a*x/(b*x^2+a)^(7/2)+3/140*C/b^2*x/(b*x^2+a)^(5/2)+1/35*C/b^2/a*x/(b*x^2+a)
^(3/2)+2/35*C/b^2/a^2*x/(b*x^2+a)^(1/2)-1/7*B/b*x/(b*x^2+a)^(7/2)+1/35*B/b/
a*x/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+
a)^(1/2)+1/7*A*x/a/(b*x^2+a)^(7/2)+6/35*A/a^2*x/(b*x^2+a)^(5/2)+8/35*A/a^3*
x/(b*x^2+a)^(3/2)+16/35*A/a^4*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24285, size = 275, normalized size = 1.29

$$-\frac{\left(\left(x^2\left(\frac{\left(176\,Fa^4b^6-15\,Da^3b^7-6\,Ca^2b^8-8\,Bab^9-48\,Ab^{10}\right)x^2}{a^4b^7}+\frac{7\left(58\,Fa^5b^5-3\,Ca^3b^7-4\,Ba^2b^8-24\,Aab^9\right)}{a^4b^7}\right)+\frac{35\left(10\,Fa^6b^4-Ba^3b^7-6\,Aa^2b^8\right)}{a^4b^7}\right)x^2+\frac{105\left(Fa^7b^3\right.}{a^4b}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a*b^9 - 48*
A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 - 4*B*a^2*b^8 - 24*A*
a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*b^7 - 6*A*a^2*b^8)/(a^4*b^7))*
x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{x^7\left(384Ab^4-a\left(6a^2bD+15a^3F+8ab^2C+48b^3B\right)\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a(aC\right.}{3a^3\left(a+bx^2\right.}$$

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - ((384
*A*b^4 - a*(48*b^3*B + 8*a*b^2*C + 6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a
+ b*x^2)^(7/2))
```

Rubi [A] time = 0.341174, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1803, 1813, 12, 264}

$$-\frac{x^7\left(384Ab^4-a\left(6a^2bD+15a^3F+8ab^2C+48b^3B\right)\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a(aC\right.}{3a^3\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - ((384
*A*b^4 - a*(48*b^3*B + 8*a*b^2*C + 6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a
+ b*x^2)^(7/2))
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4+Fx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(6b(8Ab-aB)+a\left(-aC-aDx^2-aFx^4\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a^2}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \int \frac{x^4\left(4b\left(48Ab^2-6a\right.\right.}{}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-a\left(2\right.\right.}{15a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-a\left(2\right.\right.}{15a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-a\left(2\right.\right.}{15a}$$

Mathematica [A] time = 0.165216, size = 138, normalized size = 0.72

$$\frac{8a^2b^2x^4\left(-210A+21Bx^2+Cx^4\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)+a^4\left(-105A+105Bx^2+35Cx^4+21D\right.}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6) + a
^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6 + 15*F*x^8))/(105*a^5*x*(a + b
*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 166, normalized size = 0.9

$$-\frac{384\,Ab^4x^8-48\,Bab^3x^8-8\,Ca^2b^2x^8-6\,Da^3bx^8-15\,Fa^4x^8+1344\,Aab^3x^6-168\,Ba^2b^2x^6-28\,Ca^3bx^6-21\,Da^4x^6+168}{105\,xa^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^8-15*F*a^4
*x^8+1344*A*a*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-21*D*a^4*x^6+1680*A*
a^2*b^2*x^4-210*B*a^3*b*x^4-35*C*a^4*x^4+840*A*a^3*b*x^2-105*B*a^4*x^2+105*
A*a^4)/x/(b*x^2+a)^(7/2)/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23999, size = 297, normalized size = 1.54

$$\frac{\left(\left(x^2\left(\frac{(15\,Fa^{13}b^3+6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7)x^2}{a^{14}b^3}+\frac{7\,(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6)}{a^{14}b^3}\right)+\frac{35\,(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5)}{a^{14}b^3}\right)}{105\,(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 -
 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11
*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a
^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b
*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```